

Physics 411 - Homework 5
Canonical Transformations
Due Oct. 8, 2009

Problem 1: Goldstein, Chapter 9, Problem 6.

Problem 2: After much work, you have finally solved a problem in mechanics and published it. Soon after your work is published, a new paper appears on the arXiv, claiming to solve an important mechanics problem. Reading it carefully, you suspect that it is none other than the problem you have solved, but in canonically transformed variables. Your task is to find the canonical transformation, and prove that you have scooped your rival.

Your rival's Hamiltonian is

$$H = \frac{a}{2}(p^2 - 1)q^2, \quad (1)$$

where a is a constant. (a) Find the equations of motion for the above Hamiltonian.

You know from the context of the problem that your momentum is related to his by $P = \ln p$. (b) Using the principles you have learned in class, find your coordinate Q in terms of his variables. (c) Find your Hamiltonian $K(Q, P)$. (d) Find the equations of motion in these variables so you can demonstrate that your published solution maps onto his. (e) Find the generating function for this canonical transformation. (P.S. This is basically a true story).

Problem 3, The phase space clock: A given curve in phase space represents a dynamical change of your coordinates. If you know what the Hamiltonian H is, then it is possible to use this curve in phase space to make a clock. Suppose that we define a function on phase space $s(q, p)$ that coincides with the physical time t along this curve. Compute the Poisson bracket $\{s, H\}$. Are s and H canonically conjugate variables? (Hint: The answer is not 2, despite appearances).