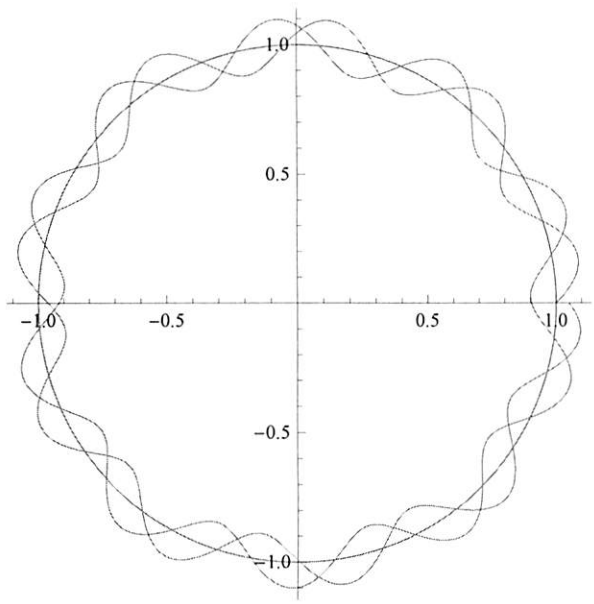
Here is a plot of a bounded, but not closed orbit

In[7]:= `PolarPlot[{1, 1 + 1 / 10 Sin[6 GoldenRatio t]}, {t, 0, 4 Pi}]`

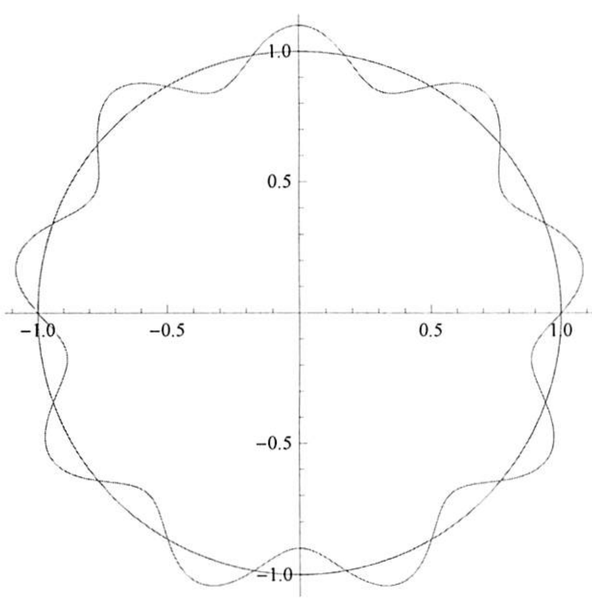
Out[7]=



- **Here is a plot of a bounded, and closed orbit - rational commensurate frequencies**

In[8]:= `PolarPlot[{1, 1 + 1 / 10 Sin[6 × 1.5 t]}, {t, 0, 4 Pi}]`

Out[8]=

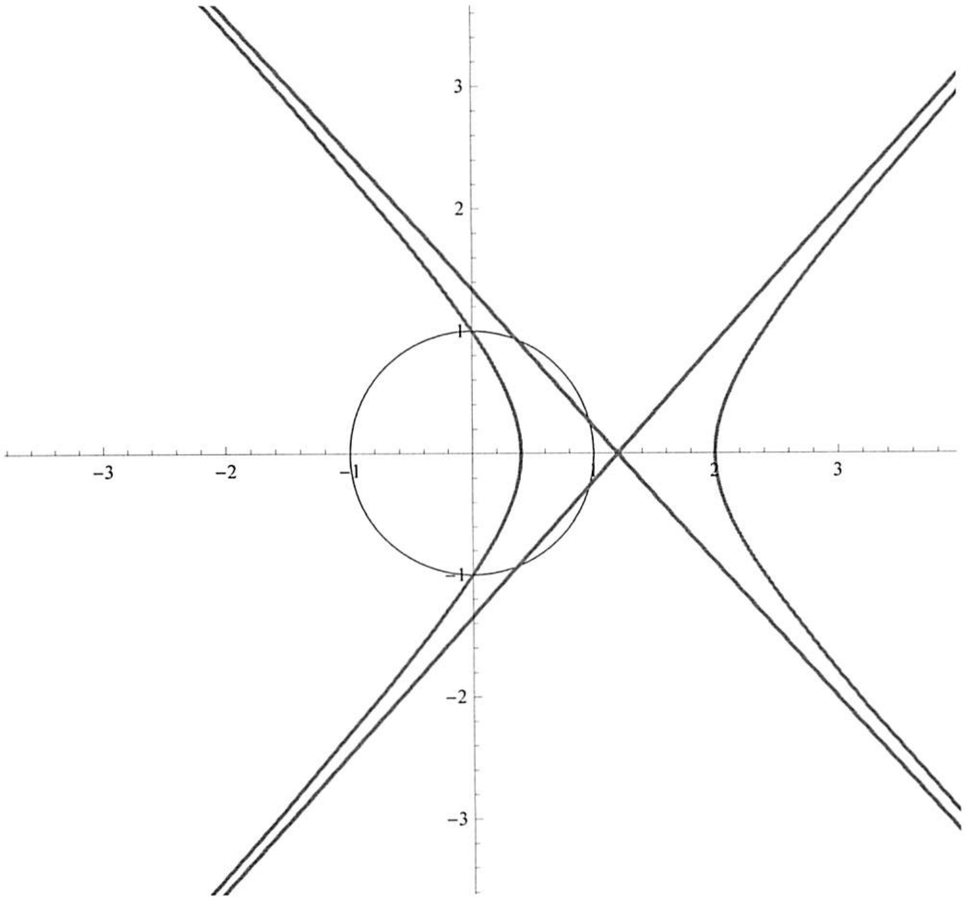


- **First, we take a positive epsilon, (positive energy) describing an unbounded, hyperbolic trajectory - the physical trajectory is the left branch.**

In[26]:=
```
epsilon = 1.5;
```

In[27]:=
```
PolarPlot[{1, (1 + epsilon Cos[t]) ^ (-1)}, {t, 0, 2 Pi}, PlotStyle → {Black, Directive[Thick]}]
```

Out[27]=

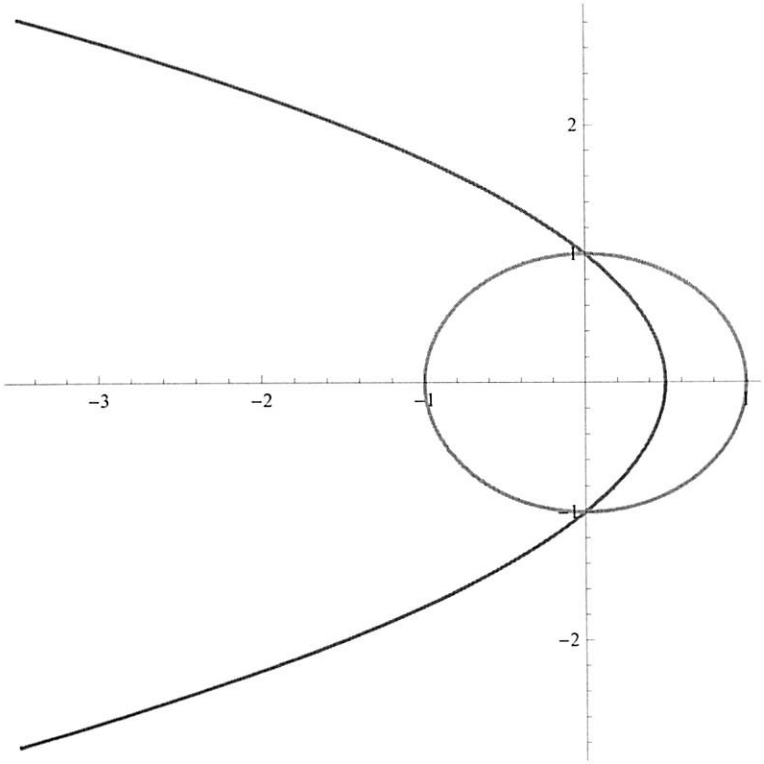


- **Next, we take a epsilon=0, (zero energy) describing an unbounded, parabolic trajectory**

In[33]:=
```
epsilon = 1;
```

In[50]:=
```
Plot[{-Sqrt[(1 - x)^2 - x^2], Sqrt[(1 - x)^2 - x^2 ], Sqrt[1 - x^2], -Sqrt[1 - x^2]},
  {x, -3.5, 1}, AspectRatio → 1, PlotStyle → Thick]
```

Out[50]=

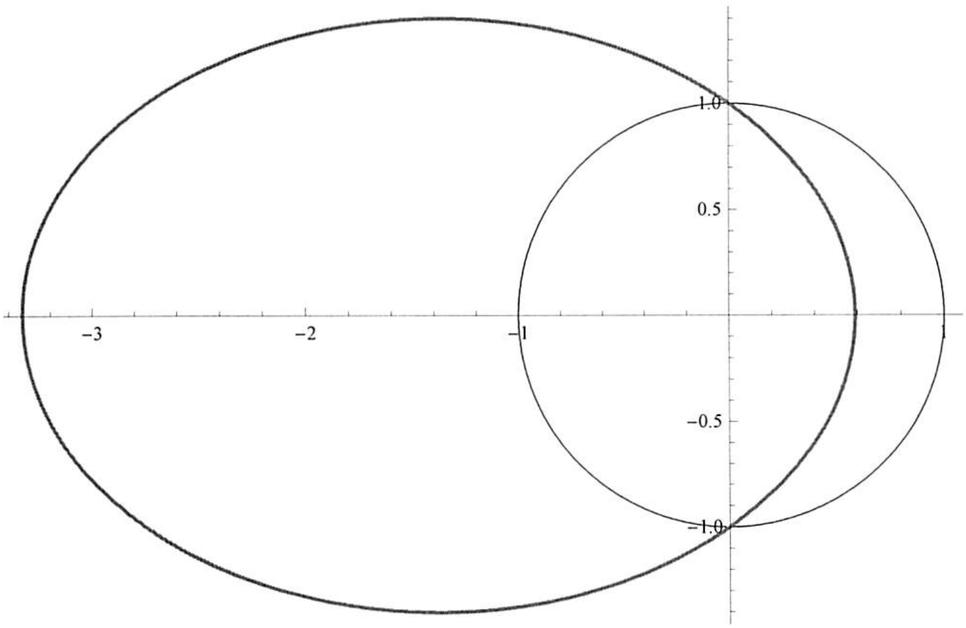


- **Next, we take a epsilon less than 1, (negative energy) describing a bounded, elliptical trajectory**

In[31]:= `epsilon = 0.7;`

In[32]:= `PolarPlot[{1, (1 + epsilon Cos[t]) ^ (-1)}, {t, 0, 2 Pi}, PlotStyle → {Black, Directive[Thick]}]`

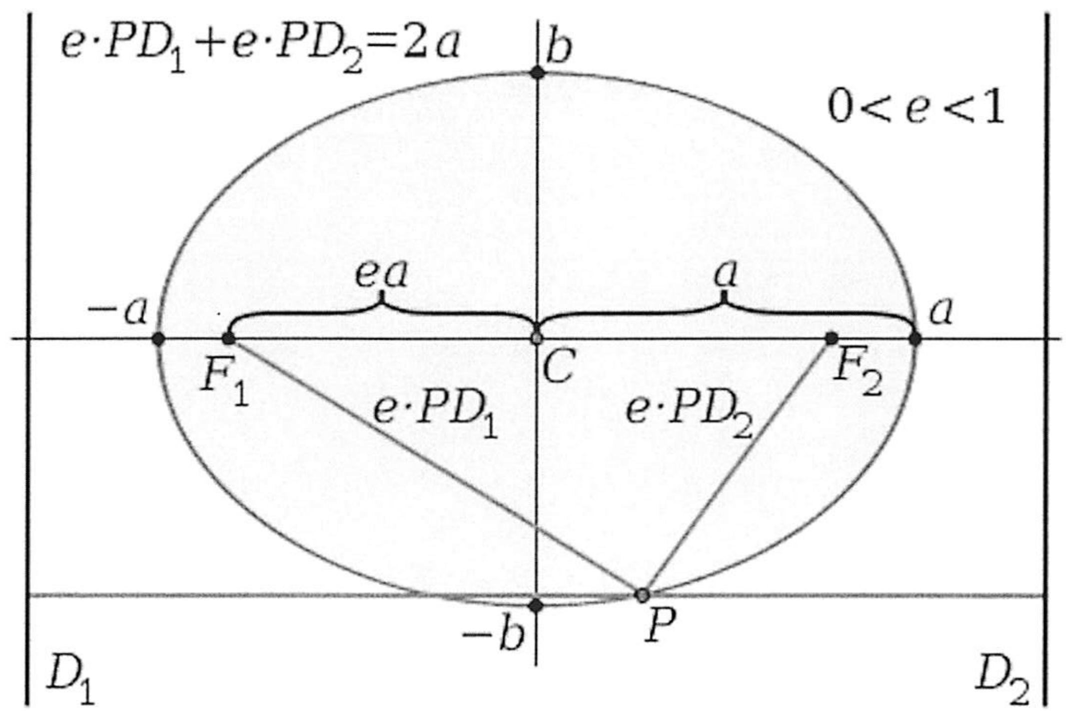
Out[32]=



- **We finally note that for epsilon = 0, we get the circles drawn above**

Semi-minor axis.

$F_1 + F_2$ are the foci of the ellipse.



Semi-major axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \; ; \qquad e = \frac{\sqrt{a^2 - b^2}}{a}$$ is the Eccentricity.