Physics 418
Homework 2 - Due Feb. 6, 2009

Problem 1: Callen 5.3-2

Problem 2: Derive the general relation

$$\kappa_S = \kappa_T - TV\alpha^2/C_P \tag{1}$$

Problem 3: Consider the classical ideal gas. The total (extensive) entropy can be written as:

$$S(U,V,N) = (N/N_0)S_0 + Nk_B ln[(U/U_0)^{3/2}(V/V_0)(N/N_0)^{-5/2}], \tag{2}$$

where U is the total internal energy, V is the total volume, and N is the number of particles. E_0, V_0, N_0, S_0, and k_B are constants. (a) Starting from the above $S(U,V,N)$, find the Helmholtz free energy $F(T,V,N)$, the Gibbs free energy $G(T,P,N)$, and the Grand Potential $\Omega(T,V,\mu)$, by the method of Legendre transforms. (b) Find the chemical potential μ, by taking an appropriate first derivative of a thermodynamic potential. Show explicitly that the chemical potential is the same as the Gibbs free energy per particle. (c) Find the pressure P, by taking an appropriate first derivative of a thermodynamic potential. Show explicitly that the pressure is the same as the negative of the Grand Potential per volume. (d) By computing the appropriate 2nd derivatives of the appropriate thermodynamic potentials, compute the specific heats C_V and C_P , the compressibilities κ_T and κ_S, and the coefficient of thermal expansion α. Show by comparison of the preceding results that the two specific heats, and the two compressibilities, obey the general relations discussed in lecture and in problem 2.

Problem 4: Consider the fundamental equation of a simple paramagnetic model, given in Callen, Eq. (3.66). Recalling that magnetic moment is thermodynamically conjugate to magnetic field, (a) calculate the functions $B_e(S,I,N)$ and $T(S,I,N)$. (b) Calculate the "magnetic Gibbs potential" $\tilde{U}(T,B_e,N) = U - B_eI - TS$. (c) Show that it is possible to recover the original fundamental relation via inverse Legendre transforms.