

**Physics 418**  
**Homework 3 - Due Feb. 13, 2009**

Problem 1: Consider the atmosphere on the planet *Isentropia*. It has two curious properties: (i) the atmosphere acts as an ideal gas made of particles of mass  $m$  such that the entropy per particle is the same throughout the atmosphere. (ii) The atmosphere extends up only to a finite height, where it abruptly stops. In the problem below, assume that the height of the gas is sufficiently small that we can use linearized gravity  $U(h) = mgh$  as we did in class. Use the principles of kinetic theory to solve this problem.

(a) Satellite measurements indicate that the density and temperature of the atmosphere at the surface of the planet are  $n(h = 0) = n_0$  and  $T(h = 0) = T_0$ . Find the density and temperature of the atmosphere as a function of the height  $h$  above the surface of the planet.

(b) Find what the height of the atmosphere is as a function of the conditions on the ground, and other given information.

(c) What is the temperature of the atmosphere at the top?

(d) Find the total number of particles in a vertical column of area  $A$ , extending from the surface to the top of the atmosphere. Compare this number to the same in the exponential atmosphere considered in class. Interpret this result physically.

Problem 2: Consider a closed RC-circuit in thermal equilibrium with its environment of temperature  $T$ . Verify that Nyquist's theorem (the current noise power of a resistor in thermal equilibrium is given by  $S_I = 2k_B T G$ , where  $G = 1/R$  is the conductance) is satisfied for this system :

(a) Assuming the equipartition law is valid, what is the variance of the voltage across the resistor (or capacitor)? (b) Calculate the total noise power associated with the voltage and current fluctuations: The noise power of the variable  $A$  is  $S_A = \int_{-\infty}^{\infty} d\tau \langle A(t + \tau)A(t) \rangle_t$ . The notation  $\langle \dots \rangle_t$  indicates an average over time  $t$ . (Hint: account for the capacitor's relaxation dynamically, and the thermal excitation statistically via part a).

Problem 3: Patheria 1.1

Problem 4: Calculate the area and volume of a sphere in  $N$  dimensions. (Patheria, Appendix C will help here!)