

Physics 418
Homework 7 - Due April 3, 2009

Problem 1: A particle of mass m is confined to a harmonic potential of frequency ω . This particle is prepared in a mixed state ρ with elements:

$$\langle q|\rho|q'\rangle = \frac{1}{\sqrt{2\pi V_q}} \exp\left\{-\frac{(q+q')^2}{2V_q} - \frac{V_p(q-q')^2}{2\hbar^2}\right\}. \quad (1)$$

Here q, q' denote position coordinates, so the elements above are in the position basis. If you make repeated measurements of the system with this identically prepared state, find the following:

- (a) What will be the variance in position and momentum: $\langle q^2 \rangle, \langle p^2 \rangle$?
- (b) What is the purity of this state, $P = \text{Tr}\rho^2$?
- (c) What is the probability of finding the particle in its first excited state?
- (d) What values of V_q, V_p correspond to a pure state?

Problem 2: Suppose one has access to data taken from measurements on a spin 1/2 system, prepared identically every time. The average of measurements taken on the $x, y,$ and z components of the spin are given by P_x, P_y, P_z respectively. What is the density matrix describing this spin? Find the von Neumann entropy.

Problem 3: Patheria, 6.22

Problem 4: Patheria, 6.1