Physics 418
Homework 8 - Due April 17, 2009

Problem 1: Find the Fermi temperature of these systems of non-interacting Fermions (are they in the classical or quantum regime?): (i) Electrons in a typical metal at room temperature. (ii) Liquid Helium (4) at $T = 1K$. (iii) Protons in the core of the sun. (iv) A neutron star.

Problem 2: Consider a degenerate Fermi gas of non-interacting, non-relativistic, particles in two dimensions (such as found in a semiconductor two-dimensional electron gas).
a) Find the density of states $g(E)$.
b) Find the Fermi energy and the $T = 0$ energy density.
c) From the particle density n, find the chemical potential as a function of temperature, $\mu(T)$, for fixed density n, by doing the relevant integral exactly. You may have to look up the integral in a table or use a Mathematical program! Using the exact expression for $\mu(T)$, find a simpler approximation that holds at low $T \ll T_F = E_F/k_B$. Does $\mu(T)$ have a power series expansion in T at low T?

Problem 3: Work out the thermodynamics (as we did in class) for a Fermi gas at $T = 0$ of noninteracting, non-relativistic, particles in three dimensions. The gas is confined by a symmetric, harmonic potential: $V(q) = (1/2)m\omega^2|\mathbf{q}|^2$. How do your results differ from the gas-in-a-box case discussed in class?

Problem 4: Consider an ideal Bose gas composed of molecules with an internal degree of freedom. Assume that this internal degree of freedom can have one of two energy values, the ground state $\epsilon_0 = 0$, and an excited state, $\epsilon_1 > 0$. Determine the Bose-Einstein condensation temperature T_c of the gas as a function of ϵ_1. Show in particular that for $\epsilon_1 \gg k_B T$,

$$\frac{T_c}{T_{c,0}} = 1 - \frac{2\exp(-\epsilon_1/k_B T)}{3\zeta(3)} \tag{1}$$

where $T_{c,0}$ is the transition temperature when ϵ_1 is infinite, and $\zeta(x)$ is the Riemann zeta function.

Problem 5: Consider photons of a given energy $\epsilon = \hbar\omega$.

(a) If $\langle n \rangle$ is the average number of such photons in equilibrium at temperature T, show that the fluctuation in the number of photons is

$$\langle n^2 \rangle - \langle n \rangle^2 = -\frac{1}{\epsilon}(d\langle n \rangle/d\beta). \tag{2}$$

Using the formula for the equilibrium value of $\langle n \rangle$, apply the above result to determine the relative fluctuation in the number of photons,

$$(\langle n^2 \rangle - \langle n \rangle^2)/\langle n \rangle^2. \tag{3}$$

Is this large or small? Can it be smaller than 1?