Introduction to the Transverse Spatial Correlations in Spontaneous Parametric Down-Conversion Through the Biphoton Birth Zone

James Schneeloch and John C. Howell
1 Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627
2 Center for Coherence and Quantum Optics, University of Rochester, Rochester, NY 14627
3 Air Force Research Laboratory, Information Directorate, Rome, NY 13441

The Biphoton Wavefunction From the beginning...

\[ \mathcal{H}_{EM} = \frac{1}{2} \int d^3 r (\mathbf{B} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}) \]

With the following assumptions:

- The pump field is bright enough to be treated classically.
- The pump field is not too bright compared to the Coulomb field binding electrons to atoms
  - i.e., intensities a good deal less than \(10^{15} \text{W/mm}^2\)
- The pump amplitude does not significantly change over the length of the crystal
  - A.K.A. the undepleted pump approximation
- The pump field is relatively narrow band
- The pump field's time dependence factors out (approximately)
- The pump field is paraxial
- The longitudinal component of \(\mathbf{k}_p\) dominates over the transverse components
- The spatial amplitude factors in to longitudinal and transverse parts
- The nonlinear crystal is embedded in a linear material of equal refractive index
- We can neglect internal reflections
- The nonlinear crystal is a good deal wider than the pump beam, and much wider than the pump wavelength
- And, we examine only the nearly degenerate part of the downconverted spectrum

We can find (in 1D for a Gaussian pump beam):

**The Double-Gaussian Approximation**

\[ \psi^{DG}(x_1, x_2) = \frac{1}{\sqrt{2\pi} \sigma_+ \sigma_-} e^{-\frac{(x_1+ x_2)^2}{2\sigma_+^2}} e^{-\frac{(x_1- x_2)^2}{2\sigma_-^2}} \]

\[ \psi \| \sigma_+ \| \sigma_- \]

**Correlation measures**

The Schmidt Number \(\mathcal{K}\)

- For the Double-Gaussian State:
  \(\mathcal{K} = \frac{1}{2} \left( \frac{\sigma_+}{\sigma_-} \right) \)
- For the Double-Gaussian State:
  \(\mathcal{K} = \frac{1}{2} \left( \frac{\sigma_+}{\sigma_-} \right) \)
- For the Double-Gaussian State:
  \(\mathcal{K} = \frac{1}{2} \left( \frac{\sigma_+}{\sigma_-} \right) \)

The Fedorov Ratio \(\mathcal{R}\)

- For the Double-Gaussian State:
  \(\mathcal{R} = 1 \left( \frac{\sigma_+}{\sigma_-} \right) \)
- For the Double-Gaussian State:
  \(\mathcal{R} = 1 \left( \frac{\sigma_+}{\sigma_-} \right) \)
- For the Double-Gaussian State:
  \(\mathcal{R} = 1 \left( \frac{\sigma_+}{\sigma_-} \right) \)

The Birth Zone Number \(N\)

- For the Double-Gaussian State:
  \(N = \left( \frac{\sigma_+}{\sigma_-} \right)^d \)
- For the Double-Gaussian State:
  \(N = \left( \frac{\sigma_+}{\sigma_-} \right)^d \)
- For the Double-Gaussian State:
  \(N = \left( \frac{\sigma_+}{\sigma_-} \right)^d \)

**Fourier Transform Properties of the Double-Gaussian**

The Heisenberg relations:

\[ \sigma_{x_1} \sigma_{x_2} = \frac{\Delta P_{x_1} \Delta P_{x_2}}{2\pi} \]

Are saturated by the Double-Gaussian state.

In addition, the conditional Heisenberg relations:

\[ \sigma_{x_1} \sigma_{x_2} \geq \frac{\Delta P_{x_1} \Delta P_{x_2}}{2\pi} \]

...are also saturated by the Double-Gaussian state.

**TABLE 1: Statistics of the Double-Gaussian**

**Temporal Correlation Width**

<table>
<thead>
<tr>
<th>Type-I SPDC</th>
<th>Type-II SPDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \equiv \frac{\sigma_1 + \sigma_2}{2})</td>
<td>(\Delta \equiv \frac{\sigma_1 - \sigma_2}{2})</td>
</tr>
</tbody>
</table>

When the joint distribution factors:

\[ p(x_1, x_2) = p(x_1) p(x_2) \]

We get the simple formula:

\[ \Delta = \sigma_1 \sigma_2 \]

For the Biphoton state:

\[ \Delta = \frac{9 L_2 \lambda_p}{10 \pi} \]

Consequence:

Thinner crystals give better (smaller) correlation widths, at the expense of brightness.

---

**DISTRIBUTION A. Approved for public release; distribution unlimited: BBABW-2016-0452**