On Position-Momentum Entanglement, Nonlocality, and Measurement

James Schneeloch
Howell Research Group
Center for Coherence and Quantum Optics
University of Rochester
To The Ones who Made this Possible

John Howell
Curtis Broadbent
Gregory Howland
Gerardo Viza
Bethany Little
Julian Martinez
Joseph Choi
Daniel Lum
Sam Knarr
Chris Mullarkey
Justin Winkler

Thank You!!
Outline:

- **Measurement**
  - The uncertainty principle
  - What position measurements do to momentum
  - Notions of uncertainty

- **Entanglement**
  - How to prove quantum entanglement experimentally
  - Witnessing entanglement with EPR-steering inequalities
  - How to demonstrate continuous-variable entanglement with discrete measurements

- **Nonlocality**
  - The Position-momentum CHSH Bell inequality
  - Bell’s strategy for demonstrating position-momentum Bell nonlocality
  - Bringing Bell’s proposal to the lab
  - Challenges and prospects
Part One: Measurement
(and reimagining the uncertainty principle)
The Uncertainty Principle

In quantum mechanics:

- Position $x$ and momentum $k$ are "complementary" observables
  - There are many other such pairs
- You can make (prepare the state of) a particle with near-definite position..
  - ..but not also with a near-definite momentum.
- The narrower you make the position of a wave, the wider its momentum spread must be.

$$\Delta x \cdot \Delta k \geq \frac{1}{2}$$
The effect of a Position measurement on Momentum

- A particle passing through a pinhole, definitely had a position within that pinhole.
- How much can we know about its momentum?
The effect of a Position measurement on Momentum

- A particle passing through a pinhole, definitely had a position within that pinhole.
- The smaller the pinhole, the better we know the position of the particle.
- But what does this do to the momentum of the particle?
The effect of a Position measurement on Momentum

Every position amplitude is a sum over momentum frequency components.

- A pinhole in momentum space excludes many high frequency components
- The image after too-small a pinhole will be significantly blurred.
- Similarly, a pinhole in position blurs the momentum amplitude distribution.
The effect of a Position measurement on Momentum

- Do sharper position measurements mean blurrier momentum measurements?
  - Not necessarily!
- Using random screens of pinholes, the momentum distribution is not blurred.
- The effect is instead seen as low level noise.

---

Original image

Random pattern

Distorted from random pattern
The effect of a Position measurement on Momentum

- A thin pinhole in position space, is a broad function in momentum space.
- The distortion in momentum is modeled as the convolution of the field with this broad transformed pinhole function.
- Convolving with broad functions makes for a blurry image.
  - Convolving with narrower functions makes less blurry images

\[
\psi(x) = \text{position amplitude of field} \\
f(x) = \text{binary pinhole function} \\
\bar{\psi}(x) = \text{distorted position amplitude}
\]

For a pinhole:

\[
f(x) = \text{rect}(a x) \\
f(k) = \frac{1}{\sqrt{2\pi a^2}} \text{sinc} \left( \frac{k}{2a} \right)
\]
The effect of a Position measurement on Momentum

- Convolving with broad functions makes for a blurry image.
  - Convolving with narrower functions makes less blurry images
- A random array of many pinholes in position space is a sharp narrow function with low level noise in momentum space.

\[
\begin{align*}
\text{For a pinhole:} & \quad f(k) \propto \text{sinc} \left( \frac{k}{2a} \right) \\
\text{For a random pinhole array of N total pixels:} & \quad f(k) \propto \text{sinc} \left( \frac{k}{2a} \right) \left( \delta(k) + \frac{b}{\sqrt{N}} \phi(k) \right) \\
& \text{..where } \phi(k) \text{ is a unit-variance Gaussian complex random variable for each value } k.
\end{align*}
\]

The perturbed momentum amplitude is then:

\[
\tilde{\psi}(k) \approx \mathcal{N} \left( \psi(k) + \frac{b}{\sqrt{N}} (\psi(k) \ast \phi(k)) \right)
\]
The effect of a Position measurement on Momentum

- With N random patterns (the same as the number of pixels)...
- ...we can retrieve the position distribution without also blurring the momentum
  - With say, least-squares optimization or compressive sensing

- What does this say about the uncertainty principle?

Notions of “Quantum” Uncertainty

Localization
- Within what tolerance can you reliably predict the outcome?
- How tightly are the random outcomes clustered about a single peak?

Information
- What is the size the set of likely outcomes?
- How many bits do you need to communicate the outcome?
Notions of Quantum Uncertainty

**Localization**
- Within what tolerance can you reliably predict the outcome?
- How tightly are the random outcomes clustered about a single peak?

\[ \sigma_x = \sqrt{\langle (x - \langle x \rangle)^2 \rangle} \]

\[ \sigma_x \cdot \sigma_k \geq \frac{1}{2} \]

**Information**
- What is the size the set of likely outcomes?
- How many bits do you need to communicate the outcome?

\[ h(x) = - \int dx \rho(x) \log(\rho(x)) \]

\[ h(x) + h(k) \geq \log(\pi e) \]
Part Two: Entanglement
(through the EPR paradox and EPR steering inequalities)
Entanglement in a Nutshell.

Entanglement is created through the interaction of quantum systems.

- The quantum state of a pair of non-interacting independent particles is separable:
  - E.g. $|\psi^{AB}\rangle = |\phi^A\rangle \otimes |\phi^B\rangle$.

- If the quantum state of a system can be made out of such non-interacting, independent pairs, that state must also be separable:
  - E.g. $\hat{\rho}^{AB} = \sum_i p_i (\rho^A_i \otimes \rho^B_i)$.

- All other states are entangled
What’s entanglement good for?

Quantum Cryptography

Quantum Teleportation

Quantum computing

Enhanced measurement

Proving Entanglement in the Lab

The **hard** way:

- Determine the joint quantum state through exhaustive tomography.

- Calculate a measure of entanglement for the given state.
  - (NP-hard, in general)

The **easy** way:

- Test an entanglement witness:
  - A statistical criterion all separable states satisfy.

- If the entanglement witness is violated,
  - Entanglement is certified.

- If the witness is not violated,
  - Entanglement is not certified
Witnessing Entanglement with EPR-Steering Inequalities

**EPR steering:** The explicitly nonlocal manipulation of a quantum state through actions on an entangled partner.

- If a pair of particle’s statistics violate an EPR-steering inequality...
  - ...they demonstrate the EPR paradox
  - ...their state must be entangled.
From the EPR paradox to EPR-steering inequalities

- **The situation:** Alice and Bob share a pair of particles A and B entangled in position and momentum.
  - A and B are space-like separated from each other at the time of measurement.

- **Locality:** The effect of measurement cannot travel faster than light.

- **Completeness:** The uncertainty principle fundamentally limits our knowledge of a quantum system.
  - Knowing everything that could locally affect a particle’s history wouldn’t change this.
From the EPR paradox to EPR-steering inequalities

- Entangled pairs of particles can have **arbitrarily strong** correlations in position and in momentum.

- The seeming paradox:
  - All “possible” information about \( x_B \) or \( k_B \) would be in Bob’s past light cone \( \lambda \).
  - Alice’s measurements couldn’t possibly give you more information about \( x_B \) or \( k_B \) than knowing everything in \( \lambda \).

\[
\begin{align*}
\sigma(x_A) \cdot \sigma(k_A) & \geq \frac{1}{2} \\
\sigma(x_A \pm x_B) \cdot \sigma(k_A \mp k_B) & \geq 0
\end{align*}
\]

but...

\[
\begin{align*}
\sigma(x_A) \cdot \sigma(k_A) & \geq \frac{1}{2} \\
\sigma(x_A \pm x_B) \cdot \sigma(k_A \mp k_B) & \geq 0
\end{align*}
\]
From the EPR paradox to EPR-steering inequalities

\[ h(x_B | x_A) \geq \int d\lambda \rho(\lambda) h(x_B | \lambda) \]
\[ h(k_B | k_A) \geq \int d\lambda \rho(\lambda) h(k_B | \lambda) \]

- Using the entropic uncertainty relation
  \[ h(x_B) + h(k_B) \geq \log(\pi e) \]

- We find that in a local universe, Alice and Bob’s measurement correlations must be limited by the (EPR-steering) inequality:
  \[ h(x_B | x_A) + h(k_B | k_A) \geq \log(\pi e) \]

- QM predicts there are no limits to these correlations!
  - EPR-steering inequalities can be violated!

\[ \sigma(x_A)\sigma(k_A) \geq \frac{1}{2} \]
but...
\[ \sigma(x_A \pm x_B)\sigma(k_A \mp k_B) \geq 0 \]
Where’s the steering?

The setup:
• When Alice measures $X$:
  • Bob finds a state *well defined in position* when conditioning on Alice’s outcome
• When Alice measures $K$:
  • Bob finds a state *well defined in momentum* when conditioning on Alice’s outcome
How does EPR-steering prove entanglement?

Problem:

- Lots of correlations can be explained classically
  - Alice and Bob could be receiving a classically correlated ensemble of states
  - Alice or Bob could have an untrusted measurement device (a “black box”)

- How do you rule out this possibility?
  - i.e., the possibility of a model of local hidden states for Alice or for Bob.
Local Hidden States?

- Local hidden variables (LHV):
  - Information existing in past light cone
- LHV models:
  \[ \rho(x_A, x_B) = \int d\lambda \rho(\lambda)\rho(x_A|\lambda)\rho(x_B|\lambda) \]
  - Ruled out by violating a Bell Inequality
- Local hidden states (LHS):
  - States determined by local hidden variables
- LHS model (for Bob):
  \[ \rho(x_A, x_B) = \int d\lambda \rho(\lambda)\rho(x_A|\lambda)\text{Tr}[\hat{\Pi}_X^B \hat{\rho}_\lambda^B] \]
  - Ruled out by violating an EPR-steering inequality (proving you can do it)
Position-Momentum EPR-steering inequalities

- Reid (1989)

\[ \sigma(x_B | x_A) \cdot \sigma(k_B | k_A) \geq \frac{1}{2} \]

- Walborn et al (2011)

\[ h(x_B | x_A) + h(k_B | k_A) \geq \log(\pi e) \]
Why use Walborn et al’s steering inequality?

\[ h(x_B | x_A) + h(k_B | k_A) \geq \log(\pi e) \]

- Entropy is a more sensitive measure of uncertainty than variances.
- The entropic uncertainty relation is tighter than the Heisenberg uncertainty relation (more states are closer to the threshold)
- Information-based uncertainty relations are easier to apply in quantum information
- You need the same information either way
How to experimentally demonstrate steering with Walborn at al’s inequality?

- **Problem:**
  - You need to know $\rho(x_A, x_B)$ to find $h(x_B | x_A)$.

$$h(x) \equiv -\int dx \rho(x) \log(\rho(x))$$

- **Solutions:**
  - (hard) Elaborate density function estimation algorithms
  - (Easy) Just use the discrete distribution!
    - **How?**
      - Discrete approximation never decreases the entropy!
Relating discrete to continuous entropy

\[ H(X) \equiv - \sum_i P(X_i) \log(P(X_i)) \]

\[ h(x) = \sum_i P(X_i) h_i(x) + H(X) \]

\[ h_i(x) \leq \log(\Delta x) \]

\[ h(x) \leq H(X) + \log(\Delta x) \]
Relating discrete to continuous entropy

• The entropy of the discrete approximation of $\rho$ is never smaller than the entropy of $\rho$, itself.

$$h(x) \leq H(X) + \log(\Delta x)$$

$$h(x_B|x_A) \leq H(X_B|X_A) + \log(\Delta x_B)$$

$$h(x_A:x_B) \geq H(X_A:X_B)$$

$$h(x_B|x_A) = h(x_A, x_B) - h(x_A)$$
A continuous variable steering inequality for discrete measurements!

- Since
  \[ H(X_B | X_A) \geq h(x_B | x_A) - \log(\Delta x_B) \]

- We can use Walborn’s inequality:
  \[ h(x_B | x_A) + h(k_B | k_A) \geq \log(\pi e) \]

- To get our first result:
  \[ H(X_B | X_A) + H(K_B | K_A) \geq \log \left( \frac{\pi e}{\Delta x_B \Delta k_B} \right) \]
Experimental success 1

- Used down-converted $325 \rightarrow 650$ nm light from BBO nonlinear crystal.
- Measured joint coincident detections to get joint probability distributions in both image and Fourier planes of the crystal.
  - Recorded at different resolutions
- Successful violation at $8 \times 8$ through $24 \times 24$ resolutions

$$H(X_B|X_A) + H(K_B|K_A) \geq \log \left( \frac{\pi e}{\Delta x_B \Delta k_B} \right)$$

<table>
<thead>
<tr>
<th>Resolution</th>
<th>Minimum $N\sigma$</th>
<th>Maximum $N\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 \times 8$</td>
<td>3.65</td>
<td>5.9</td>
</tr>
<tr>
<td>$16 \times 16$</td>
<td>8</td>
<td>11.2</td>
</tr>
<tr>
<td>$24 \times 24$</td>
<td>12.3</td>
<td>16.4</td>
</tr>
</tbody>
</table>

Lingering loopholes

- There’s a lot of an infinite distribution experimenters don’t have access to.
  - Even if we knew the exact probabilities we measure,
    - Any remaining probability outside could skew the entropy to infinity

- We cannot measure *all* the probabilities needed to get $H(X_B|X_A)$ and $H(K_B|K_A)$.
- But... we can bound these with the data we do have.
The Fano Inequality

- An upper bound for $H(X_B | X_A)$ with the probability $\eta \equiv P(X_A = X_B)$.

$$H(X_B | X_A) \leq h_2(\eta) + (1 - \eta)\log(N - 1)$$

$$h_2(\eta) \equiv - \log_2(\eta) - (1 - \eta) \log_2(1 - \eta)$$

- For continuous variables, Fano’s inequality isn’t helpful.

$$N \to \infty \implies H(X_B | X_A) \leq \infty$$
Making a continuous-variable Fano Inequality

\[ H(X_B|X_A) \leq h_2(\eta) + (1 - \eta) \log(N - 1) \]

- Add a new window variable \( W \)
  - \( W = \{0,1,2,3,\ldots\} \), (infinite number of \( \bar{N} \) pixel windows).

\[ H(X_B|X_A) \leq h_2(\eta) + H(W) + (1 - \eta) \log(\bar{N} - 1) \]

- But.. \( H(W) \leq \infty \)
- However, if the mean \( \langle W \rangle \) is finite...
  - We get a (useful) continuous-variable Fano inequality!

\[ H(X_B|X_A) \leq h_2(\eta) + \frac{h_2(\mu)}{\mu} + (1 - \eta) \log(\bar{N} - 1) \]

Here, \( \mu = P(W = 0) \), the domain probability.
Steering with Fano’s Inequality

\[ H(X_B | X_A) \leq h_2(\eta) + \frac{h_2(\mu)}{\mu} + (1 - \eta) \log(\bar{N} - 1) \]

- To use this, we need to know \( \mu \) and \( \eta \)
  - We can estimate \( \mu \) with fitting
  - Estimating \( \eta \) is more difficult
    - (you need probabilities outside your viewing window)
- So, we introduce the measured agreement probability \( \bar{\eta} \):
  \[ \bar{\eta}_x \equiv P(X_A = X_B | W = 0) \]
  \[ \bar{\eta}_x \leq \frac{\eta_x}{\mu_x} \]
Position-momentum EPR-steering with Fano steering bounds

\[ \tilde{\eta}_x \mu_x \leq \eta_x \]

For \( \tilde{\eta}_x \mu_x < \frac{1}{2} \), (and similarly for momentum) we get the steering inequality

\[
h_2(\tilde{\eta}_x \mu_x) + h_2(\tilde{\eta}_k \mu_k) + \frac{h_2(\mu_x)}{\mu_x} + \frac{h_2(\mu_k)}{\mu_k} + (2 - \tilde{\eta}_x \mu_x - \tilde{\eta}_k \mu_k) \log(N - 1) \geq \log \left( \frac{\pi e}{\Delta x_B \Delta k_B} \right)
\]
Position-momentum EPR-steering with Fano steering bounds

\[ \tilde{\eta}_x \mu_x \leq \eta_x \]

- For \( \tilde{\eta}_x \mu_x < \frac{1}{2} \), (and similarly for momentum) we get the steering inequality

\[
h_2(\tilde{\eta}_x \mu_x) + h_2(\tilde{\eta}_k \mu_k) + \frac{h_2(\mu_x)}{\mu_x} + \frac{h_2(\mu_k)}{\mu_k} + (2 - \tilde{\eta}_x \mu_x - \tilde{\eta}_k \mu_k) \log(\tilde{N} - 1) \geq \log \left( \frac{\pi e}{\Delta x_B \Delta k_B} \right)
\]

- What’s it good for?
  - You need less information
    - (only that the agreement probabilities are big enough)
  - You can compensate for finite limitations with sufficiently good data
    - e.g., finite viewing area, dead space between pixels, etc

- Tradeoff
  - Only works well for highly correlated systems
  - But... down-converted photon pairs work well for this

Aug, 19 2015
Experimental success 2!

Successful violation of steering bound even accounting for these limitations!

- Used same source of 325nm -> 650 nm down-converted photon pairs.
- Measured joint position and momentum distributions using compressive sensing techniques (faster than raster scanning)
- Obtained detector fill factors from equipment manuals.
- Obtained domain probabilities from Gaussian fitting

<table>
<thead>
<tr>
<th>Table 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\eta}_x$</td>
<td>69.4%</td>
</tr>
<tr>
<td>$\bar{\eta}_k$</td>
<td>75.1%</td>
</tr>
<tr>
<td>$\mu_x$</td>
<td>99.7%</td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>95.2%</td>
</tr>
<tr>
<td>Position fill factor</td>
<td>92%</td>
</tr>
<tr>
<td>Momentum fill factor</td>
<td>100%</td>
</tr>
</tbody>
</table>

Figures from:
Part Three: Nonlocality
(Experimental hurdles and possibilities in position-momentum)
Non-locality In a nutshell

- Alice and Bob share a spacelike-separated pair of particles A and B.
- **Locality**: information travels no faster than light.
  - What Alice and Bob’s measurements both affect is only in $\lambda_{\text{future}}$.
  - What can affect both Alice and Bob’s measurements is only in $\lambda_{\text{past}}$.

- If the Universe is local, measurement correlations can be “explained” locally:

$$\rho(x_A, x_B) = \int d\lambda \rho(\lambda) \rho(x_A | \lambda) \rho(x_B | \lambda)$$

(a model of Local Hidden Variables)
The CHSH-Bell inequality  
(for position-momentum)  

- If \( \rho(x_A, x_B) \) factors this way:
  \[
  \rho(x_A, x_B) = \int d\lambda \rho(\lambda) \rho(x_A|\lambda) \rho(x_B|\lambda)
  \]

- Then, the measurement statistics do as well:
  \[
  \langle f(x_A) f(x_B) \rangle_{(\alpha, \beta)} = \int d\lambda \rho(\lambda) \langle f(x_A) \rangle_{(\alpha, \lambda)} \langle f(x_B) \rangle_{(\beta, \lambda)}
  \]
  - \( \alpha \) is Alice’s measurement setting
  - \( \beta \) is Bob’s measurement setting
  \[
  E(\alpha, \beta) \equiv \langle f(x_A) f(x_B) \rangle_{(\alpha, \beta)}
  \]

- With the right choice of function \( f(x) \) (bounded between -1 and 1), we can get the CHSH inequality:
  \[
  |E(\alpha, \beta) - E(\alpha, \beta')| + (E(\alpha', \beta) + E(\alpha', \beta')) \leq 2
  \]
Remarks from Bell:

- Maximally entangled states don’t have to violate this Bell inequality.
- The EPR state:

\[ |\psi^{(EPR)}\rangle = \mathcal{N} \int dx_A dx_B \delta(x_A - x_B)|x_A, x_B \rangle \]

...is maximally entangled

- But its Wigner function:

\[ \mathcal{W}^{EPR}(x_A, x_B, k_A, k_B) = \mathcal{N} 2\pi \delta(x_A - x_B)\delta(k_A + k_B) \]

...is a valid probability distribution (and a local hidden variable model)

\[ \rho(x_A, x_B) = \int d\lambda \left( \rho(\lambda) \rho(x_A|\lambda) \rho(x_B|\lambda) \right) \]
Remarks from Bell:

- However, there is an entangled state that does violate the CHSH-Bell inequality:

  \[ \psi^{BV}(x_A, x_B) = N((x_A - x_B)^2 - 8\sigma^2) e^{-\left(\frac{(x_A+x_B)^2}{8\sigma_+^2}\right)} e^{-\left(\frac{(x_A-x_B)^2}{8\sigma_-^2}\right)} \]

- We call it, Bell’s wavefunction:

- It’s not unlike the Double Gaussian state:

  \[ \psi^{DG}(x_A, x_B) = N' e^{-\left(\frac{(x_A+x_B)^2}{8\sigma_+^2}\right)} e^{-\left(\frac{(x_A-x_B)^2}{8\sigma_-^2}\right)} \]

  (gives EPR state as limiting case)

- Except...
Bell’s Wavefunction

- ...it has a Wigner function with large regions of negativity.

- The corresponding Double-Gaussian state does not..
Bell’s strategy:

- Let $\psi^{BV}(x_1, x_2)$ describe a pair of entangled (massive) particles (that no longer interact).
- Time-evolve the pair with the free particle Hamiltonian

$$\hat{H}_{\text{free}} = \frac{\hat{p}_1^2}{2 m_1} + \frac{\hat{p}_2^2}{2 m_2} = \hat{H}_1 + \hat{H}_2$$

- Measurement settings are the times each particle is measured.

$$(\alpha, \beta) \rightarrow (t_1, t_2)$$

In approximation ($\sigma_+ \rightarrow \infty$), the optimal correlation measurements violate the CHSH inequality.
Bringing Bell’s strategy to the Lab:

- The 2D free-particle Schrödinger equation, and paraxial Helmholtz equation are (mathematically) identical.

\[- \frac{\partial^2 A}{\partial x^2} - \frac{\partial^2 A}{\partial y^2} = i k_p \frac{\partial A}{\partial z} \sim - \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial y^2} = i \frac{2m}{\hbar} \frac{\partial \Psi}{\partial t} \]
Bringing Bell’s strategy to the Lab:

- The 2D free-particle Schrödinger equation, and paraxial Helmholtz equation are (mathematically) identical.
- If Bell’s wavefunction could be mapped onto the biphoton state from SPDC...
  - i.e., (Spontaneous Parametric Down-Conversion)
- We could measure the sign correlations at different signal/idler propagation distances
Theoretical results:

- If Bell’s wavefunction *could* be mapped to the Biphoton state from SPDC...
- Then there would be (minute) violations!
  - Note: \((\alpha, \beta, \alpha', \beta') = (z_1, z_1, z_2, z_2)\)
Challenges and hurdles:

- Minute violations need a LOT of coincidence counts to resolve...

- Coincidence counts are Poisson Distributed...
  - You need $10^8$ coincidence counts for the uncertainty in the count rate to be one part in $10^4$
  - With good count rates at $10^4$/s...
  - It would take several days to get enough data to violate the CHSH Bell inequality
Challenges and hurdles:

- Minute violations need a LOT of coincidence counts to resolve.

- The biphoton state still needs to resemble Bell’s wavefunction
  - How to engineer the biphoton state to resemble Bell’s wavefunction?

- If we could continuously vary $\chi^{(2)}$ in a nonlinear crystal...
  - We could engineer the biphoton wavefunction however we want.
  - The next best thing...
    - Vary the duty cycle in a periodically poled crystal

\[ L_z \]

\[ H_z \]
Challenges and hurdles:

- Minute violations need a LOT of coincidence counts to resolve.

- The biphoton state still needs to resemble Bell’s wavefunction
  - How to engineer the biphoton state to resemble Bell’s wavefunction?

- Can the actual biphoton state violate the CHSH Bell Inequality?
  - Definitely maybe!
Conclusions and Future Questions
Classical information theory remains a fertile ground for new research in quantum information.

- How does the measurement–disturbance tradeoff work for other pairs (or groups) of observables?
- What are the ultimate applications of (free-space) position-momentum entanglement?
  - Position-momentum quantum cryptography?
- What can be done with multi-partite position-momentum entanglement that can’t be done with photon pairs?
Thanks for listening!
Works Cited:


Photo Credits:

Albert Einstein:

Boris Podolsky:
http://astrojem.com/imagenes_voltaire/podolsky.jpg

Nathan Rosen:

Erwin Schrödinger:
https://commons.wikimedia.org/wiki/File:Schroedinger.jpg
Contingency Slide: Experimental setup

- Experimental setup for measuring position and momentum joint distributions with compressive sensing
- Use random patterns in signal and idler arms
- Flux through patterns gives correlation between pattern and signal
  - E.g. a pattern resembling the signal will let a lot of the signal through
- With a lot of these correlations, we can reconstruct the signal by brute force
- We can do much better using compressive sensing algorithms.

experimental diagram from PRX 3 011013 (2013)
Many signals are compressible in some sparse basis
  i.e., a basis where the signal has only a few significant components.

We could sense in this sparse basis efficiently if we knew where the significant components were.

Using random measurements unbiased with the sparse basis, you can get lots of info about all significant components of the signal with each measurement.

The original signal is (ideally) the unique solution to an optimization problem.

\[
\begin{align*}
\tilde{y} &= A\tilde{x} + \phi \\
\tilde{x} &= \min_{x_0} \left( \frac{1}{2} \| \tilde{y} - A\tilde{x}_o \|_2^2 + \tau \|\tilde{x}_o\|_1 \right)
\end{align*}
\]
Okay, so what are these Demonstrations good for?

- Quantum metaphysics is fine, but...
- EPR-steering correlations are *Monogamous*!
  - i.e., the more correlated systems A and B are…
    - …the less any third system can be correlated with either of them!
  - Also, no system can be steered both by two independent parties
- Limiting a third party’s correlations, limits their information about (A and B)’s correlations.
  - Useful in *one-sided device-independent* quantum key distribution!
One-sided device-\textit{independent} Quantum Key Distribution?

- One-sided: Alice’s device is an untrusted black box with settings and outputs.
- Bob’s device is trusted.
- If Alice doesn’t trust her measurement device, but Bob trusts his…
- What about Eve?
One-sided device-independent Quantum Key Distribution?

What about Eve?
- Eve sends particles to Alice and Bob
- Bob measures in a random basis
- Bob dictates Alice’s measurement setting
- Alice measures in chosen setting.

What does Eve control?
- Eve knows the state sent to Alice and Bob
- Eve knows Alice’s measurement settings
- Eve can force Alice’s measurement device to display any outcome
- Eve does not know Alice’s measurement outcome otherwise
One-sided device-\textit{independent} Quantum Key Distribution?

- What about Eve?
- What can Eve do?
- The best Eve could do:
  - send Bob a prepared ensemble of photons
  - puppeteer Alice’s device to display expected outcomes
One-sided device-*independent* Quantum Key Distribution?

- What about Eve?
- What can Eve do?
- The best Eve could do:
  - send Bob a prepared ensemble of photons
  - puppeteer Alice’s device to display expected outcomes
- But then, Bob’s receiving local hidden states
- No steering inequality can be violated:
  - Eve could’ve sent a photon well defined in position, but Alice is told to measure in momentum.
Assuming $\langle W \rangle$ is finite:

$$H(W) \leq \frac{h_2(\mu)}{\mu}$$

$\mu \equiv P(W = 0)$, the probability that the outcomes of $X^A$ and $X^B$ are within the experimental window. Why?

- Maximum entropy decreases with increasing maximum probability
- $\mu \leq$ maximum probability