THE SUN AND OTHER BLACK BODIES

A few salient facts about the Sun
Nucleosynthesis
Blackbody radiation and temperatures of stars
The spectrum of blackbodies, and solid angle
Wien’s Law

Three-color X-ray mosaic image of the Sun, made by the NASA TRACE satellite (J. Covington, LMMS)
THE SUN: BASIC FACTS

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1.98892x10^{33} grams</td>
</tr>
<tr>
<td>Luminosity</td>
<td>3.826x10^{33} erg sec^{-1}</td>
</tr>
<tr>
<td>Radius</td>
<td>6.96x10^{10} cm</td>
</tr>
<tr>
<td>Surface rotation period, sidereal, at equator</td>
<td>2.193x10^{6} sec (25.38 days)</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>5800 K</td>
</tr>
<tr>
<td>Surface pressure</td>
<td>10 dyne cm^{-2} (10^{-5} atmospheres)</td>
</tr>
<tr>
<td>Central temperature</td>
<td>1.58x10^{7} K</td>
</tr>
<tr>
<td>Central pressure</td>
<td>2.50x10^{17} dyne cm^{-2}</td>
</tr>
<tr>
<td>Surface composition, by mass</td>
<td>H (70.4%), He (28.0%), O (0.76%), C (0.28%), Ne (0.17%), N (0.08%), others (0.32%)</td>
</tr>
</tbody>
</table>

MUCH MORE LATER

One learns a lot more about the Sun and other stars, including the reasons why the Sun has the set of properties that it does, in AST 142 and AST 241.

For our purposes, it will suffice to discuss only three of its properties now: its composition, its surface temperature, and its luminosity.
THE SUN’S COMPOSITION AND NUCLEOSYNTHESIS

The Sun is a sphere of hot ionized gas, obviously boiling at the surface, so one might think that it is well mixed. It is not.

The Sun’s size is determined by the balance of its weight and its internal pressure.

The heat that provides the pressure would leak away (in the form of sunshine) within a few million years if it were not somehow replenished.

The replenishment of heat is provided by nuclear fusion reactions taking place at the very center.

1. When atomic nuclei fuse to produce a nucleus lighter than that of iron (Fe, \(Z = 26, A = 56\)), kinetic energy is liberated (the reaction is exothermic) and added as heat to the energy of the surroundings.

By the same token, this means that the contents of stars continuously transmute themselves from light elements to heavy elements.

NUCLEOSYNTHESIS

Models of the Sun’s interior indicate that the composition at the very center is 33.6% H and 64.3% He by mass. Much of the H there has been burned to He, mostly due to the proton-proton reaction chains.

1. The interior of the Sun is so hot that the enough collisions occur to ionize H, resulting in a plasma
1. Individual protons collide, fusing together to form deuterium (fusion); deuterium continues to fuse with other protons in a chain reaction to form He.
1. The alpha particle (He) is less massive than the four protons that combined to make it, so the proton-proton chain releases energy equal to the mass difference via \(E = mc^2\)

In hotter, denser stellar cores, fusion of He (by the “triple alpha” reaction), C, O, Ne, and even heavier elements can take place at respectable rates...

...and elements heavier than Fe can be made in small quantities in such stars by the endothermic (opposite of exothermic) s-process reactions.
NUCLEOSYNTHESIS

The Universe was born (in a fireball we can see, which we call the Big Bang) with hardly any elements heavier than helium.

- This is because fusion of two normal helium nuclei ($^3\text{He}$) produces an isotope of beryllium ($^9\text{Be}$) that is quite unstable, dissolving back into helium within $10^{-16}$ sec.

Fusion of three normal helium nuclei produces a stable result:

$$3(^3\text{He}) \rightarrow ^{12}\text{C} + \gamma$$  

Triple-$\alpha$ process

But very high density and temperature are necessary in order to make three He nuclei collide simultaneously, energetically enough to fuse. And by the time the Universe cooled enough for nuclei to condense, the density was too low.

NUCLEOSYNTHESIS

In the cores of some stars, temperatures and densities can both be large enough for the triple-$\alpha$ process to proceed.

- And the carbon produced thereby can react to form other heavier nuclei.
- Thus the heavy elements that the planets — and you — are made of were synthesized by nuclear reactions in the cores of stars that lived and died billions of years ago.

Stars die before they burn more than about 10% of their total H into He, so the Universe will take a very long time to run out of hydrogen.
THE NUCLEAR-CHEMICAL EVOLUTION OF THE MILKY WAY

New stars
Younger stars
Older stars

THE TEMPERATURE OF THE SUN’S SURFACE

The Sun is a ball of hot gas; it has no sharp, solid surface. So what do we mean by the “surface” of the Sun?

- Usually, we mean the deepest place we can see in the Sun’s atmosphere; this is called the photosphere.
- The absorption of light in the Sun varies with wavelength, though, so the position of the photosphere is different for different parts of the spectrum.
- The temperature varies with depth within the Sun’s atmosphere and interior, so the physical temperature of the surface that we see is different for different wavelengths.

So, it is more convenient to characterize the Sun’s “surface” temperature in a way that depends upon the luminosity (total power output), which of course is wavelength-dependent.

- Effective temperature of the Sun: the temperature that would produce the Sun’s observed luminosity, if the Sun were a spherical blackbody with the same radius.
- You will learn all about blackbodies in PHY 143/123 and PHY 227. We will only list the properties and equations pertaining to blackbodies that we will need to use this semester, and defer the detailed explanations to those courses.
BLACKBODIES AND BLACKBODY RADIATION

A blackbody is a body that is perfectly opaque: it absorbs all of the light incident on it.

If such a body has a constant temperature, it must therefore also emit light at exactly the same rate as it absorbs light. This is called blackbody radiation.

The flux — total power emitted in all directions by a blackbody at temperature $T$ per unit area — of a blackbody is given by

$$\sigma T^4$$

Stefan's Law

$$\sigma = 5.67051 \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$$

Stefan-Boltzmann constant

$$= 5.67051 \times 10^{-8} \text{ J sec}^{-1} \text{ m}^{-2} \text{ K}^{-4}$$

DEFINITIONS: LUMINOSITY AND FLUX

Luminosity is the total power output (in the form of light) emitted in all directions by an object. Units = erg sec$^{-1}$

- The Sun’s luminosity, as we listed before, is

$$L_\odot = 3.83 \times 10^{33} \text{ erg sec}^{-1}$$

Flux is the total power (in the form of light) per unit area that passes through a (sometimes imaginary) surface. Units = erg sec$^{-1}$ cm$^{-2}$

- The flux of sunlight at the Sun’s surface is

$$f = \frac{L_\odot}{4\pi R_\odot^2} = 6.29 \times 10^{10} \text{ erg sec}^{-1} \text{ cm}^{-2}$$

- The flux of sunlight at Earth’s orbit (the solar constant) is

$$f = \frac{L_\odot}{4\pi (1 \text{ AU})^2} = 1.36 \times 10^{6} \text{ erg sec}^{-1} \text{ cm}^{-2}$$
THE SUN’S EFFECTIVE TEMPERATURE

If the Sun were a blackbody with radius equal to that of its visible photosphere ($R_\odot = 6.96 \times 10^{10}$ cm) and luminosity as observed ($L_\odot = 3.83 \times 10^{33}$ erg/s), then its (effective) temperature $T_e$ would be given by

$$L_\odot = \sigma T_e^4 4\pi R_\odot^2$$

or

$$T_e = \left( \frac{L_\odot}{4\pi R_\odot^2} \right)^{1/4} = \left( \frac{3.83 \times 10^{33} \text{ erg sec}^{-1}}{4\pi (5.67 \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-2} \text{ K}^{-4}) (6.96 \times 10^{10} \text{ cm})^2} \right)^{1/4} = 5770 \text{ K}$$

Hence the number given above for the surface temperature. This turns out not to be far off from the physical temperature typical of the photospheric region.

THE SPECTRUM OF A BLACKBODY

One reason the effective temperature comes so close to the relevant physical temperature is that the spectrum of the Sun resembles the spectrum of a blackbody.
BLACKBODY SPECTRUM

The spectrum of blackbody radiation is given by the Planck function:

$$u_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

where

$$h = 6.626 \times 10^{-27} \text{ erg sec}$$  Planck’s constant

$$k = 1.381 \times 10^{-16} \text{ erg K}^{-1}$$  Boltzmann’s constant

The dimensions of the Planck function may seem confusing at first: they are power per unit area, per unit solid angle, per unit wavelength interval; for example, erg sec\(^{-1}\) cm\(^{-2}\) ster\(^{-1}\) μm\(^{-1}\)

SOLID ANGLE?

Solid angle is to area what angle is to length. It is usually defined by a differential element in spherical coordinates:

$$d\Omega = \sin \theta \ d\theta \ d\phi$$

Angles in radians

Unit of solid angle = steradian

For small angles, the solid angle is calculated from the angular widths of the “patch” in the same manner as plane-geometrical areas. For a rectangle, the two angles

$$\Delta \Omega = \Delta \phi \ \sin \theta \ \Delta \theta$$
**REMINDER: THE SPHERICAL COORDINATES** \( r, \theta, \phi \)

\[
x = r \sin \theta \cos \phi \\
y = r \sin \theta \sin \phi \\
z = r \cos \theta
\]

**SIMPLE EXAMPLE**

What is the solid angle of a square patch of sky 15.4 arcminutes on a side? (This is the solid angle covered by the CCD camera you will use on the Mees 24-inch telescope.)

15.4 arcminutes is a small angle (<< 1 radian). Since I did not specify where the z-axis is, we are free to point it through the square's center, so \( \sin \theta = 1 \):

\[
\Delta \Omega = \Delta \theta \Delta \phi = \left( 15.4' \times \frac{1^\circ}{60'} \times \frac{2\pi \text{ radians}}{360^\circ} \right)^2
\]

\[
\Delta \Omega = 2.01 \times 10^{-5} \text{ steradians}
\]
COMPUTING SOLID ANGLES

When the angles are not small, one must integrate the differential element over the range of $\theta$ and $\phi$. You will not be doing that in AST 111, but just so you can see how it works...

The solid angle of the entire sky:

$$\Omega = \int_0^{2\pi} d\phi \int_0^\pi \sin \theta \ d\theta = -2\pi \cos \theta |_{\theta=0}^{\theta=\pi} = -2\pi (-2)$$

$$\Omega = 4\pi \text{ steradians}$$

The solid angle of a cone with an angular radius of $\pi/8$ radians:

$$\Omega = \int_0^{2\pi} d\phi \int_0^{\pi/8} \sin \theta \ d\theta = -2\pi \cos \theta |_{\theta=0}^{\theta=\pi/8} = 2\pi (1 - 0.924)$$

$$\Omega = 0.478 \text{ steradians}$$

BLACKBODY SPECTRUM

The reason for the funny dimensions of the Planck function is that it has to be integrated over wavelength, solid angle, and area in order to produce a luminosity:

$$f = \int \frac{d\Omega}{\Omega} \int_0^\infty d\lambda \ u_\lambda$$

$$L = \int_A dA \int_\Omega d\Omega \int_0^\infty d\lambda \ u_\lambda$$

In PHY 227 or AST 241, you will learn how to prove that the power per unit area emitted in all directions by a blackbody is

$$f = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \ \cos \theta \int_0^\infty d\lambda \ u_\lambda \ u_\lambda = \frac{2\pi^5 k^4}{15c^2 R^3} T^4 \equiv \sigma T^4$$

that is, Stefan’s Law follows from the Planck function.
**THE PLANCK FUNCTION**

Note that the higher the temperature, the shorter the wavelength at which the peak occurs.

Note also that visible wavelengths do not include much of the luminosity, whatever the temperature.

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**WIEN’S LAW**

We will not be using the Planck function much, and we certainly will not be integrating it. It appears now because of a useful consequence of the shape of the function.

It is fairly easy to convince oneself — as you will, in Problem Set 1 — that there is a simple relation between the temperature of a blackbody and the wavelength at which it is brightest:

\[
\lambda_{\text{max}} T = 0.2897756 \text{ cm K}
\]

\[
= 2.897756 \times 10^{-3} \text{ m K}
\]

*Wien’s law*

The shape of the spectrum tells one what the effective temperature is. This shape is easier to measure accurately than luminosity is.
EXEMPLARY

At what wavelength do you \( T = 98.6^\circ F = 37^\circ C = 310 \text{ K} \) emit the most blackbody radiation?

\[
\lambda_{\text{max}} = \frac{0.29 \text{ cm K}}{T} = \frac{0.29 \text{ cm K}}{310 \text{ K}} \\
= 9.4 \times 10^{-4} \text{ cm} = 9.4 \mu\text{m}
\]

Gamma-ray bursters emit most of their light at a wavelength of about \( 10^{-10} \text{ cm} \). If this emission were blackbody radiation, what would be the temperature of the burster?

\[
T = \frac{0.29 \text{ cm K}}{\lambda_{\text{max}}} = \frac{0.29 \text{ cm K}}{10^{-10} \text{ cm}} = 2.9 \times 10^9 \text{ K}
\] 

Pretty hot

INTRODUCTION TO THE MOON

Physical aspects of the Lunar surface
Cratering and the history of the Lunar surface
The Moon’s temperature
Rocks and minerals
THE MOON’S VITAL STATISTICS

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$7.349 \times 10^{25}$ g (0.012M⊕)</td>
</tr>
<tr>
<td>Equatorial radius</td>
<td>$1.7381 \times 10^8$ cm (0.273R⊕)</td>
</tr>
<tr>
<td>Average density</td>
<td>3.350 g cm⁻³</td>
</tr>
<tr>
<td>Albedo</td>
<td>0.12</td>
</tr>
<tr>
<td>Average surface temperature</td>
<td>274.5 K</td>
</tr>
<tr>
<td>Orbital semimajor axis</td>
<td>$3.844 \times 10^{10}$ cm</td>
</tr>
<tr>
<td>Sidereal revolution period</td>
<td>27.3217 days</td>
</tr>
<tr>
<td>Synodic revolution period</td>
<td>29.53 days</td>
</tr>
<tr>
<td>Recession rate from Earth</td>
<td>3.8 cm yr⁻¹</td>
</tr>
</tbody>
</table>

Lunation, photographed by António Cidadão

NEAR SIDE

Image by Robert Gendler

Maria ("oceans")
Highlands
Impact crater (Tycho)
Posidonius
Rilles
NEAR SIDE

Image by Robert Gendler

APOLLO LANDING SITES

Image by Robert Gendler
FAR SIDE

Almost all highlands and craters (very different from the near side)

Image from Apollo 16

VIEW FROM HIGH LATITUDE

From the Galileo planetary probe (JPL/NASA)
EXPLORE THE MOON ON YOUR OWN

As you probably already know, Google Earth covers the Moon too, providing endless hours of entertainment.
MAIN SURFACE FEATURES

Maria (dominating the near side): iron-rich dark rock in the form of flat plains, without very many large impact craters. Singular of maria = mare (pronounced MAH-ray)

Rilles (mostly in maria and crater bottoms): straight rifts that can be hundreds of kilometers long

Scarps (mostly in between craters): cliffs, usually rather shallow and scalloped compared to crater edges

Highlands (dominating the south and the far side): relatively light-colored rock in the form of mountains as high as 16 km above the maria

Canyons: surface features from seismic fracturing and collapse of material, resembling the erosional features with the same name on Earth

6 September 2018

MAIN SURFACE FEATURES

Craters (all over the place): scars from meteorite or planetesimal impact. Some exhibit rays radiating from them that are clearly splashes left over from the impact.

Crater chains: linear arrangements of closely-spaced craters, the results of secondary impacts of debris from a large primary impact

No volcanoes or volcanic craters, which would exhibit sharp cliff-like walls with flat, ash-filled floors, or cones with flat tops on steep sides.

“No” atmosphere: the pressure at the surface is only $1.3 \times 10^{-15}$ atmospheres, mostly from outgassing of noble gasses (Ar, He,...) trapped in the rocks since the Moon formed.

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UNUSUAL FEATURES

Synchronous rotation: the Moon’s revolution and rotation periods are equal.

As we will learn later this term, this condition, called tidal locking, is due to the conversion of the Moon’s rotational energy to heat (radiated away!), a process driven by the Earth’s tides on the Moon’s interior.

Mascons (dense mantle material pushed up into the crust) and other moment-of-inertia anomalies: the center of mass is about 1.7 km closer to the Earth than the center of the Moon’s nearly spherical surface.

The Moon is covered with a dusty “soil” many meters deep, produced in meteorite impacts. Common to airless, rocky planets and planetesimals, this “soil” is called the regolith.

INTERIOR STRUCTURE OF THE MOON

Determined by seismology, as on Earth: propagation speed of seismic waves depends on the local mass density \( \rho \), and different sorts of seismic waves penetrate to different depths.

- **Core**: \( \rho = 7.2 \text{ g cm}^{-3} \), radius \( R = 340 \text{ km} \). Probably partially-melted nickel-iron.
- **Lithosphere (crust)**: \( \rho = 3 \text{ g cm}^{-3} \), about 50 km thick.
- **Asthenosphere (mantle)**: \( \rho = 3.3 \text{ g cm}^{-3} \).
CRATERS

The larger larger lunar craters tend to have peaks in their centers.

- Because the centers are lower than the ground around the craters, they must be due to impact, not volcanism.
- The central peaks are due to the rebound of impact-induced seismic waves at the crust-mantle interface, where the density (and wave speed) changes abruptly.

The craters all look quite circular, despite the fact that the projectiles which made them came from all directions, not just straight down.

- It turns out that craters come out circular no matter what the angle of the collision.

CRATER IMPACT SIMULATION

Simulation by Kai Kadau, Duisberg U.
THE AGE OF CRATERED SURFACES

Most of the Moon’s craters are very old; many can be seen to be overlapped by new craters.

Because planetesimals around the Solar System – the creators of the craters – get less abundant as they are swept up by larger bodies, more heavily cratered surfaces must be older than smoother ones.

The age of any surface – that is, the time since it last solidified – can be determined by radioisotope-dating measurements on rocks from the surface. Moon rocks returned by the Apollo missions have allowed us to calibrate the ages in several spots. Lots more on how this works next class and in Problem Set 3.

The relative ages of different parts of the Lunar surface can be gauged by measuring the surface density (number per unit area) of craters (Problem Set 2):

- For example, if craters on the Moon formed once per km\(^2\) per 1000 years, and if we counted \(10^6\) craters per km\(^2\) in a given lunar region, we would infer that the age of the region is \(10^9\) years.

AGES OF CRATERED SURFACES

The maria are relatively crater-free; therefore, they must be substantially younger than the heavily-cratered highlands.

- That is to say, the places where there are now maria were melted, or more likely resurfaced by large-scale lava flows, a long time after the highlands last solidified.

As you have seen, nearly all of the maria are on the near side of the Moon.

- Thus, the large-scale lava-flow resurfacing must have something to do with the synchronous rotation and the decrease from the Moon’s original spin rate, but no one knows precisely what that link is.
THE TEMPERATURE OF THE MOON

Suppose the Moon is heated only by sunlight and cooled only by its own blackbody radiation. What would its temperature be?

To calculate this, we first need the flux (power per unit area) of sunlight at the Moon’s distance from the Sun ($r = 1$ AU, like the Earth)

$$ f = \frac{L_\odot}{4\pi r^2} $$

The Moon casts a circular shadow with radius $R = 1738$ km, for which the area is

$$ A = \pi R^2 $$

THE MOON’S TEMPERATURE

Thus, the power intercepted by the Moon, if we consider it to be black, is

$$ P_{\text{in}} = fA = \frac{L_\odot R^2}{4r^2} $$

The power the Moon gives off as blackbody radiation is

$$ P_{\text{out}} = f'A' = \sigma T^4 4\pi R^2 $$

Energy is conserved, so

$$ \frac{L_\odot R^2}{4r^2} = \sigma T^4 4\pi R^2 $$

$$ T = \left( \frac{L_\odot}{16\pi \sigma r^2} \right)^{1/4} = 278 \text{ K} $$
THE MOON’S TEMPERATURE

This is not far from what we measure with a thermometer!

The temperature at the equator at mid-day is about 380 K and at midnight is about 120 K. The global average temperature is 274 K.

The Moon does not seem likely to have an internal source of heat.

MINERALS AND ROCKS

Minerals are crystalline inorganic compounds which, in the form of small bits, are the building blocks of rocks and, in large, pure, defect-free form, are collected as gems.

Rocks are solid polycrystalline substances, usually made of a mixture of several minerals. Their basic types:

- Igneous rocks: formed from a melt and cooled slowly enough to form minerals
- Sedimentary rocks: formed from tiny-particle debris of other rocks, deposited in close-packed form by sedimentation in water, and rendered hard over time by pressure of overlying rock
- Metamorphic rock: igneous or sedimentary rock that has been subjected to sufficiently high temperatures and pressures for substantial plastic deformation (though not melting)

The Moon is all igneous rock, except maybe the core.
SILICATE MINERALS

Hundreds of minerals have been identified in Moon rocks (thousands in Earth rocks). Among them, the most abundant family of minerals by far is the silicate group.

Silicates are minerals for which the basic unit of the crystal structure is the tetrahedral molecule SiO$_4$.

Silicates are abundant because silicon and oxygen are abundant, and because the SiO$_4$ unit is very robust.

Carbon is abundant too, of course, and the CO$_4$ unit is robust as well, leading to a family of minerals called the carbonates.

But much more of the carbon remains in other molecular forms, so carbonates wind up less abundant than silicates.

ABUNDANCES OF THE ELEMENTS IN THE SUN

Data from the Clemson University Nuclear Astrophysics Group
ABUNDANCES OF THE ELEMENTS IN THE SUN

Data from the Clemson University Nuclear Astrophysics Group

The most important components of minerals

Important for radioisotope dating