Radioisotope dating
The age of moon rocks
Properties of Mercury
Comparison of Mercury and the Moon
Water on Mercury and the Moon
Tidal locking and Mercury’s eccentric orbit
Moment of inertia and differentiation in terrestrial planets
IMPORTANT EXAMPLE: THE RB-SR SYSTEM

Rubidium (Rb) is an alkali; it can replace the much-more-abundant sodium and potassium in minerals (e.g. feldspars). It has one stable isotope, $^{85}$Rb, and one long-lived radioisotope, $^{87}$Rb.

Strontium (Sr) is an alkaline and can replace magnesium and calcium in feldspars. It has four stable isotopes: $^{84}$Sr, $^{86}$Sr, $^{87}$Sr, and $^{88}$Sr. $^{87}$Sr is fairly rare.

$^{87}$Rb beta-decays into $^{87}$Sr:

$$^{87}\text{Rb} \rightarrow ^{87}\text{Sr} + e^- + \bar{\nu}_e + \text{energy}$$

Commonly used:

$$N = \frac{n}{S} = \frac{^{87}\text{Rb}}{^{86}\text{Sr}}$$

$$D = \frac{d}{S} = \frac{^{87}\text{Sr}}{^{86}\text{Sr}}$$

THE USE OF RADIONUCLIDES TO FIND OUT HOW LONG AGO AN IGNEOUS ROCK WAS LAST MELTED

There are many radioisotopes with half-lives spread from thousands to billions of years, all accurately and precisely measured in the laboratory.

We can measure the abundances of stable and radioactive nuclides “simply” by taking rocks apart into the minerals of which they are made, and in turn taking the minerals apart into atoms, counting the number for each element and isotope in a mass spectrometer.

This gives values of $N$ and $D$, a pair for each mineral. Plot the $D$s against the $N$s: the slope of the resulting line depends upon how many half-lives have passed since it froze, and the intercept depends upon the initial relative abundance of the daughter nuclide.
ISOCHRON DIAGRAM

Minerals after aging by a fixed number of half lives

Measure slope to find age. Measure intercept to find initial relative abundance of the daughter nuclide.

Minerals after freezing

$D = n^{87\text{Sr}}/n^{86\text{Sr}}$, for example

$N = n^{87\text{Rb}}/n^{86\text{Sr}}$, for example

Same $D$: daughter and stable ref. are chemically identical

EVOLUTION OF ISOCHRON DIAGRAM

Simulation by Jon Fleming
A SIMPLE TWO-MINERAL SYSTEM

Call the initial relative abundances $N_0$ and $D_0$. The relative abundance of daughter nuclides as a function of time is:

$$D = D_0 + (N_0 - N) = D_0 + N(e^{At} - 1)$$

Suppose we have a rock consisting of two minerals, $A$ and $B$, with equal initial relative abundances of the daughter nuclide. We can measure the present abundances of $A$ and $B$:

$$D_A = D_0 + N_A(e^{At} - 1) \quad D_B = D_0 + N_B(e^{At} - 1)$$

This can easily be solved for $t$ in terms of measurable quantities:

$$t = \frac{1}{\lambda} \ln \left( \frac{D_A - D_B}{N_A - N_B + 1} \right)$$

EXAMPLE TWO MINERAL SYSTEM

The rate at which $^{87}\text{Rb}$ decays into $^{87}\text{Sr}$ is

$$\lambda = 1.39 \times 10^{-11} \text{ yr}^{-1}$$

Samples of two minerals from the same plutonic rock from northern Ontario are analyzed in a mass spectrometer, with these results:

<table>
<thead>
<tr>
<th>Sample</th>
<th>$^{87}\text{Rb}/^{86}\text{Sr}$</th>
<th>$^{87}\text{Sr}/^{86}\text{Sr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0755</td>
<td>0.7037</td>
</tr>
<tr>
<td>2</td>
<td>0.3280</td>
<td>0.7133</td>
</tr>
</tbody>
</table>

How old is the rock?
EXAMPLE TWO MINERAL SYSTEM

Solution:

\[ t = \frac{1}{\lambda} \ln \left( \frac{D_A - D_B}{N_A - N_B} + 1 \right) \]

\[ = \frac{1}{1.39 \times 10^{-11} \text{yr}^{-1}} \times \ln \left( \frac{0.7133 - 0.7037}{0.328 - 0.0755} + 1 \right) \]

\[ = 2.7 \times 10^9 \text{yr} \]

The y-intercept gives the value of \( D \) that the rock had at the time it froze:

\[ D_0 = \frac{d}{s} = \frac{87 \text{Sr}}{86 \text{Sr}} = 0.7008 \]

RESULTS FOR EARTH AND MOON

The Moon began to solidify about 4.5 billion years ago.

The highlands are clearly older than the maria, as the cratering records also show.

The Moon solidified long before the Earth did.
VISITS TO MERCURY

There have been two:
- NASA’s MESSENGER (MErcury Surface, Space ENvironment, GEochemistry and Ranging) Satellite orbited with Mercury for about four years before crashing into the planet’s surface on April 30, 2015.

And BepiColombo (ESA/JAXA) is on its way (expected to arrive in 2025).

MERGENCY’S VITAL STATISTICS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$3.302 \times 10^{26}$ g ($0.055 M_\oplus$)</td>
</tr>
<tr>
<td>Equatorial radius</td>
<td>$2.4397 \times 10^8$ cm ($0.383 R_\oplus$)</td>
</tr>
<tr>
<td>Average density</td>
<td>5.427 g/cm$^3$</td>
</tr>
<tr>
<td>Albedo</td>
<td>0.106</td>
</tr>
<tr>
<td>Average surface temperature</td>
<td>442.5 K</td>
</tr>
<tr>
<td>Orbital semimajor axis</td>
<td>$5.791 \times 10^{12}$ cm ($0.387$ AU)</td>
</tr>
<tr>
<td>Orbital eccentricity</td>
<td>0.2056</td>
</tr>
<tr>
<td>Sidereal revolution period</td>
<td>87.908 days</td>
</tr>
<tr>
<td>Sidereal rotation period</td>
<td>58.6462 days</td>
</tr>
</tbody>
</table>
MOON VS. MERCURY

From the appearance, albedo, and reflectance spectra, we conclude that the surfaces are similar in composition: pyroxene–bearing rocks (no feldspar).

From the density, we conclude that Mercury has more than its share of iron in the form of a core with a radius about ¾ that of the planet.

From the magnetic field, we conclude that Mercury’s core is probably liquid for a dynamo mechanism to operate.

Diagrams by UCAR (U. Michigan)
**PLANETARY BODY TEMPERATURES**

All planetary bodies gradually cool over time as a result of heat transfer and radiative loss, because internal heat sources cannot be replenished.

This causes the **lithosphere** (outer, rigid layer of the mantle that conducts heat via conduction) to thicken as the size of the **asthenosphere** (inner layer of the mantle that conducts heat via convection) shrinks.

- Proceeds more rapidly for smaller bodies
- Volcanic and tectonic activities will also diminish

If the cooling proceeds to the point where the asthenosphere no longer convects, then the entire mantle becomes the lithosphere.

The timing of this process depends on the body’s composition and size.

---

**TECTONIC ACTIVITY: MERCURY V. MOON**

From observations of rilles and scarps, we conclude that both have a “plastic” mantle, but Mercury has been tectonically active and the Moon has not.

Recently, volcanoes have been seen on Mercury within the giant Caloris impact basin. The moon has none.

-MESSENGER (JHU-APL/NASA)-
SURFACE FEATURES OF MERCURY

Mercury’s surface consists of **highlands** and **lowland plains**.
- The highlands consist of heavily cratered areas mixed with intercrater plains, relatively smooth clearings.

Heavily cratered regions contain fewer craters than the Moon, especially at small scales (< 50 km). This is most likely due to early major resurfacing, preferentially filling the smallest craters (the source of the intercrater plains).

WATER ON AIRLESS SOLAR SYSTEM BODIES

It is easy for hydrogen to escape from small bodies like the Moon and Mercury, and easy for sunlight to dissociate water into H and O. So, is there any water at or just underneath the surface of Mercury and the Moon? There are two good reasons to think that there should be, despite the difficulties:

1. **The solar wind.** The Sun is constantly spewing out about $10^{-14} M_\odot/yr$ in the form of the Solar wind, mostly as protons and electrons.
   - Traveling at high speeds, the protons bury themselves several cm below an airless surface that they encounter.
   - It does not take long for each proton to become H, nor for two of these to make water with a nearby O.
   - This is probably the origin of the widespread water detected on comets by the NASA Deep Impact and on the Moon by the ISRO Chandrayaan-1 spacecraft.
   - Quantity: about one liter per ton of rock or regolith. What is closest to the surface seems to come and go with darkness and sunlight.
MOON TEMP, WATER CONTENT

Temperature (left) and water concentration (right) on the Moon one quarter of a lunar day apart. From Jessica Sunshine and the EPOXI team.

Polar Ice: Mercury v. Moon

2. **Comet and asteroid impacts.** In principle, comets and asteroids could deliver larger amounts of water to the Moon and Mercury than the Solar wind can.

- It would not last long in the sunshine on either body – it would evaporate, dissociate, and in short order the hydrogen would escape.
- But because the Moon and Mercury both have very small obliquity (tilt of rotation axis from the orbital axis), there are permanently shaded parts of craters near the north and south poles, where some of the delivered water can last – in Mercury’s case, practically forever.
POLAR ICE: OBSERVATIONS WITH ARECIBO

In 1999, radar astronomers at the Arecibo Observatory made a long-wavelength, radar-polarimetric study of the north pole of Mercury and found that the perpetually-shadowed crater floors were very reflective in a way most easily explained by clean water ice.

For ice detection, this technique is less controversial than other means previously employed on the Moon. But it was shocking nonetheless: ice on Mercury?

POLAR ICE: ARECIBO AND THE MOON

Arecibo also tried to confirm previous “detections” of water ice on the Moon. They were able to show that there is not any surface ice in the Moon’s polar craters.

Recently confirmed from space by the JAXA Kaguya/SELENE and NASA LRO satellites.
POLAR ICE: BELOW THE SURFACE

What about subsurface ice mixed with dirt in the Lunar polar craters? To expose this was the job of NASA's Lunar Crater Observation and Sensing Satellite (LCROSS).

Make a small crater within a permanently-shadowed region with the impact of a spent rocket stage.

Analyze the composition of the ejecta from measurements by satellites directly overhead.

A permanent shadow on the Moon (yellow contour) compared to neutron signal (blue) and the LCROSS impact site (red dot). (NASA and ISR/RAS)

POLAR ICE: RESULTS FROM LCROSS

Results show water ice present at the level of about 50 liters of water per ton of ejecta in this polar-crater floor.

This would be on the high end of soil water content for the driest deserts on Earth, like the Sahara and Atacama.

There is as much molecular hydrogen (H\textsubscript{2}) in the soil as water ice, molecule for molecule. Much of the neutron signal was from this H\textsubscript{2}.

The water ice comes with very large amounts of other volatiles like CO, calcium, magnesium, and mercury.

<table>
<thead>
<tr>
<th>Species</th>
<th>Abundance in ejecta (percent by mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H\textsubscript{2}O</td>
<td>5.4</td>
</tr>
<tr>
<td>H\textsubscript{2}</td>
<td>1.4</td>
</tr>
<tr>
<td>CO</td>
<td>5.7</td>
</tr>
<tr>
<td>Hg</td>
<td>1.2</td>
</tr>
<tr>
<td>Ca</td>
<td>1.6</td>
</tr>
<tr>
<td>Mg</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Colaprete et al. (2010), Gladstone et al. (2010)
POLAR ICE: COMPARISON TO PREVIOUS RESULTS

This amount of water is consistent with the Arecibo radar observations.

There is enough to be scientifically interesting, and the mixture of volatiles constrained models of delivery and the thermal history of the crater floors.

An entire permanently shaded region contains $10^6 - 10^7$ gallons, which would fill a large municipal water tower. This would go a long way toward the needs of a Lunar base, but it must be laboriously extracted and purified.

CHEMICAL COMPOSITION OF MERCURY’S SURFACE

Mercury formed from materials with less oxygen than those that formed other terrestrial planets

- Surface depleted in Fe and Ti by an order of magnitude
- Mg/Si ratio is 2-3 times higher than the Moon and other terrestrial planets
- Al/Si and Ca/Si are half as large
- Extremely high amount of S/Si (volatile)
TIDAL LOCKING AND MERCURY'S ORBIT

In celestial mechanics, tidal locking means that the heat dissipated during an orbit, by variations in the tidal forces, is minimized.

Tidal forces are the gravitational force differences between the near and far sides of a body due to their different distances away from a second body, and from one side to another of a body due to different directions toward the second body.

The result is a stretch along the line between the two bodies and a compression in the perpendicular directions.

We will derive formulas for tidal forces — and much more on tides — later this semester.

AN INTRODUCTION TO TIDAL FORCES

Earth pulls all things toward its center, and pulls harder on things which are closer.

To you, it seems as if you are stretched in the direction of the Earth’s center and squeezed in the other direction.

Figure from Thorne, Black Holes and Time Warps
TIDAL LOCKING & THE MOON’S ORBIT

A body in circular orbit, with rotation period equal to revolution period, suffers no change in tidal forces as it travels its orbit. Thus, there is no heat dissipated from stretching and shrinking.

The Moon’s orbit about Earth has a fairly small eccentricity (0.055), so it can be considered circular to good approximation.

The Moon’s revolution and revolution periods are identical, so it is tidally locked.

MERCURY’S ELLIPTICAL ORBIT

Mercury’s orbit has a much larger eccentricity (0.2056) and thus is more distinctly elliptical.

Reminder of the properties of ellipses:

<table>
<thead>
<tr>
<th>Equation</th>
<th>( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foci at ( x = \pm c )</td>
<td>( c = \sqrt{a^2 - b^2} )</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>( e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} )</td>
</tr>
</tbody>
</table>
| Focal lengths (aphelion and perihelion distances) | \( f_a = a + c = a(1 + e) \)  
\( f_p = a - c = a(1 - e) \) |