Astronomy 102: Black Holes, Time Warps, and the Large-Scale Structure of the Universe

What do black holes, wormholes, time warps, spacetime curvature, hyperspace and the Big Bang have in common?

Explanations with their origins in Einstein’s theories: the special theory of relativity (1905) and the general theory of relativity (1915).

This semester we will discuss all of these exotic phenomena, mostly qualititatively, in the context of Einstein’s theories.
Our primary goals in teaching Astronomy 102

- to demystify black holes, the Big Bang, and relativity, so you can evaluate critically the things you find about them in the media;
- to show you how scientific theories are conceived and advanced in general.

In doing so we aim primarily at non-science majors.

This semester’s AST 102 class, plotted by concentration.
Our primary goals in teaching Astronomy 102 (continued)

We hope that by the end of the course you will understand and retain enough to be able to offer correct explanations of black holes and such to your friends and family, and that you will retain a permanent, basic understanding of how science works.

This semester’s AST 102 class, plotted by cohort.
Human features of Astronomy 102

People:
- Laura Arnold, lecturer
- Dan Watson, professor
- William Bock, teaching assistant
- Theodore Lane, teaching assistant
- You, student

Expectations:
- Be respectful!
- Get to know your classmates and work with them.
- Ask questions.
- Seek assistance if needed.
Electronic features of Astronomy 102

- **Computer-projected lectures**, for greater ease in presentation of diagrams, astronomical images and computer simulations, and for on-line accessibility on our...

- **Web site**, including all lecture presentations, schedule, practice exams, much more.
  - **Primary reference for course.**

- **Personal response system**, for in-lecture problem-solving. *(Required; available at the UR Bookstore.)*

- **WeBWorK**, a computer-assisted personalized homework and exam generator.
Printed (or possibly electronic) features of AST 102

Textbooks (one required, three recommended)

Onerous features of Astronomy 102

- The minimum of mathematics required to tell our story
- Six problem sets, all using WeBWorK, comprise 15% of your grade.
- Three exams, also all using WeBWorK, comprise 80% of your grade (but no comprehensive final exam).

But grades are assigned on a straight scale, not a curve.
90% of success is showing up.

All members of the class are expected to attend all of the lectures, and encouraged to attend one recitation per week.

- Class participation accounts for a small part of your grade (5%) and is based upon answering in-class PRS questions.
- You will very probably get a better grade if you go to class, as is demonstrated by these average test score and average attendance data from past AST 102 classes.
- You may attend any recitation you like, whether you’re registered for it or not.
Mid-Lecture Break

This will be a regular feature of Astronomy 102 lectures.

- Homework #1 soon to be on WeBWork, due 10 September at 5:30 PM

During the break, please **turn on** your PRS clicker, wait for it to find the course **AST102-01**, and then press the green-arrow Enter key (            ) to join the class PRS session.

Test PRS question

In what city would you rather live?

A. Boston           B. Chicago               C. New York
D. San Francisco           E. Washington

1 September 2011
Today in Astronomy 102: How big is that?

Before discussing black holes, the Big Bang, and other celestial objects and phenomena, we need to become

- familiar with distances, time scales, masses, luminosities and speeds of astronomical importance, and
- proficient at **unit conversion**.

---

Million light years

1 2 3 4 5
## Sizes and distances in astronomy

<table>
<thead>
<tr>
<th></th>
<th>centimeters</th>
<th>kilometers</th>
<th>miles</th>
<th>light years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of a hydrogen atom</td>
<td>$1.1 \times 10^{-9}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter of a human hair</td>
<td>$8.0 \times 10^{-3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter of a penny</td>
<td>1.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter of Rochester</td>
<td>$2.0 \times 10^6$</td>
<td>20</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Diameter of the Earth</td>
<td>$1.3 \times 10^9$</td>
<td>$1.3 \times 10^4$</td>
<td>$7.9 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>Diameter of the Moon</td>
<td>$3.5 \times 10^8$</td>
<td>$3.5 \times 10^3$</td>
<td>$2.1 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>Diameter of Jupiter</td>
<td>$1.4 \times 10^{10}$</td>
<td>$1.4 \times 10^5$</td>
<td>$8.8 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>Diameter of the Sun</td>
<td>$1.4 \times 10^{11}$</td>
<td>$1.4 \times 10^6$</td>
<td>$8.6 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>Diameter of the Milky Way galaxy</td>
<td>$1.6 \times 10^{23}$</td>
<td></td>
<td></td>
<td>$1.7 \times 10^5$</td>
</tr>
<tr>
<td>Distance to Buffalo</td>
<td>$1.0 \times 10^7$</td>
<td>100</td>
<td>62</td>
<td></td>
</tr>
<tr>
<td>Distance to the Moon</td>
<td>$3.8 \times 10^{10}$</td>
<td>$3.8 \times 10^5$</td>
<td>$2.4 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>Distance to the Sun</td>
<td>$1.5 \times 10^{13}$</td>
<td>$1.5 \times 10^8$</td>
<td>$9.2 \times 10^7$</td>
<td></td>
</tr>
<tr>
<td>Distance to the next nearest star, $\alpha$</td>
<td>$3.8 \times 10^{18}$</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Centauri</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance to the center of the Milky Way</td>
<td>$2.6 \times 10^{22}$</td>
<td></td>
<td></td>
<td>$2.7 \times 10^4$</td>
</tr>
<tr>
<td>Distance to the nearest galaxy</td>
<td>$1.6 \times 10^{23}$</td>
<td></td>
<td></td>
<td>$1.7 \times 10^5$</td>
</tr>
</tbody>
</table>
Typical lengths and important conversions

- Diameter of normal stars: millions of kilometers (km)
- Distance between stars in a galaxy: a few light-years (ly)
- Diameter of normal galaxies: tens of kilo-light-years (kLly)
- Distances between galaxies: a million light-years (Mly)
- 1 ly = $9.46052961 \times 10^{17}$ cm = $9.46052961 \times 10^{12}$ km
  = $5.87862537 \times 10^{12}$ miles ≈ 6 trillion miles
- 1 km = $10^5$ cm; 1 kly = $10^3$ ly; 1 Mly = $10^3$ kly = $10^6$ ly.

**Example:** The Andromeda galaxy (a galaxy a lot like our Milky Way) lies at a distance $D = 2.5$ Mly. How many centimeters is that?

$$D = 2.5 \text{ Mly} \times \frac{10^6 \text{ ly}}{1 \text{ Mly}} \times \frac{9.46 \times 10^{17} \text{ cm}}{1 \text{ ly}} = 2.4 \times 10^{24} \text{ cm}$$
More detail on Unit Conversion

Previous example: repeated multiplication by 1. One may always multiply anything by 1 without changing its real value.

The unit conversions always give a couple of useful forms of 1. Take, for example, the conversion $1 \text{ ly} = 9.46 \times 10^{17} \text{ cm}$:

$$\frac{9.46 \times 10^{17} \text{ cm}}{1 \text{ ly}} = 1 = \frac{1 \text{ ly}}{9.46 \times 10^{17} \text{ cm}}$$

Choose forms of 1 that cancel out the units you want to get rid of, and that insert the units to which you wish to convert. This sometimes takes repeated multiplication by 1, as in the previous example:

$$D = 2.5 \text{ Mly} \times \frac{10^{6} \text{ ly}}{1 \text{ Mly}} \times \frac{9.46 \times 10^{17} \text{ cm}}{1 \text{ ly}} = 2.4 \times 10^{24} \text{ cm}$$
More detail on numerical answers

Note the answer was written as $2.4 \times 10^{24}$ cm.

- Not just $2.4 \times 10^{24}$. Numerical answers in the physical sciences and engineering are incomplete without units.
- And not $2.36513240 \times 10^{24}$ cm, even though that’s how your calculator would put it. Numerical answers should be rounded off: **display no more than one more significant figure than the least precise input number**.

- If we had been told that the distance to the Andromeda galaxy is 2.5000000 Mly, then the conversion factor would have to have been put in with more significant figures, and the right answer would have been $2.36513240 \times 10^{24}$ cm.
There is nothing sacred about centimeters, grams and seconds.

Units are generally chosen to be convenient amounts of whatever is being measured. Examples:

- The **light-year (ly)** is far more convenient than the centimeter for expression of length in astronomy; on large scales we even use millions of ly (Mly).
- The convenient unit of **mass** in astronomy is the **solar mass**: the mass of the Sun.

Values of physical quantities are **ratios** to the values of the unit quantities.
How is that so far?

How comfortable are you, with these concepts, and doing these calculations?

A. Pretty uncomfortable  B. OK but need reminders
C. Perfectly comfortable  D. Expert  See you on Thursday
# Masses in astronomy

<table>
<thead>
<tr>
<th></th>
<th>Grams</th>
<th>Pounds</th>
<th>Solar masses $(M_\odot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen atom</td>
<td>$1.67 \times 10^{-24}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Penny (uncirculated)</td>
<td>3.2</td>
<td>0.0071</td>
<td></td>
</tr>
<tr>
<td>Ton</td>
<td>$1.02 \times 10^6$</td>
<td>2240</td>
<td></td>
</tr>
<tr>
<td>Earth</td>
<td>$6.0 \times 10^{27}$</td>
<td>$1.3 \times 10^{25}$</td>
<td>$3.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Moon</td>
<td>$7.4 \times 10^{25}$</td>
<td></td>
<td>$3.7 \times 10^{-8}$</td>
</tr>
<tr>
<td>Jupiter</td>
<td>$1.9 \times 10^{30}$</td>
<td></td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Sun</td>
<td>$2.0 \times 10^{33}$</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Milky Way galaxy</td>
<td>$6 \times 10^{45}$</td>
<td></td>
<td>$3 \times 10^{12}$</td>
</tr>
</tbody>
</table>
Typical masses and important conversions

- Smallest stars: 0.08 solar masses ($M_\odot$)
- Normal stars: around one $M_\odot$
- Giant stars: tens of $M_\odot$
- Normal galaxies: $10^{11} - 10^{12} M_\odot$
- Clusters of galaxies: $10^{14} - 10^{15} M_\odot$
- $1 M_\odot = 2.0 \times 10^{33}$ grams = solar mass = mass of the Sun
- 1 pound = 454 grams

Example: Vega, the brightest star in the Northern sky, has a mass of about $2.5 M_\odot$. What is its mass in grams?

$$M = 2.5 M_\odot \times \frac{2 \times 10^{33} \text{ gm}}{1 M_\odot} = 5.0 \times 10^{33} \text{ gm}$$
How many Earths in the Sun?

That is, by what factor is the Sun more massive than the Earth?

Find the masses a couple of pages back, work it out and send the answer on your clicker.
## Times and ages in astronomy

<table>
<thead>
<tr>
<th></th>
<th>seconds</th>
<th>hours</th>
<th>days</th>
<th>years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth’s rotation period</td>
<td>$8.64 \times 10^4$</td>
<td>24</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Moon’s revolution period</td>
<td>$2.3606 \times 10^6$</td>
<td>655.73</td>
<td>27.322</td>
<td></td>
</tr>
<tr>
<td>Earth’s revolution period</td>
<td>$3.1558 \times 10^7$</td>
<td>$8.7661 \times 10^3$</td>
<td>365.25</td>
<td>1</td>
</tr>
<tr>
<td>Century</td>
<td>$3.16 \times 10^9$</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Recorded human history</td>
<td>$1.6 \times 10^{11}$</td>
<td></td>
<td></td>
<td>5000</td>
</tr>
<tr>
<td>Milky Way Galaxy’s</td>
<td>$7.5 \times 10^{15}$</td>
<td></td>
<td></td>
<td>$2.4 \times 10^8$</td>
</tr>
<tr>
<td>rotation period (at Sun’s orbit)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age of the Sun and Earth</td>
<td>$1.44 \times 10^{17}$</td>
<td></td>
<td></td>
<td>$4.56 \times 10^9$</td>
</tr>
<tr>
<td>Total lifetime of the Sun</td>
<td>$4.7 \times 10^{17}$</td>
<td></td>
<td></td>
<td>$1.5 \times 10^{10}$</td>
</tr>
<tr>
<td>Age of the Universe</td>
<td>$4.4 \times 10^{17}$</td>
<td></td>
<td></td>
<td>$1.4 \times 10^{10}$</td>
</tr>
</tbody>
</table>
Typical timespans and important conversions

- Planetary revolution period: around 1 year
- Life expectancy, normal stars: around $10^{10}$ years
- Life expectancy, giant stars: $10^6 - 10^8$ years
- Rotation period of normal galaxies: $10^7 - 10^9$ years
- 1 year = $3.16 \times 10^7$ seconds
- 1 hour = 3600 seconds

**Example:** How many seconds is a typical human lifespan (US)?

$$t = 75 \text{ years} \times \frac{3.16 \times 10^7 \text{ seconds}}{1 \text{ year}} = 2.37 \times 10^9 \text{ seconds}$$
The fundamental dimensions

Distance, time and mass are fundamental dimensions.

- Distances along each of the three different perpendicular directions of space determine the location of a given body with respect to others.
- Time determines the instant in the given body has that location.
- A given body’s mass determines how strongly the force of gravity influences it.
- Each given body has an additional fundamental dimension like mass, corresponding to each of the forces of nature. Electric charge, for example, dictates how strongly the electrostatic force influences a given body.
The fundamental dimensions (continued)

The dimensions of all other physical quantities are combinations of these fundamental dimensions.

- For instance: the dimension of velocity, and velocity’s magnitude speed, is distance divided by time, as you know.

- The dimension of energy is mass times distance squared, divided by time squared.
  - i.e. mass times the square of the dimension of speed

- Units are the scales of the quantities that go with the qualities that are dimensions.

Thus: four fundamental dimensions for location (three space, one time), and in principle four for response to forces (gravity, electricity, and the strong and weak nuclear forces).
### Speeds in astronomy

<table>
<thead>
<tr>
<th>Description</th>
<th>cm per second</th>
<th>km per second</th>
<th>miles per hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>NYS Thruway speed limit</td>
<td>3.0×10^3</td>
<td>3.0×10^{-2}</td>
<td>65</td>
</tr>
<tr>
<td>Earth’s rotational speed at the equator</td>
<td>4.7×10^4</td>
<td>0.47</td>
<td>1050</td>
</tr>
<tr>
<td>Speed of Earth in orbit</td>
<td>3×10^6</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Speed of Sun in orbit around center of Milky Way</td>
<td>2.5×10^7</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>Speed of Milky Way with respect to local Universe</td>
<td>5.5×10^7</td>
<td>550</td>
<td></td>
</tr>
<tr>
<td>Speed of light</td>
<td>2.9979×10^{10}</td>
<td>2.9979×10^5</td>
<td></td>
</tr>
</tbody>
</table>
Typical speeds and important conversions

- Planetary orbits in a solar system: tens of km/s
- Stellar orbits in a normal galaxy: hundreds of km/s
- Speed between nearby galaxies: hundreds of km/s
- Speed of light: $2.99792458 \times 10^{10}$ cm per second
- Conversion factors: use those given for distance and time.

**Example:** One mile is equal to 1.61 kilometers. What is the speed of light in miles per hour?

\[
c = 2.9979 \times 10^{10} \frac{\text{cm}}{\text{sec}} \times \frac{\text{km}}{10^5 \text{ cm}} \times \frac{\text{mile}}{1.61 \text{ km}} \times \frac{3600 \text{ sec}}{\text{hour}}
\]

\[
= 6.70 \times 10^8 \frac{\text{mile}}{\text{hour}} \quad (670 \text{ million miles per hour})
\]
Work, heat and energy in astronomy

Hydrogen atom binding energy: $1.6 \times 10^{-12}$ erg

Dietary calorie: $4.2 \times 10^{10}$ erg

Burn 1 kg anthracite coal: $4.3 \times 10^{14}$ erg

Detonate H bomb (1 megaton): $4.2 \times 10^{22}$ erg

Earth-Sun binding energy: $5.3 \times 10^{40}$ erg

Sun’s fuel supply at birth: $2 \times 10^{51}$ erg

Supernova (exploding star): $10^{53}$ erg
Units of energy

In AST 102 our usual unit of energy will be the erg:

\[
1 \text{ erg} = \frac{1 \text{ gram} \times (1 \text{ cm})^2}{(1 \text{ second})^2} = \text{gm cm}^2 \text{ sec}^{-2}
\]

which is the unit of energy in the CGS (centimeter-gram-second) system of units.

Possibly you are more familiar with the International System (SI, a.k.a. MKS for meter-kilogram-second) unit of energy, the joule:

\[
1 \text{ joule} = \frac{1 \text{ kg} \times (1 \text{ m})^2}{(1 \text{ second})^2} = \text{kg m}^2 \text{ sec}^{-2} = 10^7 \text{ erg}
\]

The others we have listed will find some uses too.
### Luminosity (total power output) in astronomy

<table>
<thead>
<tr>
<th>Description</th>
<th>Luminosity (ergs per second)</th>
<th>Power (watts)</th>
<th>Solar Luminosities</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 W light bulb</td>
<td>$1.0 \times 10^9$</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>150 horsepower car engine</td>
<td>$1.2 \times 10^{12}$</td>
<td>$1.2 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>Large city</td>
<td>$10^{15}$</td>
<td>$10^8$</td>
<td></td>
</tr>
<tr>
<td>H bomb (1 megaton, 0.01 second)</td>
<td>$4.2 \times 10^{21}$</td>
<td>$4.2 \times 10^{14}$</td>
<td>$1.1 \times 10^{-12}$</td>
</tr>
<tr>
<td>Sun</td>
<td>$3.8 \times 10^{33}$</td>
<td>$3.8 \times 10^{26}$</td>
<td>1</td>
</tr>
<tr>
<td>Largest stars</td>
<td>$4 \times 10^{38}$</td>
<td>$4 \times 10^{31}$</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Milky Way galaxy</td>
<td>$8 \times 10^{43}$</td>
<td></td>
<td>$2 \times 10^{10}$</td>
</tr>
<tr>
<td>3C 273 (a typical quasar)</td>
<td>$4 \times 10^{45}$</td>
<td></td>
<td>$10^{12}$</td>
</tr>
</tbody>
</table>

For the astronomical objects, the power is emitted mostly in the form of light; hence the name.
Typical luminosities and important conversions

- Normal stars: around one solar luminosity \( (L_\odot) \)
- Giant stars: thousands to hundreds of thousands of \( L_\odot \)
- Normal galaxies: \( 10^9 - 10^{10} \) \( L_\odot \)
- Quasars: \( 10^{12} - 10^{13} \) \( L_\odot \)
- \( 1 \ L_\odot = 3.8 \times 10^{33} \ \text{erg/s} = \text{luminosity of the Sun} \)
- \( 1 \ \text{watt} = 10^7 \ \text{erg/s} \)

**Example:** Vega, the brightest star in the Northern summer sky, has a luminosity of about \( 1.9 \times 10^{35} \ \text{erg/s} \). What’s that in solar luminosities?

\[
L = 1.9 \times 10^{35} \ \text{erg/s} \times \frac{1 \ L_\odot}{3.8 \times 10^{33} \ \text{erg/s}} = 50 \ L_\odot
\]
Rates

Speed and luminosity are examples of rates.

- **Speed** $v$ is the rate of change of position $x$ with time $t$:

  $$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v \Delta t \text{ if } v \text{ is constant.}$$

  - **position interval**
  - **time interval**

- **Luminosity** $L$ is the rate of change of energy $E$ with time $t$:

  $$L = \frac{\Delta E}{\Delta t} \Rightarrow \Delta E = L \Delta t \text{ if } L \text{ is constant.}$$
Speed as a rate

**Example:** The radius of the Earth’s orbit around the Sun is $1.5 \times 10^{13}$ cm. What is its orbital speed (assumed constant)?

\[
v = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{\Delta t} = \frac{2 \times 3.14159 \times (1.5 \times 10^{13} \text{ cm})}{1 \text{ year}} \times \left(\frac{1 \text{ year}}{3.16 \times 10^7 \text{ seconds}}\right)
\]

\[
= 3.0 \times 10^6 \text{ cm/sec} \times \left(\frac{\text{km}}{10^5 \text{ cm}} \times \frac{1 \text{ mile}}{1.61 \text{ km}} \times \frac{3600 \text{ sec}}{\text{hour}}\right) = 66,800 \text{ mph}.
\]

**Example:** How long should it take to get to Buffalo from here, at the Thruway speed limit?

\[
\Delta t = \frac{\Delta x}{v} = \frac{60 \text{ miles}}{65 \text{ miles/hour}} = 0.92 \text{ hour} \times \left(\frac{60 \text{ minutes}}{\text{hour}}\right) = 55 \text{ minutes}.
\]
Now you try, with PRSs

There are eight furlongs in a mile, and two weeks in a fortnight. Suppose we take the furlong to be our unit of length, and a fortnight to be our unit of time.

Then, what are the units of speed?
A. Furlong fortnights  
B. Fortnights per furlong  
C. Furlongs per fortnight  
D. Furlongs per second.
And again.

There are eight furlongs in a mile, and two weeks in a fortnight. Suppose we take the furlong to be our unit of length, and a fortnight to be our unit of time.

What is the NYS Thruway speed limit in this new system of units?
A. 1.5 furlongs per fortnight
B. $1.5 \times 10^5$ furlongs per fortnight
C. $8 \times 10^{-4}$ furlong fortnights
D. 42 fortnights per furlong
Luminosity as a rate

**Example**: How long could the Sun live at its current luminosity, considering the fuel supply with which it was born?

\[
\Delta t = \frac{\Delta E}{L} = \frac{2 \times 10^{51} \text{ erg}}{3.8 \times 10^{33} \text{ erg/sec}} = 5.3 \times 10^{17} \text{ sec} \times \left( \frac{\text{year}}{3.16 \times 10^7 \text{ sec}} \right)
\]

\[
= 1.7 \times 10^{10} \text{ years} \quad (17 \text{ billion years}).
\]

It has already lived 4.56 billion years.

**Example**: What is your “luminosity” in erg/sec, if you eat 3000 calories a day and don’t gain or lose weight?

\[
L = \frac{\Delta E}{\Delta t} = \frac{3000 \text{ Cal}}{1 \text{ day}} \times \left( \frac{1 \text{ day}}{86400 \text{ sec}} \times \frac{4.2 \times 10^{10} \text{ erg}}{\text{Cal}} \right) = 1.5 \times 10^9 \text{ erg/sec}.
\]
Remember the How Big Is That sheet

Many important physical quantities that we will use frequently are collected on the “How Big Is That?” sheet, found under the “Constants and Equations” tab on the AST 102 Web site.

- You will always have access to this page while you’re doing homework or exams. Thus you don’t have to memorize all the numbers.

- However, to use the sheet effectively, and to understand our astronomical discussions, you must become familiar enough with them to know about how big most of them are.
  - It would do you good to memorize at least the “typical” values of things, on the previous pages.