Today in Astronomy 102: electron degeneracy pressure and white dwarfs

- Electron degeneracy pressure, relativistic degeneracy pressure, and white dwarf stars.
- The origin of white dwarf stars.
- Planetary nebulae.
- Prevention of black holes, but only those less massive than $1.4 M_\odot$.

Triple-star system 40 Eridani (Dan Purrington, Tulane U.). The leftmost star, 40 Eri B, is a white dwarf.
White dwarf stars

White dwarfs are stars similar in mass and temperature to normal stars, but are much fainter and much smaller - the size of planets. Discovered in 1862, they were a hot topic in astronomy in the 1920s. Thousands are known today.

Sirius, the brightest star in the sky, has a companion star which is a white dwarf. They orbit each other with a period of about 50 years. Sirius A is vastly brighter than Sirius B at visible wavelengths; the contrast is smaller in this X-ray image.

Chandra X-ray Observatory image (NASA/CfA)
Sirius B: a fairly typical white dwarf

- From its measured distance from Sirius A and their orbital period (plus Newton’s laws), we know that the mass of Sirius B is $1.00 \, M\odot$.

- From its observed color (blue-white), we know that its temperature is rather high: $T = 29,200 \, K$, compared to $5800 \, K$ for the Sun and $10000 \, K$ for Sirius A.

- Its luminosity is only $0.003 \, L\odot$, much less than that of Sirius A ($13 \, L\odot$).

- From all of this information, astronomers can work out the diameter of Sirius B; the result is $9.8 \times 10^3 \, km$, slightly smaller than that of the Earth ($1.3 \times 10^4 \, km$).

The mass of a star, in the size of a planet.
Theory of white dwarfs

Fowler applied his theory of degeneracy pressure, soon after he invented it (1926), to white dwarf stars. His result:

- Stars supported by degeneracy pressure instead of gas pressure would have sizes close to that determined from astronomical observations of Sirius B.

Soon thereafter, Edmund Stoner (1929) noticed that this result implies a peculiar relation between mass and size:

- higher-mass degenerate stars are smaller than lower mass ones, the opposite how normal stars behave.
  - Reason: more mass means more pressure is required to balance gravity, and more degeneracy pressure requires more tightly-confined electrons (smaller star).
Stoner’s first white dwarf theory

Measurements of white dwarf mass and circumference by Provencal et al. 1998:
40 Eridani B
Sirius B

Earth’s circumference
Theory of white dwarfs (continued)

- For stars heavier than about a solar mass, Stoner (prompted, but incorrectly, by Anderson) noticed from his theory that the confinement imparted so much energy to the electrons in the center of the star that the electron speeds are close to the speed of light.

- Fowler’s theory of degenerate matter did not take Einstein’s special theory of relativity into account; therefore Stoner (and independently Frenkel) started over to combine relativity and quantum mechanics into a new theory of relativistic degeneracy pressure.

Recall that electrons, like everything else, can’t move faster than light.
Theory of white dwarfs (continued)

- The more massive the degenerate star, the closer the electron speeds get to the speed of light.
- The closer the speeds get to $c$, the harder it is to accelerate the electrons further.
- Thus, the electron degeneracy pressure doesn’t keep increasing as much with tighter confinement: the electrons reach a point where they cannot move any faster. There is a maximum to the electron degeneracy pressure, and a corresponding maximum weight – and maximum mass, about $1.7 M_\odot$ – that degeneracy pressure can support (Stoner 1930,1932; independently by Landau 1931, Chandrasekhar 1931).
- If the weight cannot be supported by electron degeneracy pressure, the degenerate star will collapse to smaller sizes.
Meanwhile, Subrahmanyan Chandrasekhar appeared on the scene, initially as Fowler’s grad student.

- He combined Stoner’s theory of degeneracy pressure with a much more detailed theory of stellar structure, and, armed only with an adding machine, produced a numerical solution correct for all white-dwarf masses (*Chandrasekhar 1935*).

- The result confirmed that white-dwarf circumference decreases smoothly to zero as mass increases to a maximum, which turns out to be $1.44 \, M_\odot$. 

Chandra (AIP)
Chandrasekhar’s white dwarf calculations

Measurements of white dwarf mass and circumference by Provencal et al. 1998:

- 40 Eridani B
- Sirius B

Earth’s circumference

Maximum mass, $1.4 M_\odot$
Learn to read these graphs.

What’s the mass of a white dwarf with circumference $2 \times 10^{10}$ cm?

A. 0.01 solar mass  
B. 0.011 solar mass  
C. 0.02 solar mass  
D. 0.025 solar mass
Learn to read these graphs.
What’s the mass of a white dwarf with circumference $1 \times 10^{10}$ cm?
1. 0.1 solar mass
2. 0.11 solar mass
3. 0.15 solar mass
4. 0.2 solar mass
Mid-lecture break.

- Homework #3 is available on WeBWorK; it’s due 21 October, 5:30PM.

Simulation of the interaction of the white dwarf AE Aquarii B with its normal-star companion, by R. West and G. Wynn, U. Leicester.
How to make a white dwarf star

Start with a normal star like the Sun. Fusion of protons into helium in the star’s center generates heat and pressure that can support the weight of the star. The Sun was mostly made of hydrogen (=1 proton + 1 electron) when it was born, and started with enough hydrogen to last like this for more than 10 billion years.

The Sun is not really yellow, but it’s popular to draw it that way.
PRS quiz:

What color is the Sun?
A. White.
B. Yellow, no matter what you just said.
C. Orange.
D. Red.
How to make a white dwarf star (continued)

When it begins to run out of hydrogen in its center, not enough heat and pressure are generated to balance the star’s weight, so the core of the star gradually begins to collapse.
How to make a white dwarf star (continued)

As the core collapses it gets hotter, though no extra heat has been generated, simply because it compresses. It gets so hot that light from the core causes the outer parts of the star to expand and get less dense, whereupon the star looks cooler from the outside. The star is becoming a **red giant**.
How to make a white dwarf star (continued)

Eventually the core gets so hot that it is possible for helium to fuse into carbon and oxygen. Extra heat and pressure are once again generated and the core stops collapsing; it is stable until the helium runs out, which takes a few million years. Meanwhile, the outer parts of the star aren’t very stable.
Eventually the core is all carbon and oxygen, no additional heat and gas pressure is generated, and the core begins collapsing again. This time the density is so large – the electrons so close together – that electron degeneracy pressure begins to increase significantly as the collapse proceeds.
How to make a white dwarf star (continued)

Electron degeneracy pressure eventually brings the collapse of the core to a halt, before it gets hot enough to fuse carbon and oxygen into magnesium and silicon. The unstable outer parts of the star fall apart altogether; they are ejected and ionized by light from the core, producing a planetary nebula.
Planetary nebulae

The death of solar-type stars produces some of the most photogenic nebulae. Here are HST images of some pretty ones; lots of others can be found at

Astronomy Picture of the Day by using Search and entering “planetary nebula.”

(Credits: Hubble Heritage Team and Hubble Helix Nebula Team, STScI/NASA.)
Planetary nebulae (continued)

Hourglass Nebula, MyCn18

Butterfly Nebula, M2-9

Planetary nebulae (continued)

Credits: Hubble Heritage Team and R. Sahai (STScI/JPL/NASA).
How to make a white dwarf star (continued)

The planetary nebula’s material expands away from the scene in a few thousand years, leaving behind the hot, former core of the star, now about the size of Earth. Its weight supported against further collapse by electron degeneracy pressure, it will do nothing but sit there and cool off, for eternity.
How to make a white dwarf star (continued)

When brand new, this degenerate star is quite hot and looks white (like Sirius B) or even blue in color, leading to the name **white dwarf**. The oldest “white dwarfs” in our galaxy, age about 12 billion years, have had enough time to cool down to temperatures in the few thousands of degrees, and thus look red. (Despite this they are still called white dwarfs.)
Experimental confirmation of the theory: today thousands of white dwarf stars are known. Sure enough, all stellar masses under $1.4 \, M_\odot$ are represented, but no white dwarf heavier than this has ever been found.

Nominally for this work, Chandrasekhar was awarded a Nobel Prize in Physics (1983). The NASA Chandra X-ray Observatory (CXO) is named in his honor. And Stoner is, unfortunately, largely forgotten.

Seven white dwarfs (circled) in a small section of globular cluster M4 (By H. Richer and M. Bolte. Left: Kitt Peak National Observatory 36”; right: Hubble Space Telescope).
Theory of white dwarfs (continued)

- Important result of the theory: maximum mass for white dwarfs, which turns out to be $1.4 \, M_\odot$. Electron degeneracy pressure cannot hold up a heavier mass.

- Implication: for stars with core mass less than $1.4 \, M_\odot$, core collapse is stopped by electron degeneracy pressure before the horizon size is reached.

- However, for stars with cores more massive than $1.4 \, M_\odot$, the weight of the star overwhelms electron degeneracy pressure, and the collapse can keep going. **What can stop heavier stars from collapsing all the way to become black holes after they burn out?**
Final collapse of burned-out stars: white dwarf or black hole?

The Sun and smaller stars will become white dwarfs after they burn out and, lacking the gas pressure generated by their nuclear heat source, collapse under their weight. What about Sirius A, which weighs a good deal more than the limit?