CHAPTER 5 USING NEWTON'S LAWS

FRICTION

RECALL THAT CONTACT FORCES ARE EITHER PERPENDICULAR (NORMAL) OR PARALLEL TO SURFACES

FRICTION IS A CONTACT FORCE THAT OPPOSES MOTION. THERE ARE TWO "TYPES" OF FRICTION.

STATIC FRICTION OPPOSES MOTION WHEN THERE IS NO SLIPPING $F_{s} = \mu_{s} N$

KINETIC FRICTION OPPOSES MOTION WHEN THERE IS SLIPPING BETWEEN TWO SURFACES $F_{k} = \mu_{k} N$

$\mu_{s}, \mu_{k}$ ARE THE STATIC AND KINETIC COEFFICIENTS OF FRICTION.

TYPICALLY $1 > \mu_{s} > \mu_{k}$
CHAPTER 5 USING DIRECTED FORCES

Friction

Friction is empirical, not fundamental.

Close up of the interface between two surfaces.

At the microscopic level, contact forces are due to electromagnetic interactions between molecules.

Example: An inclined plane with friction.

How big can \( \theta \) get before the block starts sliding? (\( \mu_s \): coefficient of static friction)

\[ F_s = \mu_s N \]

\[ \sum F_x = mg \sin \theta - F_s = ma_x \quad (1) \]

\[ \sum F_y = -mg \cos \theta + N = ma_y = 0 \quad (2) \]

We want \( a_x = 0 \) so

from (1) \( mg \sin \theta - \mu_s N = 0 \rightarrow mg \sin \theta = \mu_s N \)

from (2) \( N = mg \cos \theta \)

Divide these: \( \frac{mg \sin \theta}{mg \cos \theta} = \frac{\mu_s N}{N} \rightarrow \tan \theta = \mu_s \) at max.
Example: Two boxes are tied together by a massless string. Box 1 has mass \( m_1 = 10 \) kg and is resting on a table. Box 2 has mass \( m_2 = 5 \) kg and is hanging from a pulley.

What is the magnitude of the acceleration felt by the boxes? (Assume \( \mu_k = 0.2 \).)

FBD's:

\[ \Sigma F_x = T - F_k = m_1 a \quad \text{(Equilibrium)} \]

\[ \Sigma F_y = N - m_1 g = 0 \Rightarrow N = m_1 g \]

\[ T = m_2 g - m_2 a \]

Combine relations:

\[ T - F_k = T - \mu_k N = T - \mu_k m_1 g = m_1 a \]

\[ (m_2 g - m_2 a) - \mu_k m_1 g = m_1 a \]

Solve for \( a \):

\[ m_2 g - \mu_k m_1 g = (m_1 + m_2) a \]

\[ a = \frac{m_2 g - \mu_k m_1 g}{m_1 + m_2} \]

Now check limits:

- \( m_2 > m_1 \) \( a = \frac{-\mu_k m_1 g}{m_1} \) -> Acceleration is downward.

- \( m_2 > m_1 \) \( a = \frac{m_2 g}{m_2} = g \) -> Free-fall for Box 2.
**Uniform Circular Motion**

By this we mean an object moving in a circle with constant speed. Since speed \(= |\mathbf{v}| \) is constant, \( \mathbf{a} \) and \( \mathbf{v} \) must be perpendicular.

- Speed is constant, but direction is changing, so \( \mathbf{a} \neq 0 \)
- \( \mathbf{a} \) points to the center of the circle.
- Magnitude of \( \mathbf{a} = \frac{v^2}{r} \) (\( r \) = radius of circle)

Note: Centripetal force is not a new force, it is the resultant of all forces acting on the object.

\[
F_{\text{net}} = m \frac{v^2}{r}
\]

If motion is "uniform circular".

Period - time required to get complete \( n \) circle

\[
T = \frac{2\pi r}{v}
\]

Frequency - number of revolutions per second

\[
f = \frac{1}{T} = \frac{v}{2\pi r}
\]
**Example 1** As we sit here on the rotating Earth, how "non-inertial" as our frame? What is our acceleration?

![Diagram of Earth and radius vector](image)

\[ r = R_\Theta \cos \theta \]

\[ \theta = \text{latitude} \]

\[ r \] is the radius of the circle in which we travel.

\[ V = \frac{2\pi r}{T} \]

\[ a = \frac{V^2}{r} = \left( \frac{2\pi}{T} \right)^2 r \Rightarrow a = \left( \frac{2\pi}{T} \right)^2 R_\Theta \cos \theta \]

\[ R_\Theta = 6.38 \times 10^6 \text{ m}, \quad \theta = 43^\circ, \quad T = 86400 \text{ s} \]

\[ a = \left( \frac{2\pi}{86400} \right)^2 (6.38 \times 10^6) (\cos 43^\circ) \approx 0.02 \text{ m/s}^2 \]

This is small! Only \( \approx 0.2\% \) of \( g \).

**Example 2** Problem 5.82 in Giancoli.

Carnival ride in a spinning cylinder whose floor drops away.

\[ 0.5 \text{ rev/s} \]

We need to find \( M_s \).

![Force diagram](image)

\[ \sum F_x = N = m a_x = m \frac{V^2}{r} \]

\[ \sum F_y = F_y - m g = m a_y = 0 \quad F_y = m g = M_s N \]

\[ N = \frac{m u^2}{r} = M_s \]

\[ M_s = \frac{g r}{V^2} \]

\[ M_s = \frac{(9.8 \text{ m/s}^2)(5.5 \text{ m})}{(17.3 \text{ s})^2} \rightarrow M_s = 0.2 \]
EXAMPLE 1

A car moves in a circle of radius \( r = 30 \text{ m} \). The coefficient of static friction is \( \mu_s = 0.80 \) between the tires and the road. What is the maximum speed the car can take a corner without slipping?

\[
\Sigma F_x = F_s = \mu_s N = mg = \frac{mv^2}{r}
\]

\[ \mu_s N = \frac{mv^2}{r} \]

\[ \Sigma F_y = N - mg = ma_y = 0 \rightarrow N = mg \]

\[ \mu_s N = \frac{mv^2}{r} = \mu_s mg \]

\[ v^2 = \mu_s gr \]

\[ v = \sqrt{\mu_s gr} \]

\[ v = \sqrt{(0.8)(10^{-6}) (30 \text{ m})} = 15 \text{ m/s} \]

Speed does not depend on mass!