CHAPTER 7  WORK AND ENERGY

IN PHYSICS THE TERM "WORK" SPECIFICALLY REFERS TO DESCRIBES WHAT HAPPENS WHEN AN OBJECT MOVES AS THE RESULT OF A FORCE.

WORK IS DEFINED AS THE PRODUCT OF THE MAGNITUDE OF THE OBJECT'S DISPLACEMENT AND THE FORCE PARALLEL TO THE DISPLACEMENT.

\[ W = F \cdot d \]

\[ F \]

\[ d \]

\[ \theta \]

So, for vectors \( \vec{F}, \vec{d} \):

\[ W = |F| |d| \cos \theta = \vec{F} \cdot \vec{d} \]

WORK HAS UNITS OF \( \frac{kg \cdot m}{s^2} = N \cdot m = Joule(s) \).

\[ \text{Remember,} \]

\[ \vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z = |A||B| \cos \theta \]

THE DOT PRODUCT OF TWO VECTORS IS A SCALAR.

\[ |A| |B| \cos \theta = |A| (|B| \cos \theta) \]

"PROJECTION" OF \( \vec{B} \) THAT LIES ALONG \( \vec{A} \).

\[ \text{So } W = \vec{F} \cdot \vec{d} = |d| |F| \cos \theta \]

PROJECTION OF FORCE THAT IS PARALLEL TO DISPLACEMENT.
**Example**

To pull a sled 50 m, Bob applies a force of 30 N at an angle 30° above horizontal. How much work does Bob do?

\[ W = F \cdot d \cdot (\cos \theta) \cdot d \]

\[ W = (30 N) \cdot (\cos 30°) \cdot (50 m) \]

\[ W = 1300 J \]

In general (3-D): \[ W = \mathbf{F} \cdot \Delta \mathbf{r} \]

**Variable Forces**

For \[ W = \mathbf{F} \cdot \Delta \mathbf{r} \], if we allow \( \Delta \mathbf{r} \) to become very small \((\Delta \mathbf{r} \to d\mathbf{r})\), than any force, even if it is changing as a function of \( r \), can be assumed constant across \( d\mathbf{r} \).

Then we can define

\[ dW = \mathbf{F} \cdot d\mathbf{r} \]

As the infinitesimal amount of work done during displacement \( d\mathbf{r} \)

Now

\[ W = \int_a^b dW = \int_a^b \mathbf{F} \cdot d\mathbf{r} \]

"Path integral"
Kinetic Energy

Moving things have kinetic energy (energy of motion).

Translational kinetic energy: \[ KE = \frac{1}{2} MV^2 \]

Notice that this also has units of Joules.

In order to increase an object's kinetic energy, one must do work. In fact, for "conservative" (see next chapter) systems, the change in an object's KE is exactly the net work done on the object.

\[ W_{\text{net}} = KE_f - KE_i \] (Work-Energy Principle)

Example: In 1-D, consider the work needed to accelerate a mass \( m \) from \( V_i \) to \( V_f \).

\[ V^2 - V_i^2 = 2a(x-x_i) = 2a(dx) \]

\[ \Rightarrow a = \frac{V_f^2 - V_i^2}{2dx} \]

Since \( F_{\text{net}} = ma \), \( F_{\text{net}} = m\left(\frac{V_f^2 - V_i^2}{2dx}\right) \)

\[ F_{\text{net}} = \frac{mV_f^2 - mV_i^2}{2} \frac{1}{dx} \]

\[ \Rightarrow (F_{\text{net}})(dx) = \frac{1}{2} mV_f^2 - \frac{1}{2} mV_i^2 \]

\[ W_{\text{net}} = KE_f - KE_i \]