**Problem 1**

**The mass of the Milky Way galaxy is approximately** $M_{\text{galaxy}} \approx 10^{12}$ kg. If the Sun has a mass of $M_\odot \approx 2 \times 10^{30}$ kg, and is 27,000 lightyears from the galactic center, approximately how long does it take the Sun to orbit the galaxy once? (1 light year $= 10^{16}$ m)

**Solution:** Assuming a circular orbit,

$$F_G = G \frac{M_\odot M_{\text{MW}}}{r_{\text{orbit}}^2} = M_\odot \frac{v^2}{r_{\text{orbit}}}$$

So

$$v^2 = G \frac{M_{\text{MW}}}{r_{\text{orbit}}}$$

But

$$v = \frac{2\pi r_{\text{orbit}}}{T} \rightarrow T = \frac{4\pi^2 r_{\text{orbit}}^3}{G M_{\text{MW}}}$$

So

$$T = \sqrt{\frac{4\pi^2 r_{\text{orbit}}^3}{G M_{\text{MW}}}} \approx 3.5 \times 10^{15} \text{ seconds}$$

**$T \approx 110 \text{ Myr}$**

The true period is actually around 2.5 Myr. Why might our calculation result in two short a period?

(Hint: The Milky Way has a radius of about 60 kpc)
Problem 2

A car is traveling around a banked (θ = 15°) circular track of radius 50 m. At what speed does the car feel no force due to friction?

Solution:

* Note that $F_x$ will point down the ramp if $v$ is more than the desired velocity, and up the ramp if $v$ is less than the desired value.

At the right $v$, friction vanishes, so all that's left is $mg$ and $N$

\[ \Sigma F_x = N \sin \theta = m \frac{v^2}{r} \rightarrow v^2 = \frac{mN}{m} \sin \theta \]

\[ \Sigma F_y = N \cos \theta - mg = 0 \rightarrow N = \frac{mg}{\cos \theta} \]

\[ \theta \]

\[ v^2 = \left( \frac{50 \text{ m}}{50 \times 10 \text{ m/s}^2} \right) \left( 90^\circ - 15^\circ \right) \]

\[ v = \sqrt{11,600 \text{ m}^2/\text{s}^2} \]

\[ v = 11,60^\circ/\text{s} \]
Problem 3

A massless pulley is suspended from the ceiling by two springs of constants $k_1 = 200 \text{ N/m}$ and $k_2 = 400 \text{ N/m}$.

A massless tray hangs from the pulley at an unweighted equilibrium position of $y = l_i = 1\text{ m}$.

If a 20 kg mass is then placed on the tray find the new equilibrium position of the tray ($l_f$).

Solution

The new position $l_f = l_i + \Delta y$

But because of the pulley we know that $\Delta y$ is half of the length added to the string/spring circuit.

So we need to find out how much each spring stretches.
PROBLEM 3 (cont)

To find out how much each
stretched, we must know the force
pulling on each. This is just going
to be the tension in the string.

We can draw a force diagram
for the weight.

\[ T = \frac{mg}{2} \]

So each spring is being pulled
by a force of \( \frac{mg}{2} \).

So then for spring 1

\[ F_1 = -k_1 \Delta y_1 = -\frac{mg}{2} \quad \text{(for equilibrium)} \]

\[ \Rightarrow \Delta y_1 = \frac{mg}{k_1} = \frac{200N}{400N/m} = 0.5 \text{ m} \]

And for spring 2

\[ F_2 = -k_2 \Delta y_2 = -\frac{mg}{2} \Rightarrow \Delta y_2 = \frac{mg}{2k_2} = \frac{200N}{400N/m} = 0.25 \text{ m} \]

Then \( \Delta y = \Delta y_1 + \Delta y_2 = 0.75 m = 0.375 m \)

So \( l_f = l_i + 0.375 m = 1.375 m \)