P113 University of Rochester NAME Soln key - SPM
S. Manly Fall 2012

Final Exam (December 18, 2012)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

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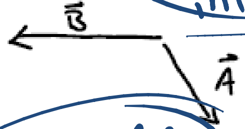
Problem 1 (12 pts, show ~~your work~~):

Below you will find six sketches along with a request to specify the direction of something. For each part, indicate the direction of the quantity as requested. Choose from the directions (as you look at your paper): to the right, to the left, up toward the top of the paper, down toward the bottom of the paper, into the paper, out of the paper.

(a)



indicate direction of $\vec{A}\times\vec{B}$

into paper

(b)

indicate direction of $\vec{A}\times\vec{B}$

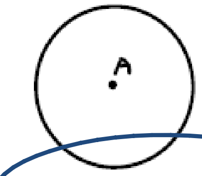
into paper

(c)

indicate the direction of $\vec{L}$

To right

(d)



wheel spins about axis at point A in a clockwise direction from your point of view. Indicate direction of $\vec{\omega}$

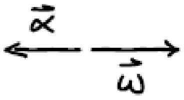
into paper

(e)

if system has $\vec{\omega}$ as shown, indicate direction of $\vec{L}$

To right

(f)



if system has $\vec{\alpha}$ and $\vec{\omega}$ as shown, indicate the direction of the torque $\vec{\tau}$ on the system

$\vec{\tau} = I\vec{\alpha}$

Torque is to left

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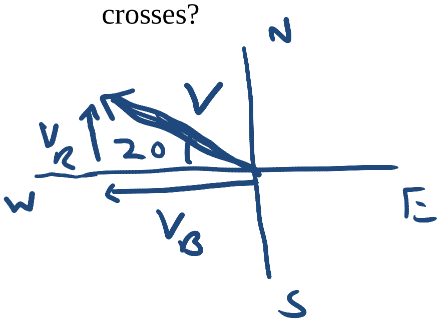
Problem 2 (10 pts, show your work):

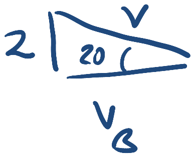
After finishing final exams, Billy Ray Eastman takes his sweetheart, Emma Mae Lattimore, out on the Genesee River for a ride in his new motor boat. Emma Mae is looking fine with the wind blowing through her long, lovely hair and Billy Ray's intentions are not good. But, for more of that part of the story, you'll have to take Human Sexuality 101 in the Psychology Department. Here we are more concerned with how Billy Ray and Emma Mae and the boat get from one place to another. And, of course, we are going to ignore the wind resistance of Emma Mae's hair in our calculations.

Billy Ray attempts to cross the Genesee by pointing his boat directly west toward the other shore. The river at this point flows north with a speed of 2 m/s. Billy Ray determines that his boat is moving at [redacted] a direction 20 degrees north of west.

How fast is Billy Ray's boat moving in a westerly direction?

How long will it take Billy Ray to cross the river if it is 100 meters wide at the point where he crosses?





$V \sin 20 = 2$

$V = 5.8 \text{ m/s}$

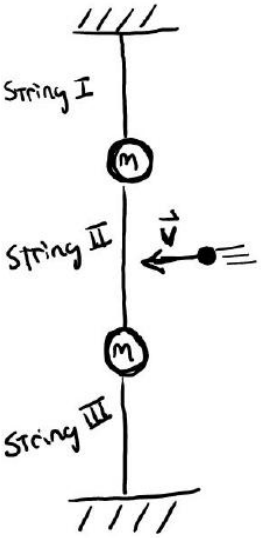
$(5.8)\cos 20 = V_B$ $\boxed{V_B = 5.45 \text{ m/s}}$

$x = x_0 + v_0 t + \frac{1}{2} a t^2 \rightsquigarrow 100\text{m} = (5.45)\, t$

$\boxed{t = 18 \text{ seconds}}$

Problem 3 (10 pts):

Two identical masses are attached to three identical strings as shown in the sketch to the right. The top string is attached to the ceiling and the bottom string is attached to the floor. The strings are taut. A bar moves slowly from right to left through the room and encounters the mid-point of the middle string. The bar keeps moving slowly to the left until one of the strings breaks. Which string breaks (top, middle, or bottom)? Briefly defend your answer with text and/or sketches.



String I breaks. The tension in String I is the greatest since it must support the weight of the other two masses plus the additional tension in the string caused by the passing of the bar through the string

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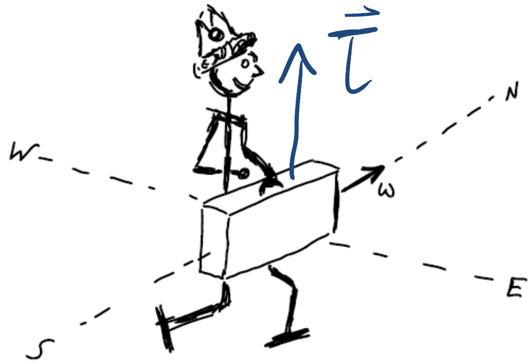
Problem 4 (8 pts, defend your choice):

Although it isn't widely known, 2012 is considered Elfen Spring in some quarters. This year "Santa's Elves" and the Abominable Snowman have joined forces to fight for their freedom from the oppressive regime of the evil Kris Kringle.
Below is a sketch of the elf separatist, Hermey the Sneaky, as he carries a small, guided anti-aircraft missile across the ice in hopes that he can use it to take out Santa's sleigh on Christmas Eve. Hermey's missile is enclosed in the case that he is shown carrying. The missile contains a spinning wheel as part of the guidance system. The wheel spins with an angular velocity, **ω**, oriented in a northerly direction as shown in the sketch. If Hermey turns west, the front end of the case containing the missile



[✓ a) Lifts upward]
b) Dips down
c) Does nothing whatever unusual
d) Pulls to the east

Briefly defend your choice.

Hermey Turning west means there is an upward torque on the case. $\vec{\tau} = \frac{d\vec{L}}{dt}$ and $\vec{L}$ is along $\vec{\omega}$ so $\vec{\omega}$ and the front of the case will lift up

*

Problem 5 (10 pts, ~~show your work~~):

If you want to impress your family over Christmas break, you can place a golf ball on top of a basketball and drop the pair simultaneously from rest. (I think this is not something you should try indoors … otherwise you may find it will impress your parents in a way that you might rather avoid!) When the basketball hits the ground, the golf ball pops high into the air. Briefly explain why this happens through text, ~~equations~~, and/or sketches.

*

when the basket ball collides with the ground it rebounds elastically and begins moving upward with the same velocity with which it hit the ground. This sets up an elastic collision between the upward moving basket ball and the downward going golf ball. Thru momentum conservation and the fact that the mass of the basketball is much larger than that of the golf ball, it means after the collision the golf ball will move upward with a large speed … popping high into the air.

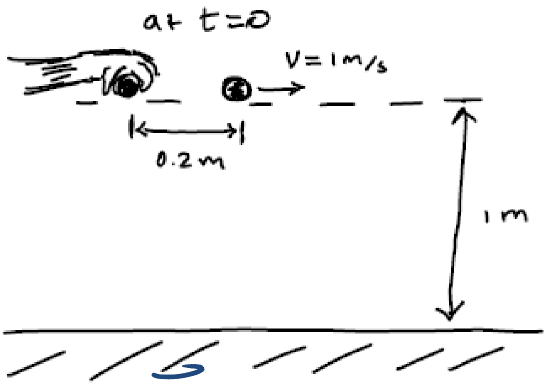
1)	/12
2)	/10
3)	/10
4)	/8
5)	/10
6)	/10
7)	/6
8)	/8
9)	/13
10)	/13
tot	/100

Mention masses

*

Problem 6 (10 pts, show your work):

Consider the situation in the sketch to the right. A ball of mass M is dropped at t=0 from a height of 1meter. At the same time an identical ball at a position 0.2 m horizontally from the first ball moves away from the first ball at a speed of 1 m/s.



(a) 2 pts - Which ball hits the ground first? Or do they hit the ground at the same time?

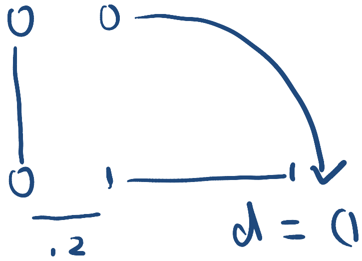
They hit at the same time
the acceleration for both is g

(b) 4 pts - How long does it take the dropped ball to hit the ground after it is released?

$y=y_0+\underset{\overset{\uparrow}{0}}{v_0}t+\frac{1}{2}\underset{\overset{\uparrow}{g}}{a}t^2$ $\qquad -1=-\frac{9.8}{2}t^2$

$\boxed{t=0.45\,s}$

(c) 4 pts - How far apart are the points of impact on the ground for the two balls?

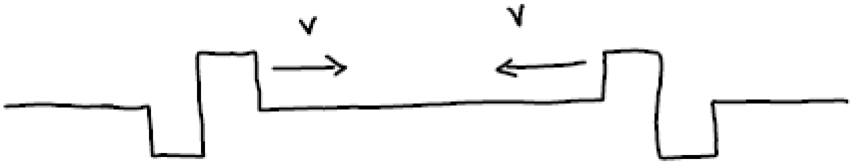


Total separation = 0.2 + 0.45 = 0.65 m

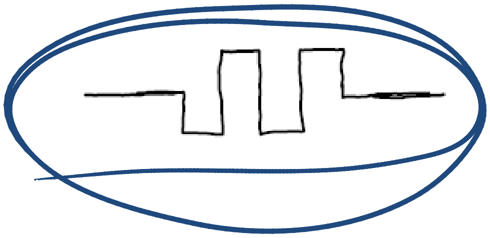
$d=(1\,m/s)(t)=(1)(.45)=.45$

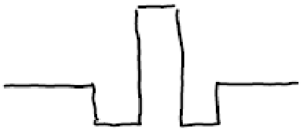
Problem 7 (6 pts, no partial credit):

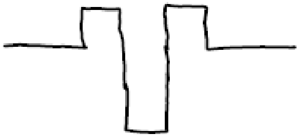
Consider the waves moving on a string as shown in the sketch below.



At a later time, as the waves pass each other, which of the following shapes (of the 4 shown) will NOT be observed instantaneously at some point during the passage? Circle that shape.







Problem 8 (8 pts, show your work):

Below is a sketch of a transverse wave on a string at time t=0. The wave moves to the left with time from your point of view. The divisions in x and in y are 1 meter apart. The bit of string at point P executes simple harmonic motion in the y direction with a period of 0.5 seconds.

What is the speed of the wave?

$\lambda = 16 m$ $T = 0.5 s$

$$V = \frac{\lambda}{T} = \frac{16}{0.5} = 32 \, {}^{m}/_{s}$$

Write down a mathematical description of the wave, i.e. y(x,t) =?

Ampl. = 2 m

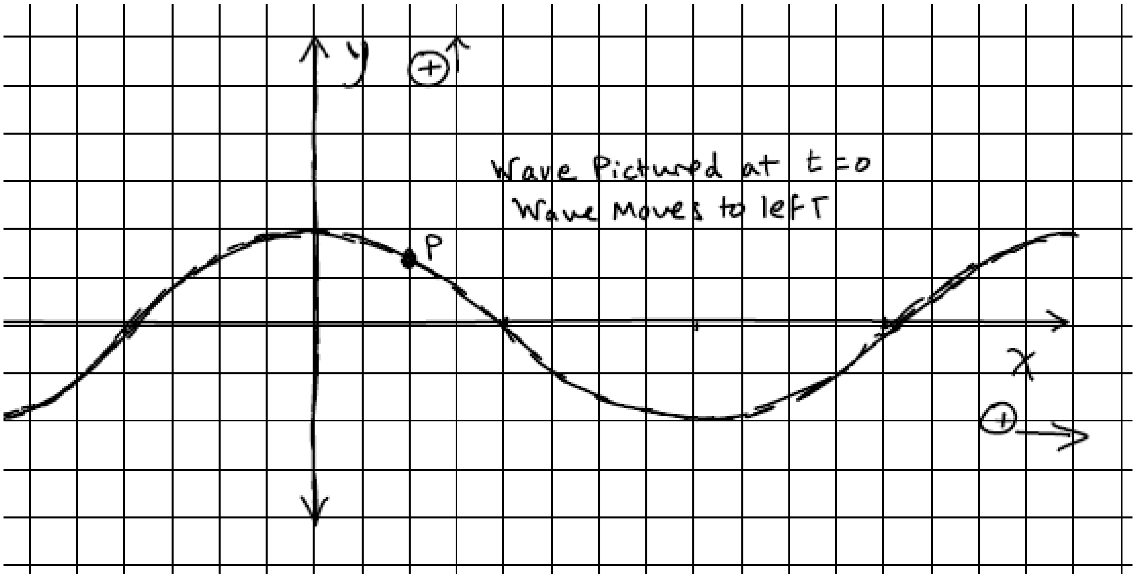
at $t=0$ → looks like a Cos

$$y(x,t) = 2 \cos\left(\frac{2\pi}{16} x + \frac{2\pi}{.5} t\right)$$

or

$$y(x,t) = 2 \cos\left(0.39 x + 12.6 t\right)$$

NOT necessary to simplify



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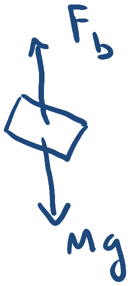
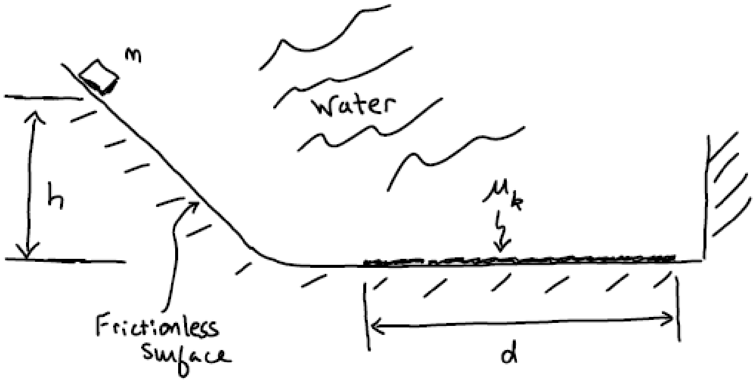
Problem 9 (13 pts, show your work):

Deep in the bottom of a pool of water, a mass of ~~2~~ 3.5 kg slides down a frictionless inclined plane that makes an angle of 60 degrees with the horizontal. The mass begins from rest at a distance h = 1.5 meters above the bottom of the pool. After reaching the bottom of the inclined plane, the mass slides across a horizontal surface into a wall. The horizontal surface is frictionless along the path of the mass except for a distance d=2 meters where the mass encounters friction as it slides on the floor of the pool ($\mu_k = 0.3$). Assume the mass has a volume of 0.001 m^3 and has a density of 3500 kg/m^3. Assume the density of water is 1000 kg/m^3. Neglect the viscosity of the water, i.e. assume the mass passes through the water without any resistance (not a good assumption in real life, but who cares?).

How fast is the mass going when it strikes the wall?





$F_B = [.001\ m^3][1000\ \frac{kg}{m^3}]\ 9.8$

$= 9.8\ N$

$Mg = (3.5kg)(9.8) = 34.3\ N$

Net Force Down $= 34.3 - 9.8 = 24.5\ N$

Find V at bottom of inclined plane

use Energy conservation

$F_{down} \cdot h$ = PE converted to KE

$F_{down}\ h = \frac{1}{2} m v^2$

$\sqrt{\frac{(24.5)(1.5)(2)}{3.5}} = 4.6\ M/s$

at bott of inclined plane

use here

Energy loss due to friction

$= \underset{0.3}{\mu_k}\ \underset{24.5}{N}\ \overset{2}{d}$ = 14.7 Joules

E just before wall all KE

$\frac{1}{2} M V_{wall}^2 = \frac{1}{2} M V^2 - 14.7$

$\boxed{V_{wall} = 12.7\ M/s}$

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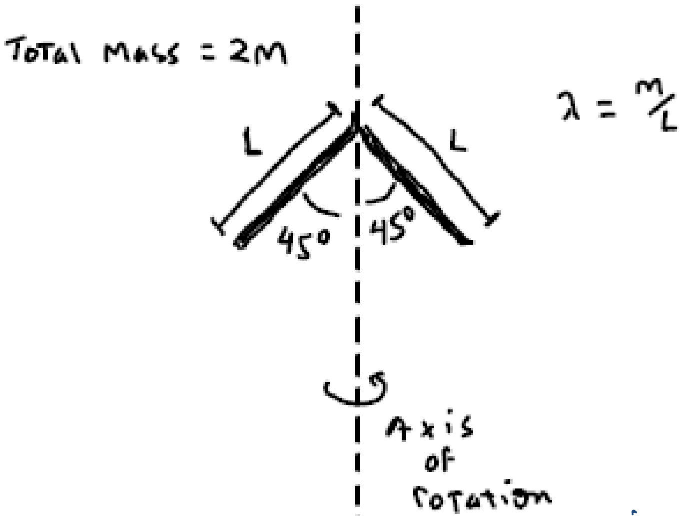
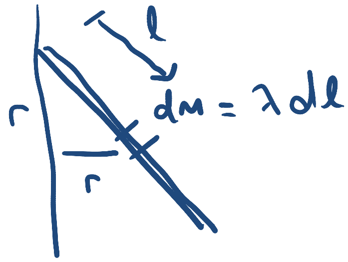
Problem 10 (13 pts, show your work):

Consider the two thin rods that make up a rigid body rotating about a vertical axis as shown in the sketch below. Each rod in the rigid object has length L and mass M and has a mass uniform mass per unit length, λ, equal to M/L. Each of the two rods in the rigid body make an angle of 45 degrees with the vertical. Determine the moment of inertia of this object about the vertical axis shown in terms of the variables M and L.



$$I = \int r^2\,dm$$

one side //



$$l^2 = r^2 + r^2$$

$$l = \sqrt{2}\,r$$

$$\frac{1}{\sqrt{2}}\,dl = dr$$

$$I_{1\,side} = \int_0^L \frac{l^2}{2}\,\lambda\,\frac{dl}{\sqrt{2}}$$

$$= \frac{M}{L(2)\sqrt{2}}\int_0^L l^2\,dl = \frac{M}{2\sqrt{2}\,L}\,\frac{L^3}{3} = \frac{1}{6\sqrt{2}}ML^2$$

I for full body is 2×1 side

so $I_{Full} = \frac{\sqrt{2}}{6}ML^2$

add Note *

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$v = v_o + at$$

$$x = x_o + v_o t + \frac{1}{2}at^2$$

$$x = x_o + \left(\frac{v_o + v}{2}\right)t$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$x - x_o = \int_{t_o}^{t} v\,dt$$

$$v - v_o = \int_{t_o}^{t} a\,dt$$

$$\sum \vec{F} = m\vec{a}$$

$$F_{\text{friction}} = \mu_k N$$

$$F_{\text{friction}} = \mu_s N$$

$$F_{\text{centripetal}} = \frac{mv^2}{r}$$

$$\vec{F}_{\text{spring}} = -k(\vec{x} - \vec{x}_o)$$

$$\text{work} = \int F \cdot ds$$

$$\text{power} = \frac{dw}{dt}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\theta = \theta_o + \left(\frac{\omega + \omega_o}{2}\right)t$$

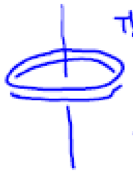
$$\omega \ \ = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$KE_{\text{translation}} = \frac{1}{2}MV^2$$

$$KE_{\text{rotation}} = \frac{1}{2}I\omega^2$$

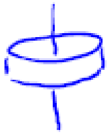
$$I = \sum m_i r_i^2 = \int r^2 dm = \int r^2 \rho\, dv$$

$$X_{cm} = \frac{\sum x_i m_i}{M} = \frac{\int x\,dm}{M}$$



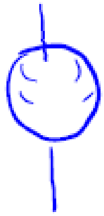
Thin hoop radius R

$I = MR^2$



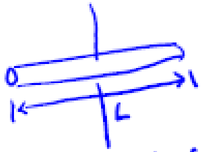
Solid cylinder Radius R

$I = \frac{1}{2}MR^2$



uniform Sphere Radius R

$I = \frac{2}{5}MR^2$



uniform rod of length L

$I = \frac{1}{12}ML^2$

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$I = I_{cm} + mh^2$$

$$\vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$\text{circumference of circle} = 2\pi r$$

$$\text{area of circle} = \pi r^2$$

$$\vec{F}_{grav} = -\frac{Gm_1 m_2}{r^2}\hat{r}$$

$$\text{quadratic equation} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{circumference of circle} = 2\pi r$$

$$\text{area of circle} = \pi r^2$$

$$\text{quadratic equation} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \lambda \nu$$

$$T = \frac{1}{f}$$

$$\omega^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$D(x,t) = A\sin(kx \pm \omega t + \varphi)$$

$$\beta = 10\log\frac{I}{I_o}$$

$$f' = \frac{f}{\left(1 \pm \frac{v_s}{v}\right)}$$