P121 University of Rochester NAME Soln key - SLM
S. Manly Spring 2003

Final Exam (May 8, 2003)

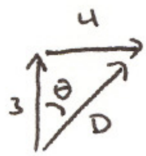
Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless stated otherwise. Good luck!!

For problems 1-3 you are to circle the correct/best answer. There will be no partial credit. Please use great care on these and check your work.

Problem 1 (12 pts, 3 parts, 4 pts each):

A ball moves with a speed in the north direction of 3 m/s and a speed in the in the east direction of 4 m/s (simultaneous with the north motion). The velocity of the ball is

a) 7 m/s.
b) 7 m/s in a direction of 53.1 degrees east of north.
c) 5 m/s in a direction of 53.1 degrees east of north.
d) 5 m/s in a direction of 36.9 degrees west of south.
e) 5 m/s in a direction of 53.1 degrees east of south.



$3^2+4^2=D^2$

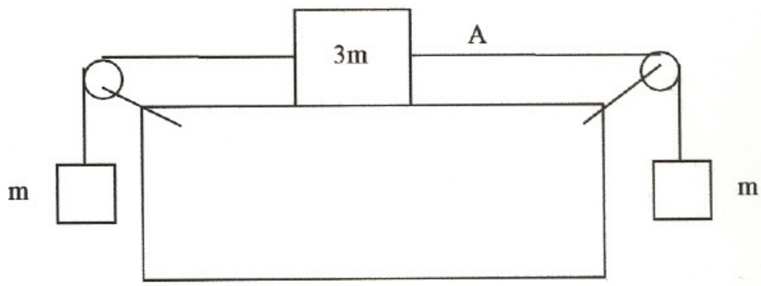
$D=5$

$\tan\theta = \frac{4}{3}$

$\theta = 53.1°$

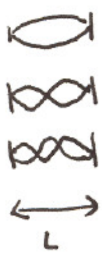
A stationary block with mass 3m sits on a frictionless surface. It is attached via massless cords, as shown below, to two blocks of mass m. The pulleys are massless and frictionless. The tension in the cord at point A is

a) zero.
b) 5mg.
c) 2mg.
d) mg.
e) 4mg.



The fundamental frequency of a vibrating string (fixed at both ends) is f. If the tension in the string is doubled, the fundamental frequency becomes

a) f/2
b) f/$\sqrt{2}$
c) f
d) $\sqrt{2}$f
e) $\sqrt{2}$/f



$\frac{1}{2}\lambda = L$

$1\lambda = L$

$\frac{3}{2}\lambda = L$

$n\frac{\lambda}{2} = L$

$\lambda = \frac{v}{\nu}$

$\frac{nv}{2\nu} = L$

$\frac{nv}{2L} = \nu$

$v = \sqrt{\frac{T}{\mu}}$ $\nu_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

if $T \to 2T$

$\nu \to \sqrt{2}\,\nu$

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1)	/12
2)	/12
3)	/12
4)	/9
5)	/9
6)	/9
7)	/9
8)	/10
9)	/9
10)	/9
tot	/100

Problem 2 (12 pts, 3 parts, 4 pts each):

The torque exerted on a spinning, perfectly spherical and uniform density orbiting communications satellite by the gravitational pull of the earth is

a) directed toward the earth.
b) the cause of satellite precession toward the earth.
c) directed parallel to the earth's axis of rotation and toward the north pole.
d) directed toward the satellite.
e) zero.

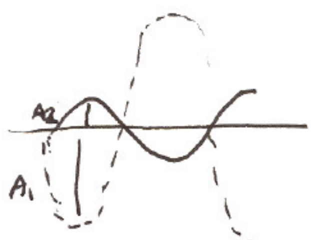
Spherical + uniform Density
gravitational force acts at center of Satellite
Which is ~~center~~ Axis of rotation

If two identical waves with the same phase are added, the result is

a) a wave with the same frequency but twice the amplitude.
b) a wave with the same amplitude but twice the frequency.
c) a wave with zero amplitude.
d) a wave with zero frequency.
e) This problem cannot be solved without knowing the wavelength.

Two waves with the same frequency and wavelength but with different amplitudes are added. If $A_1=2A_2$ and the waves are 180 degrees out of phase, then the amplitude of the resultant wave is

a) zero
b) the same as A_1
c) the same as A_2.
d) equal to A_1+A_2.
e) coherent.



Problem 3 (12 pts, 3 parts, 4 pts each):

Two identical cylindrical disks have a common axis. Initially one of the disks is spinning. When the two disks are brought into contact, they stick together. Which of the following is true?

KE NOT conserved get new KE using Ang. Momentum cons.

a) The total kinetic energy and the total angular momentum are unchanged from their initial values.
b) Both the total kinetic energy and the total angular momentum are reduced to half of their original values.
c) The total angular momentum is unchanged, but the total kinetic energy is reduced to half its original value.
d) The total angular momentum is reduced to half its original value, but the total kinetic energy is unchanged
e) The total angular momentum is unchanged, and the total kinetic energy is reduced to one-quarter of its original value.

Angular moment. cons →

$KE = \frac{1}{2} I \omega^2$

$L = 2I\omega'$

$I\omega = 2I\omega'$

$\omega = 2\omega'$

$KE_{init} = \frac{1}{2} I \omega^2$

$KE_f = \frac{1}{2} I \omega'^2 = \frac{1}{2} I \left(\frac{\omega}{2}\right)^2 = KE_I \left(\frac{1}{4}\right)$

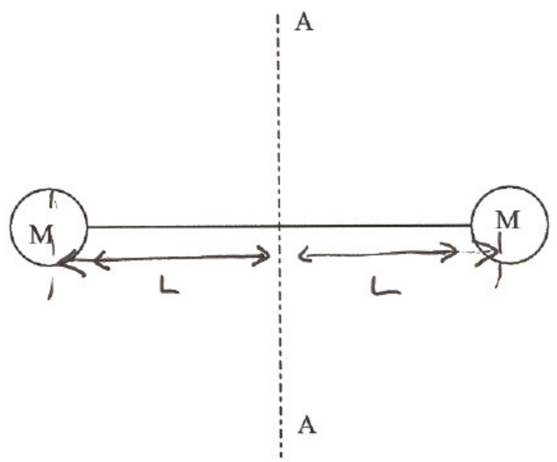
~~But NOT elastic~~ $\cancel{KE' = \frac{1}{2} 2I \omega'^2}$ $\cancel{KE' = \frac{1}{2} I \omega^2 (1/4)(2)}$

An 80-kg man on ice skates pushes a 40-kg boy, also on skates, with a force of 100 N. The force of the boy on the man is

a) 200 N.
b) 100 N. — Newton's 3RD law: Action-reaction pair
c) 392 N.
d) 784 N.
e) zero unless the boy pushes back.

The moment of inertia of a set of dumbbells, considered as two point masses M separated by a distance 2L, about the axis AA, is

a) $(1/2)ML^2$
b) ML^2
c) $2ML^2$
d) $(1/4)ML^2$
e) $4ML^2$



$I = \sum MR^2 = 2ML^2$

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Problem 4 (9 pts):

A plastic cube of mass 3M collides elastically with a smaller plastic cube of mass 1M. Initially the 3M mass is moving a 4 m/s and the 1M mass is at rest. After the collision both masses move in the same direction as the 3M mass. Calculate the velocities of the 3M mass and the 1M mass after the collision. Assume this event takes place on a smooth frictionless surface such as ice.

Initial situation Final situation

Elastic Collision => KE Conserved

Also Momentum is conserved

Pcons $(3M)4 = 3MV_{f3} + MV_{f_1} \Rightarrow 12 = 3V_{f3} + V_{f_1}$ ①

KE cons $\frac{1}{2}(3M)4^2 = \frac{1}{2}(3M)V_{f3}^2 + \frac{1}{2}MV_{f_1}^2 \Rightarrow 24 = \frac{3}{2}V_{f3}^2 + \frac{1}{2}V_{f_1}^2$ ②

① → $V_{f_1} = 12 - 3V_{f3}$

Sub into ②

$24 = \frac{3}{2}V_{f3}^2 + \frac{1}{2}(12 - 3V_{f3})$

$24 = \frac{3}{2}V_{f3}^2 + \frac{1}{2}(144 - 72V_{f3} + 9V_{f3}^2)$

$24 = \frac{3}{2}V_{f3}^2 + 72 - 36V_{f3} + \frac{9}{2}V_{f3}^2$

$0 = 48 - 36V_{f3} + 6V_{f3}^2$

$0 = 8 - 6V_{f3} + V_{f3}^2$

Now solve quadratic eqn for V_{f3}

$V_{f3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(8)}}{2}$

$V_{f3} = \frac{6 \pm 2}{2} = \cancel{4 \text{ or } 2 \text{ m/s}}$ 4 or 2 m/s

If $V_{f3} = 4$ m/s $\Rightarrow V_{f_1} = 0$ m/s ✗

MAKES NO SENSE

If $V_{f3} = 2$ m/s $\Rightarrow V_{f_1} = 6$ m/s

So $\boxed{V_{f3} = 2 \text{ m/s} \quad V_{f_1} = 6 \text{ m/s}}$

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Problem 5 (9 pts):

A spherical 2 kg mass is held in static equilibrium by two cables as shown below in the sketch. Cable 2 is 3m long as measured from the center of the mass to the point where it is attached to the ceiling.

a) Determine the tension in the two cables.

$\Sigma F_y = 0 = Mg - T_2 \cos 15 \Rightarrow T_2 = \frac{Mg}{\cos 15} = \frac{(2)(9.8)}{\cos 15} = 20.3\ N$

$\Sigma F_x = 0 = T_2 \sin 15 - T_1 \qquad T_1 = (20.3N) \sin 15 = 5.3\ N$

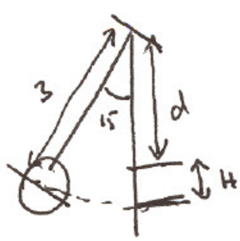
Suppose cable 1 is cut. The mass moves as a pendulum in simple harmonic motion.

b) What is the period of the motion of the pendulum?

$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{3}{9.8}} = 3.5\ s$

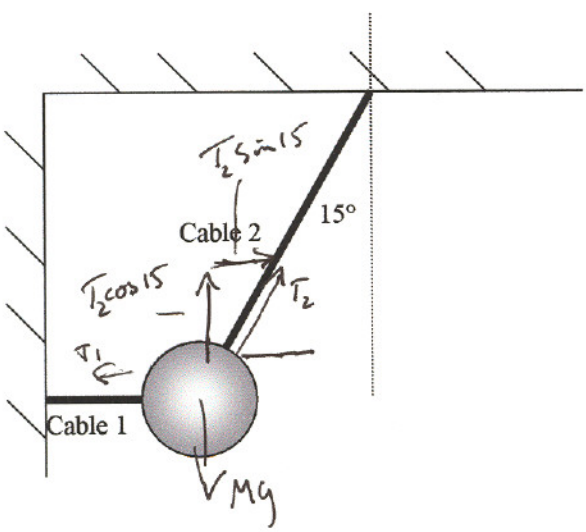
c) How fast does the mass move at the lowest point in its motion?

Use E conservation



$\cos 15 = \frac{d}{3} \qquad d = 2.9\ M$

$\therefore H = 0.1\ M$



$\Delta \text{grav PE} = MgH = \underbrace{\frac{1}{2} MV^2}_{\text{AT BOTTOM OF MOTION}}$

$V = \sqrt{2gH}$

$$\boxed{V = \sqrt{2(9.8)(0.1)} = 1.4\ M/s}$$

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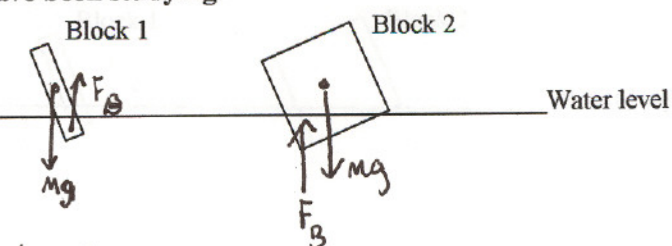
Problem 6 (9 pts):

Two rectangular, uniformly dense blocks are floating in water. Each of them is tipped by someone and released. At the instant they are released the blocks are as shown in the sketch. Please explain, using text, equations or drawings as appropriate, what will happen to the blocks in the next few moments. Will either of them right themselves or will either tip over? Defend your answer using concepts we have been studying.

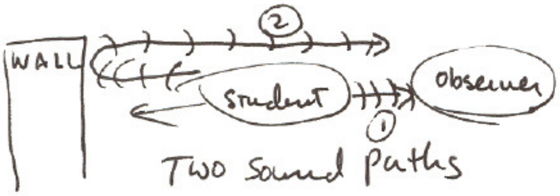


In Both cases, two Forces Act on the blocks, the force of gravity Acting at the Center-of-gravity (at the geometrical center of the block) And the bouancy force which Acts off to one side of the center-of-gravity because it Acts at the center-of-Mass of the displaced water volume. In the case of block 1, the Bouancy force provides a torque that will act to cause the block to tip over. In the case of Block 2, the bouancy force provides a torque that will restore the block to the upright position.

Problem 7 (9 pts):

Some say that if Dante were alive today, he might write about a hell where the Devil is a physics laboratory instructor who studies the inelastic collisions of innocent and hard-working undergraduate students. One student gets thrown at high velocity against a brick wall (and survives unharmed to do it again and again, of course) while the rest of the class has to make observations. The particularly sinful students get to do the error analysis!

Anyway, suppose Biff Jones is thrown by the Devil toward the wall. Biff emits a constant yell at a pure frequency of 400 Hz. While Biff is in flight, the observing students hear a sound that oscillates up and down in intensity with a frequency of 5 Hz. What is the speed with which the Devil throws Biff at the wall? ***(Ignore air resistance. Assume the velocity of sound is 343 m/s. Consider this as a one-dimensional problem.)***



The student's yell is Doppler shifted down in frequency for the direct path to the observer. For the Path with the reflection off the wall, The student's yell is doppler shifted up in frequency.

Sound along the two separate paths interfere and create beats.

beats at 5 Hz means $f_{1} - f_{2} = 5\ Hz$ so $f_{1} = 400 - 2.5 = 397.5\ Hz$

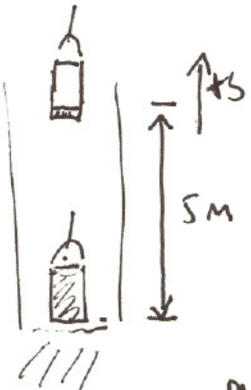
$f_{2} = 400 + 2.5 = 402.5\ Hz$

402.5

$$f_{2} = 400\left(\frac{1}{1 - \frac{V_{student}}{343}}\right) \Rightarrow \boxed{V_{student} \cong 2.1\ m/s}$$

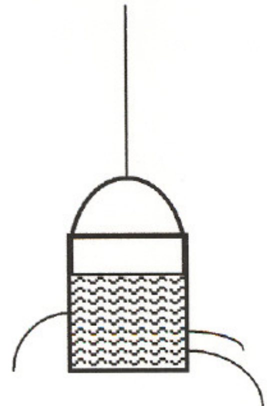
Problem 8 (10 pts):

Jill goes to the top of the hill to fetch a pail of water. (By herself, of course, since her worthless boyfriend, Jack, is sprawled out on the hill with his broken crown.) She lifts a pail with a total volume of 4 liters (=4x10^{-3} m^3) up a distance of 5 m. Unfortunately the pail is leaking. It starts out full of water at the bottom of the well. But, by the time the pail reaches a height of 5 m it contains only 1 liter (1x10^{-3} m^3) of water. How much energy does Jill use to lift the pail of water the full 5 m? Assume she lifts the pail slowly at constant velocity and assume the pail's rate of loss of water is constant.



$$E_{jill} = \int_{bottom}^{Top} F\cdot ds$$

$$= \int_0^5 m(s)\, g\, ds$$



Mass changes with height!

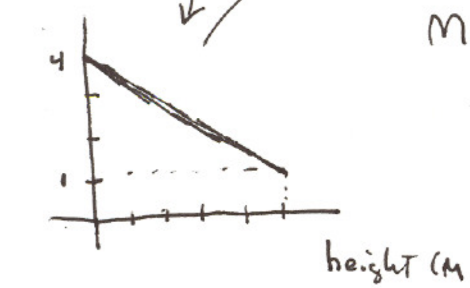
MUST use info given to determine Mass as function of height s.

look AT graph

Volume ~~A(s)~~ $= -\frac{3}{5}s + 4$

1 liter $= 1\times10^{-3}\,M^3 \times 1000\,\frac{kg}{M^3}$

$= 1$ kg water



$$M_{kg} = \left(-\frac{3}{5}s + 4\right) kg$$

$$\therefore\ E_{jill} = \int_0^5 g\left(4 - \frac{3}{5}s\right) ds$$

$$E_{jill} = 4g\int_0^5 ds - \frac{3}{5}g\int_0^5 s\,ds$$

$$E_{jill} = 20g \text{ (joules)} - \frac{3}{5}g\,\frac{25}{2} \text{ (joules)} = \frac{25}{2}g \text{ joules} = 122.5 \text{ joules}$$

Problem 9 (9 pts):

At the end of a concert, famous hip-hop artist Vanilla Spongebob takes off his bejeweled 0.02 kg navel ring and throws it into the crowd. The ring leaves his hand at an angle of thirty degrees above the horizontal. It leaves his hand with a velocity of 5 m/s and strikes the floor 3 meters below where his hand releases the ring.

a) How high above the floor does the ring rise?

$y = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

5, 30

$V_y = V\sin 30 = 2.5\ M/s$

$V_x = V\cos 30 = 4.3\ M/s$

$\underline{\underline{x}}$: $a_x = 0$, $d = V_x t$

$\underline{\underline{y}}$: $a_y = -9.8\ M/s^2$

$V = V_0 + at$

Time to top:

$0 = 2.5 - 9.8t$ ∴ $0.255\ s$

$Y_{top} = 3 + 2.5(.255) - \frac{1}{2}9.8(.255)^2$

$Y_{top} = 3.32\ M$

rises to 3.32 M above floor

b) How far horizontally from Vanilla Spongebob does the ring strike the floor?

HAve Time to Top calculated Above $= 0.255\ s$

Time from Top to bottom

$y = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

$0 = 3.32 + 0 - \frac{9.8}{2}t^2$

$t_{\text{Top to Bott}} = 0.823\ s$

TOTAL Time in Air $= t_{\text{To Top}} + t_{\text{top to Bott}}$

$= 0.255 + 0.823 = 1.08\ s$

$d = V_x t_{TOTAL}$

$= (4.3)(1.08) = 4.64\ M$ = horizontal dist ring goes

Suppose Vanilla Spongebob happens to give a concert to a group of astronauts on Mars. Assuming he throws an identical navel ring in an identical way into the crowd of astronauts. The mass of Mars is 6.42x10^{23} kg. The radius of Mars is 3.4x10^{6} m. The distance of Mars from the Sun is 2.28x10^{11} m. The gravitational constant, G=6.67x10^{-11} (n)(m^2)/kg^2.

c) How high above the floor does the ring rise on Mars?

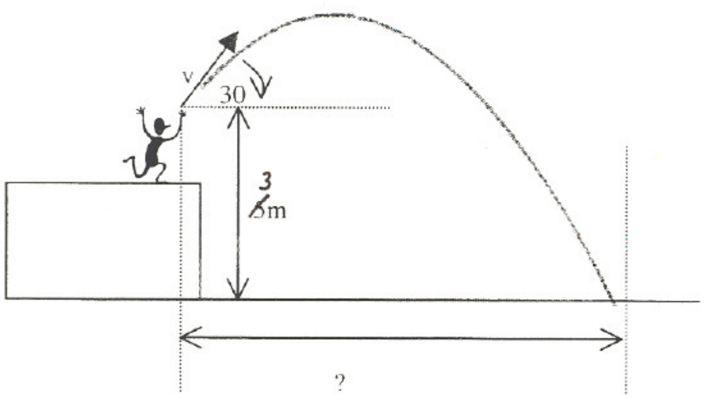
$|g_{Mars}| = \frac{M_{Mars} G}{R_{Mars}^2} = 3.7\ M/s^2$

so, following calcs above substituting g_{Mars} for g

Time to Top $= \frac{2.5}{3.7} = 0.67\ s$

Height at top $= 3 + 2.5(0.67) - \frac{1}{2}3.7(.67)^2 = 3.8\ M$

d) How far horizontally from Vanilla Spongebob does the ring strike the floor?



Time from top to Bottom

$0 = 3.8 + 0 - \frac{3.7}{2}t^2$

$t_{Top \to Bott} = 1.4\ s$

TOTAL Time in Air $= 0.67 + 1.4 \cong 2.1\ s$

TOTAL Horiz. Dist $= (2.1)(4.3)$

$= 9\ m$

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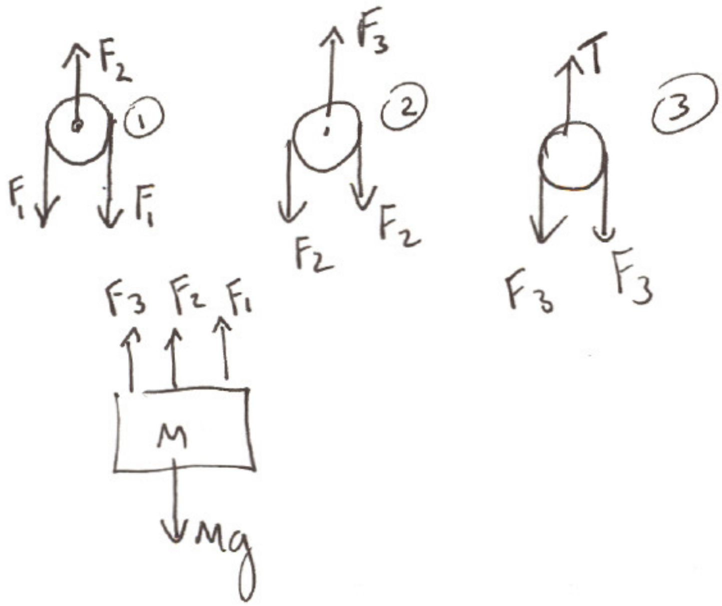
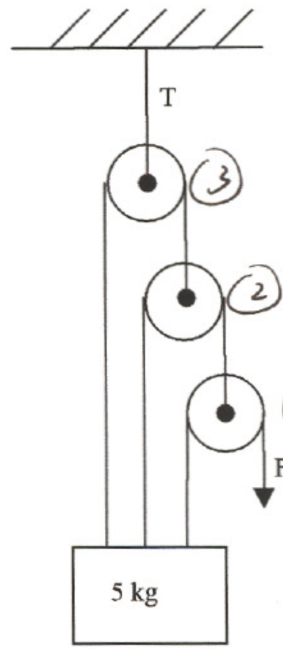
Problem 10 (9 pts):

Consider the system shown below in the sketch. The force F holds this mass and pulley system in static equilibrium. Assuming the pulleys to be frictionless and massless and the string to be massless, Determine the force F and the tension T in the upper cable.





For MASS $\quad \Sigma F_y = 0$

$$Mg = F_1 + F_2 + F_3 = F_1 + 2F_1 + 4F_1 = \boxed{7F_1 = Mg}$$

for pulley 3 $\quad \Sigma F_y = 0$

$$T = 2F_3 \Rightarrow \boxed{T = 8F_1}$$

for pulley 2 $\quad \Sigma F_y = 0$

$$F_3 = 2F_2 \Rightarrow F_3 = 4F_1$$

for pulley 1 $\quad \Sigma F_y = 0$

$$F_2 = 2F_1$$

$$F_1 = Mg/7 = F$$

$$T = \frac{8}{7}Mg$$