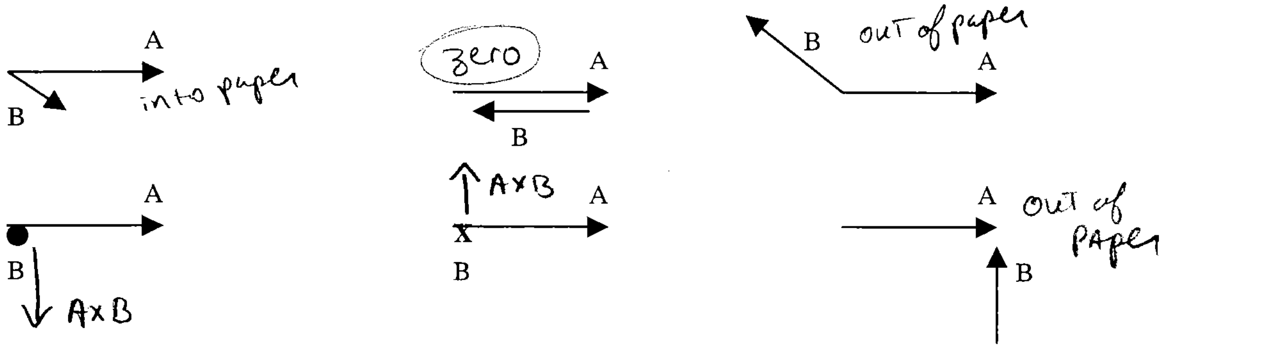
P113 S. Manly — University of Rochester, Fall 2000 — NAME Soln Key - SLM

Exam 3 (November 16, 2000)

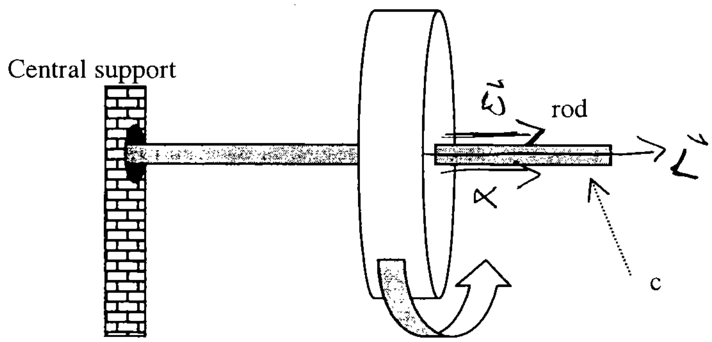
Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (24 pts):

Indicate the direction of the cross product, AxB, on each of the following 6 sketches. If it is zero, write "zero" next to the sketch.



Consider the massive spinning wheel drawn to the right of this text. The wheel spins rapidly without friction about a rod that is supported on one end by the "central support" The wheel/rod are free to rotate about the central support axis in a horizontal plane perpendicular to the plane of the paper. Consider an instant when the wheel and rod are oriented as shown with the rod in the plane of the paper.



a) Draw the angular velocity vector for the wheel about the rod.

b) Draw the angular momentum vector for the wheel about the rod.

c) What is the direction of the precession for the wheel and rod about the central support? Into paper

counterclockwise as seen from above

d) What happens to the system if a weight is hung at position c on the rod?

The precession rate increases

e) Suppose someone increases the magnitude of the angular velocity of the wheel. Draw the direction of the angular acceleration vector.

f) If the wheel is now moving faster, is there a change in the precession velocity?

$$mgr = \tau = \frac{dL}{dt} = |\vec{L}| \frac{d\Omega}{dt}$$

$\frac{d\Omega}{dt} \sim$ precession rate goes as $\frac{1}{|\vec{L}|}$

$\therefore$ precession velocity is less or slower

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Problem 2 (30 pts)

a) Passengers in cars are required to wear seat belts. However, children in school buses are not. Please justify this (without invoking cost) using concepts you've studied recently in physics.

The Mass of a bus is very large. So, in a collision the change in Momentum which can come about with a small change in velocity relative to a passenger car. In the end it is the net change in velocity over a short time (or acceleration) that hurts people. Therefore you are less likely to be hurt in a collision if you are riding in a massive object relative to a lighter object.

b) Helicopters always have two rotors. They either have one large rotor in the horizontal plane accompanied by a small one on the tail in the vertical plane or they have two large rotors in the horizontal plane that spin in opposite directions. Please explain why they are designed this way.

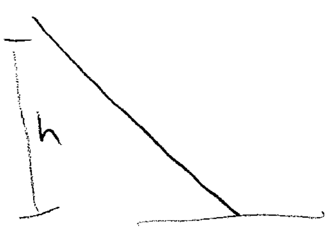
If you change the angular velocity of the large rotor (or it's direction), you are changing the rotor's Angular momentum. The rest of the helicopter will react to cancel out this change in Angular momentum (with fatal consequences) unless the small rotor is used to balance the change in Angular momentum.

c) Consider a solid cylinder, a solid sphere and a solid cube. If these three objects are simultaneously allowed to slide or roll down an inclined plane. In what order will they arrive at the bottom? Assume the cube slides without friction and that the cylinder and sphere roll without slipping. Explain your answer.

cube <u>before</u> sphere <u>before</u> cylinder

because

$I_{cube} = 0 < I_{sphere} < I_{cylinder}$



Energy cons $\Rightarrow$ $mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

The larger I, the more energy is tied up in rotation rather than translation

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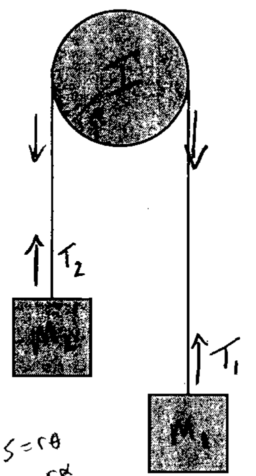
1)	/24
2)	/30
3)	/26
4)	/25
tot	/105

Problem 3 (26 pts):

Two masses are connected via a cord over a frictionless pulley with a moment of inertia I. The cord does not slip on the pulley. The mass on the right is M_1 kg and the mass on the left is M_2 kg.

a) Find an expression for the linear acceleration of M_1 in terms of the masses and I.



$s = r\theta$
$a = r\alpha$

$m_1 g - T_1 = m_1 a \rightarrow T_1 = m_1(g-a)$ I

$T_2 - m_2 g = m_2 a \rightarrow T_2 = m_2(a+g)$ II

$T_1 R - T_2 R = I\alpha = I\frac{a}{R}$ III

Pulley Disk
$I = \frac{1}{2}MR^2$

$m_1(g-a) - m_2(a+g) = \frac{Ia}{R^2}$ I, II → III

$(m_1 - m_2)g - (m_1 + m_2)a = \frac{Ia}{R^2}$

$(m_1 - m_2)g = \left(\frac{I}{R^2} + m_1 + m_2\right)a$

$$a = \frac{(m_1 - m_2)g}{\left(\frac{I}{R^2} + m_1 + m_2\right)}$$

b) Starting from rest, what is the angular velocity of the pully wheel after 3 seconds?

Know a → $\alpha = \frac{a}{R}$, know α

$\omega = \omega_0 + \alpha t$

$\omega = \frac{a}{R} t = \frac{a}{R}(3\,s)$

c) Through what angle (in radians) does the pulley turn in the first 3 seconds of motion?

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = \frac{1}{2}\alpha t^2$

Actual problem needs input of #'s given in exam

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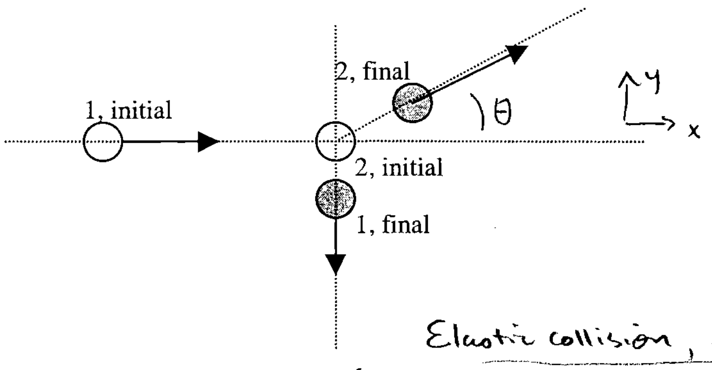
Problem 4 (25 pts):

Let's do a little nuclear physics. Yes, I know we haven't covered that chapter yet. The background is for your education, perspective and entertainment. The problem stands alone and requires no knowledge of basic nuclear physics. If the background bothers you, skip it. It is not necessary to solve the problem.

Background: *A nuclear chain reaction occurs if one brings together more than a certain amount of an isotope of uranium. What happens is that one of the uranium nuclei spontaneously splits into two smaller nuclei releasing some energy and two subatomic particles called neutrons. This process is called nuclear fission. If there is enough uranium in the sample, the two neutrons released in the first fission each hit other uranium nuclei and cause both nuclei to undergo fission, each in turn releasing two neutrons and a little energy. Each of the four neutrons produced in the second set of fission reactions hit other uranium nuclei, and so forth. The energy release grows enormously fast. If this process is regulated by stopping some fraction of the neutrons from hitting uranium nuclei, it is known as controlled fission. This is the basis for the operation of a nuclear power plant. If there is no regulation and the number of neutrons is allowed to grow unchecked, the reaction is said to have gone "critical". If the geometry of the sample is correct, the release of energy is explosive. This is the basis of operation for an atomic bomb. One way to control a chain reaction is to insert a substance that can slow down the neutrons so they don't have enough kinetic energy to split uranium nuclei. One substance that can be used to do this is "heavy" water. Heavy water is regular water that has one or both of the hydrogen nuclei replaced by deuterium, which is an isotope of hydrogen. Hydrogen has a nucleus made up of one proton and deuterium has a nucleus made up of a proton and a neutron. Below you will do a calculation that will demonstrate why heavy water can be used to control a nuclear chain reaction.*

The problem: Consider a particle of mass m (a neutron), moving with an initial velocity v, that collides elastically with a different particle of mass 2m (a deuterium nucleus), initially at rest. Suppose the incoming particle (the neutron) is scattered by 90 degrees relative to its original direction. Prove the incoming particle loses approximately 60% of its incoming kinetic energy in the collision.



P_x conservation

$$mv = (2m)v_2\cos\theta \quad \text{I}$$

P_y conservation

$$0 = -mv_1 + (2m)v_2\sin\theta \quad \text{II}$$

Elastic collision, KE conservation

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}(2m)v_2^2 \quad \text{III}$$

$$v^2 = v_1^2 + 2v_2^2$$

Have 3 eqns now. Find v_1 in terms of v

$\text{I} \rightarrow v^2 = 4v_2^2\cos^2\theta$

$\text{II} \rightarrow v_1^2 = 4v_2^2\sin^2\theta$

Add together

$$v^2 + v_1^2 = 8v_2^2$$

Sub in III for v_2

$$v^2 = v_1^2 + \frac{v^2}{4} + \frac{v_1^2}{4}$$

$$\frac{3}{4}v^2 = \frac{5}{4}v_1^2$$

$$v_1^2 = \frac{3}{5}v^2$$

$$KE_{initial} = \frac{3}{5}KE_{final}$$

for incoming particle