P113 University of Rochester NAME Soln key - SM
S. Manly Fall 2001

Exam 2 (November 1, 2001)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (15 pts):

Consider the two vectors:

$\vec{A} = 6\hat{i} + 3\hat{j} + 2\hat{k}$

$\vec{B} = 7\hat{i} - 4\hat{j} + 9\hat{k}$

5pts a) Compute the vector scalar product of these two vectors, $\vec{A} \bullet \vec{B}$

$$\vec{A}\cdot\vec{B} = (6)(7) + (3)(-4) + (2)(9) = 42 - 12 + 18 = 48$$

$$\boxed{\vec{A}\cdot\vec{B} = 48}$$

5pts b) What is the magnitude of $\vec{A}$?

$$|\vec{A}| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{36+9+4} = \boxed{7 = |\vec{A}|}$$

5pts c) What is the opening angle (in degrees) between the two vectors?

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

compute $|\vec{B}| = \sqrt{7^2+4^2+9^2} = 12.1$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{48}{(7)(12.1)} \Rightarrow \boxed{\theta = 55.4^\circ}$$

Problem 2 (~~15~~ 12 pts):

3 ~~4~~pts a) You compress a spring with spring constant, k=30 N/m, by 4 cm relative to its natural length. Have you done positive, negative or zero work?

$\vec{F}\cdot d\vec{x}$ is + $\Rightarrow$ + work

3 ~~4~~pts b) What is the magnitude of the work you did in part (a)

Work done = PE in spring $= \frac{1}{2}kx^2 = \frac{1}{2}(30\ \frac{N}{m})(.04\ m)^2 = 0.024$ J

3 ~~4~~pts c) You now stretch the same spring 4 cm beyond its natural length. Have you done positive, negative or zero work?

$\vec{F}\cdot d\vec{x}$ is + $\Rightarrow$ + work

3 ~~4~~pts d) What is the magnitude of work you did on the spring in part (c)?

Same as part (b) $\Rightarrow$ 0.024 J

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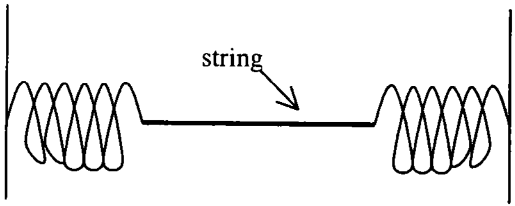
Problem 3 (~~10~~ 13 pts):

A string is tied between two identical springs, stretching each 5 cm beyond their natural length. Each spring has a spring constant of 100 N/m and is attached to a wall on the side away from the string. What is the tension in the string?



1)	/15
2)	/15
3)	/10
4)	/15
5)	/15
6)	/15
7)	/15
tot	/100

$$|\vec{F}|_{spring} = Kx = (100\ \tfrac{N}{M})(.05\ M) = 5\ N$$

This is the Tension in The String because it is the Force Needed to extend the springs!

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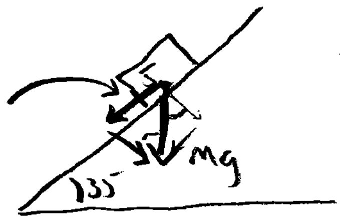
Problem 4 (15 pts):

a) A 5 kg mass in the shape of a block slides down a frictionless inclined plane that makes an angle of 35 degrees with the horizontal. What is the acceleration of the block down the plane?

mg sin 35

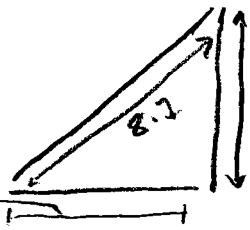


$a_{\text{down plane}} = g\sin 35 = (9.8\ m/s^2)\sin 35$

$\boxed{a = 5.6\ m/s^2}$

b) Suppose the inclined plane is 8.7 m long and 5 m high and the block moves from the top to the bottom of the inclined plane. What is the speed of the block at the bottom of the inclined plane?

IF TAKE this as 8.7 m → okay



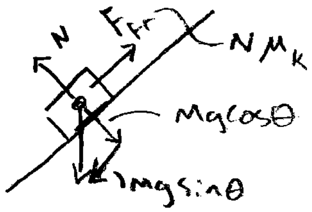
5m

use Energy conservation

$\frac{1}{2}mv^2 = mgh \qquad v = \sqrt{2gh} = \sqrt{2(9.8)5}$

$\boxed{v = 9.9\ m/s}$

c) Suppose the same block now slides down the same inclined plane. However, now there is friction between the surface of the inclined plane and the block. Assume μ_k=0.15. Now what is the acceleration of the block down the inclined plane.



$\Sigma F_y = 0 = N - Mg\cos 35 \Rightarrow N = mg\cos 35$

$\Sigma F_\perp = ma = mg\sin\theta - \underbrace{F_{FF}}_{N\mu_K} = Mg\sin 35 - (mg\cos 35)(.1$

$\boxed{a = 4.4\ m/s^2}$

More room
★ ↓

d) For the case in part (c), what is the speed of the block at the bottom of the inclined plane?

use energy cons.

$$\underbrace{KE_{\text{bottom}}}_{\frac{1}{2}Mv^2} + \underbrace{\cancel{PE_{\text{BOTT}}}}_{0} = \underbrace{\cancel{KE_{\text{Top}}}}_{0} + \underbrace{PE_{\text{Top}}}_{mgh} - \underbrace{(\text{Work done by friction})}_{(F_{FF})(8.7)}$$

$v_{Top} = 0$

$$\frac{1}{2}mv^2 = mgh - \underbrace{(mg\cos 35)}_{N}\underbrace{(.15)}_{\mu_k}(8.7)$$

$$v = \sqrt{2gh - 2g\cos 35(.15)(8.7)} = \boxed{8.8\ m/s = v}$$

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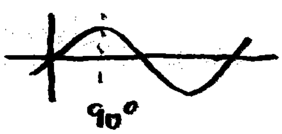
Problem 5 (15 pts):

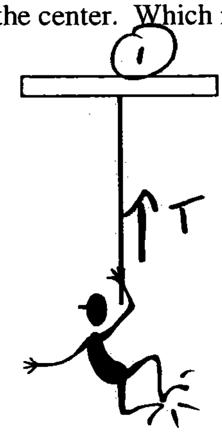
Drop (b)

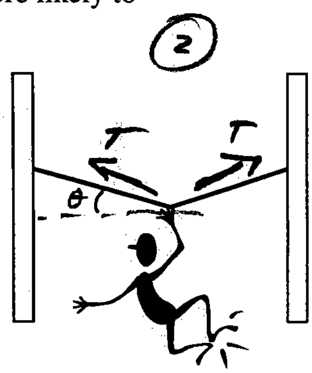
a) Elmer Fudd wears a hat in a cartoon. He steps into an elevator and presses the down button. The elevator descends so quickly that Elmer's hat comes off his head. Can this happen in real life? Explain why or why not.

This cannot happen unless Elmer is Tied or glued to the elevator and it accelerates downward with $|a| > g$. without this Both Elmer and his hat will accelerate downward together, if $|a|_{elevator} < g$.
If $|a|_{elevator} \geq g$, Elmer and his hat are in freefall (at $|a|_{elevator}$) and will both accelerate downward at g.

b) Two identical twins have identical ropes. One twin ties his rope to the branch of a tree and hangs (without swinging) from the end of the rope. The other twin ties his rope between two trees and hangs (without swinging) from the rope in the center. Which rope is more likely to break? Explain your answer.







① $T = mg$

② $2T\sin\theta = mg$

$$T = \frac{Mg}{2\sin\theta}$$

If θ is small

$T_{②} > T_{①}$ ∴ Case ② More likely to break

~~[illegible]~~

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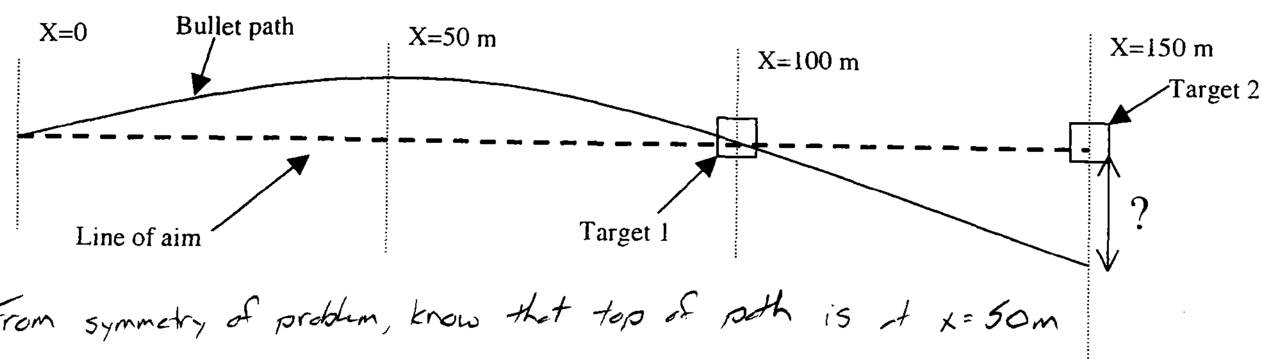
Problem 6 (15 pts):

Sadly, Afghanistan is currently a laboratory in applied physics. Each side in this conflict employ snipers equipped with high-powered rifles and telescopic gun sights. The gun sights are adjusted to aim high to compensate for the effect of gravity. This makes the gun accurate only in a specific range.

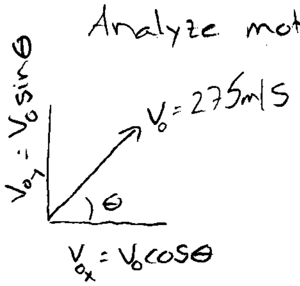
If a gun is sighted to hit targets that are at the same height as the gun and 100 meters away (for example, target "1" below), how low will the bullet hit if the gun is aimed directly at a target that is 150 meters away (such as target "2" below)? Assume the muzzle velocity (speed of the bullet as it emerges from the gun barrel) is 275 m/s and that the bullet makes an angle θ with the horizontal as it leave the gun. Ignore air resistance.
Helpful hint you may need: $2\sin\theta\cos\theta = \sin 2\theta$.



From symmetry of problem, know that top of path is at $x = 50\,m$

$\therefore V_y = 0$ at $x = 50$

Analyze motion to top of path:



along x: $x = \cancel{x_o} + V_{ox}t + \cancel{\tfrac{1}{2}a_x t^2}$
$x_o = 0$ $\quad a_x = 0$

$\Rightarrow 50 = V_o\cos\theta\, t$

$$\boxed{t = \frac{50}{V_o\cos\theta}} \quad (1)$$

along y: $V_y = V_{oy} + a_y t$

where $a_y = -g$ and $V_y = 0$ at $x = 50$

$\Rightarrow 0 = V_o\sin\theta - gt$

$$\boxed{t = \frac{V_o\sin\theta}{g}} \quad (2)$$

Equations (1) and (2) both represent t to x = 50. Solve for θ!

$$\frac{50}{V_o\cos\theta} = \frac{V_o\sin\theta}{g} \Rightarrow 50g = V_o^2\sin\theta\cos\theta \Rightarrow \frac{100g}{V_o^2} = \sin 2\theta$$

$$\Rightarrow \sin 2\theta = \frac{100(9.8)}{(275)^2} = 0.013$$

and so, $\theta = 0.37°$ (cont'd

Problem 6 : (cont'd)

Now, use this θ to find the time t its the bullet to reach $x = 50$

Using ① $t = \frac{50}{V_0 \cos\theta} = \frac{50}{(275)\cos(0.37)} = 0.18 \text{ sec}$

Now, we know that V_x never changes $(a_x = 0)$

∴ time to $x = 150$ is just $3(t \text{ to } 50)$

So, $t_{x=150} = (3)(0.18 \text{ sec}) = 0.55 \text{ sec}$

Find y at $t = 0.55 \text{ sec}$

$$y = y_0 + V_{0y} t + \tfrac{1}{2} a_y t^2$$

$$y = 0 + (275)\sin(0.37)(.55) + \tfrac{1}{2}(-9.8)(0.55)^2$$

$$y = 0.98 - 1.48$$

$$y = -0.5 \text{ m}$$

So, bullet hits 0.5m below the target at 150m.

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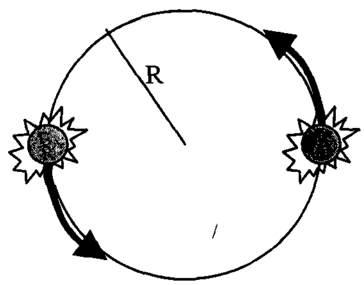
Problem 7 (15 pts):

My friend Zodork comes from another planet. In fact, his home planet has two suns. In other words, Zodork's world is situated in a binary star system. Each of the two stars in this system has a mass M. Each star moves in a circle of radius R in such a fashion that the other star is always situated exactly on the other side of the circle at all times. Find and circle the correct expression for the orbital period of one of Zodork's suns. That is to say, find the time it takes for one of the stars to go completely around the circular orbit one time. (Note: This has nothing to do with Kepler's equations and the sections of chapter 12 that we skipped.) You must prove this and show your work in order to receive credit.

a) $\frac{GM^2}{R^2}(2\pi R)$ b) $\frac{GM^2}{R^2}(\pi R^2)$ $\boxed{\text{c) } \frac{2\pi R}{\sqrt{\frac{GM}{4R}}}}$ d) $\frac{\pi R^2}{\sqrt{\frac{GM}{4R}}}$

e) $\frac{2\pi R}{\sqrt{\frac{GM}{R}}}$ f) $\frac{\pi R^2}{\sqrt{\frac{GM}{R}}}$

Zodork



$$\frac{Mv^2}{R} = \frac{GMM}{(2R)^2}$$

Centripetal force holding one sun on circle

Gravitational Attraction of other sun

$$v = \sqrt{\frac{GM}{4R}}$$

$$\text{Time} = \frac{\text{distance}}{\text{Velocity}} = \frac{2\pi R}{\sqrt{\frac{GM}{4R}}}$$