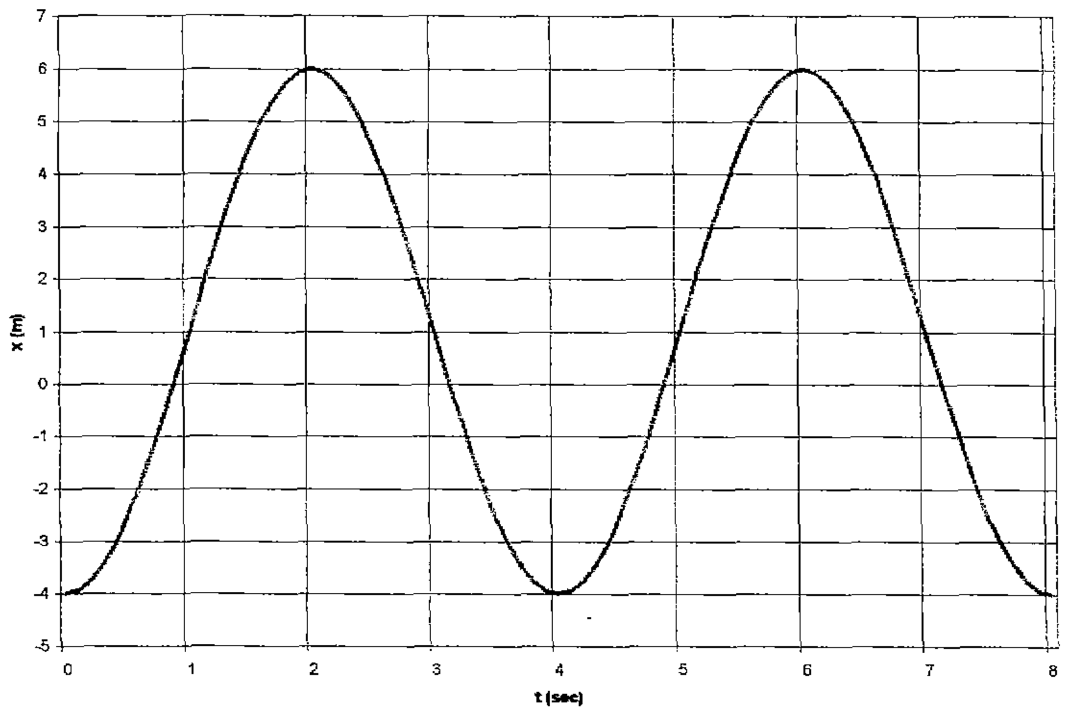
P113 NAME________________________________ University of Rochester
S. Manly Fall 1999

Exam 4 (December 3, 1999)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless noted otherwise.

Problem 1 (20 pts, 4 pts for each part) – no partial credit:

A 2 kg mass is attached to a massless, ideal spring. It is constrained to oscillate along one dimension on a horizontal frictionless surface. A mark has been placed at an arbitrary location on the table to designate the x=0 position for a graph of the motion. As the mass oscillates without loss of energy it traces the path shown on the graph below.



(Circle the correct answer.)

Part I: What is the amplitude of the motion for this oscillator?
6 m (B) –4 m (C) 5 m (D) 2 m

Part II: What is the period of the oscillator?
(A) 2 s (B) 4 s (C) 6 s (D) 8s

Part III: Of the following times, when is the mass moving the slowest?
(A) 1s (B) 2.5 s (C) 4.8 s (D) 6 s

Part IV: Of the following times, when does the mass have its greatest acceleration?
(A) 1s (B) 2.5 s (C) 4.8 s (D) 6 s

Part V: What is the spring constant of the spring?
(A) 19.74 N/m (B) 2 N/m (C) 4.93 N/m (D) 68.35 N/m

Problem 2 (20 pts) :

A round wooden ball is dropped from a height of 2 meters into a large pool of water. It plunges to a depth of 1.5 meters before rising in the water.

a) *(14 points)* Calculate the density of the wood in the ball.

Helpful approximations: Assume the ball passes through the air and the water without friction. Also, neglect the complicated situation that occurs during the time the ball begins to enter the water until it is completely submerged.

b) *(6 points)* With the approximations above, your answer to part (a) cannot be correct. Do you think your estimate is too high or too low? Why?

Problem 3 (20 pts):

Consider a mass m on a frictionless table sandwiched between two springs with spring constants k_1 and k_2, respectively. The springs each have a natural length L. The springs are collinear (stretched out along the same line) and the mass oscillates horizontally on the table. Choose the expression that gives the correct period for the motion. Show below how you arrived at your answer.

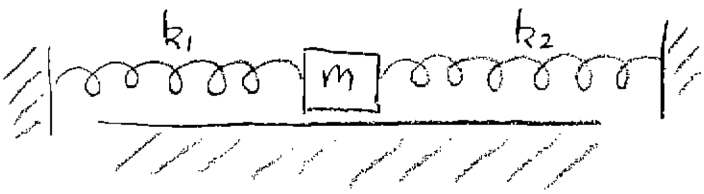
(a) $T = 2\pi\sqrt{\frac{m}{k_1 - k_2}}$

(b) $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

(c) $T = 2\pi\sqrt{\frac{m}{k_2 - k_1}}$

(d) $T = 2\pi\sqrt{\frac{m}{g}}$

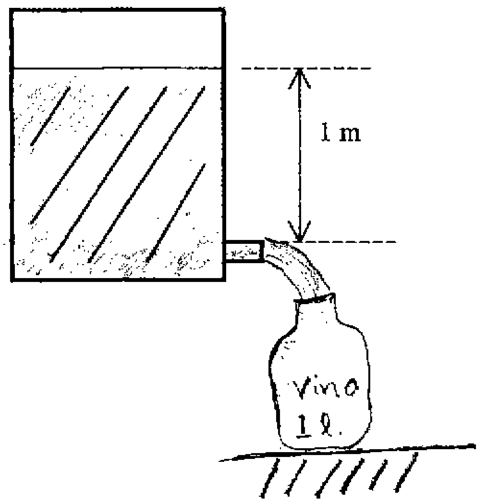
(e) $T = 2\pi\sqrt{\frac{L}{g}}$



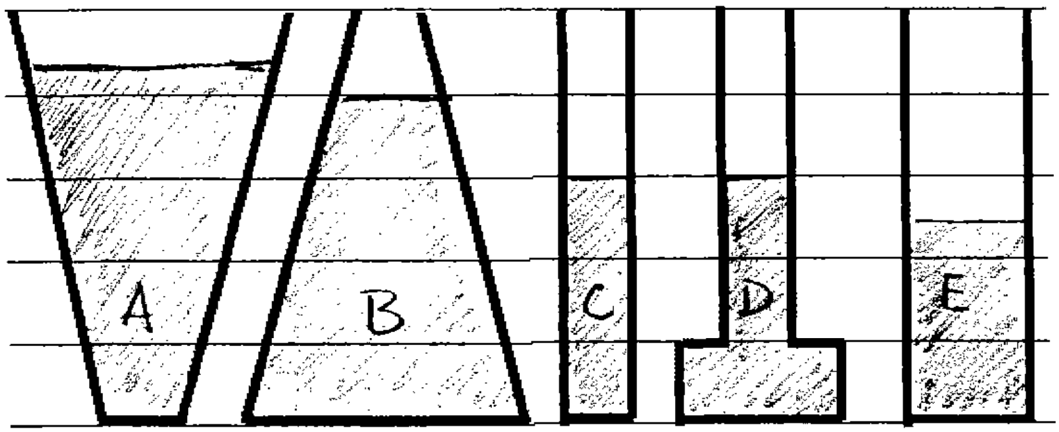
Problem 4(20 pts):

I have a friend who makes wine ... *not Chaz, he just consumes!* Suppose my friend had a big vat of wine shaped like a bucket with straight sides (pictured below). The vat has a radius of 0.5 m. One meter below the top level of the wine in the vat is a spigot used to fill jugs of wine. The spigot has round cross section and a radius of 1 cm. How long would it take my friend to fill a one liter jug of wine.

Helpful hints: 1 milliliter = 1 cm³. If needed, assume the density of wine is equal to that of water. Assume the movement of the level of the wine in the vat is negligible



Problem 5 (20 pts, 5 pts each part, no partial credit for part a):



(a) If the same fluid fills the containers shown, the pressures at the bases are related according to which of the following expressions?

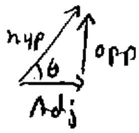
(i) $P_C<P_A<P_E<P_D<P_B$
(ii) $P_E<P_D<P_B<P_C<P_A$
(iii) $P_C=P_E<P_D<P_B<P_A$
(iv) $P_E<P_D=P_C<P_B<P_A$
(v) None of these is correct.

(b) In old World War II naval movies, the aircraft carriers always "turn into the wind" before launching aircraft off the bow of the ship. Why is this important? *("Turn into the wind" means turn so the wind is blowing across the ship from front to back)*

(c) You take a walk by the canal and see a ship filled with iron ore floating in a canal lock. Due to an emergency on board, the captain has much of the iron ore thrown overboard. What happens to the level of the water in the lock? Why?

(d) A hollow spherical shell with a non-negligible mass is filled with water through a small hole. It is hung by a long thread and set into motion as a simple pendulum. As the water slowly flows out of the hole at the bottom, one finds that the period of oscillation first increases and then decreases. Explain.

EXAM 4 - Formula Sheet



$\sin\theta = \frac{opp}{Hyp}$ $\quad \cos\theta = \frac{Adj}{hyp}$ $\quad \tan\theta = \frac{opp}{Adj}$

$V_x = V_{0x} + a_x t$

$x = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$

$V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$

$x = x_0 + \left(\frac{V_{0x} + V_x}{2}\right) t$

$x - x_0 = \int_{t_0}^{t} v\,dt$

$v - v_0 = \int_{t_0}^{t} a\,dt$

$\frac{dx^n}{dx} = n x^{n-1}$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$F_{Friction} = \mu_s N$

$F_{Friction} = \mu_k N$

$\sum \vec{F} = m\vec{a}$

$K.E. = \frac{1}{2} m v^2$

$\vec{F}_{Spring} = -k(\vec{x} - \vec{x}_0)$

$F_{radial} = m\frac{v^2}{R}$

$Power = \frac{dW}{dt}$

$Work = \int \vec{F} \cdot d\vec{s}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\omega = \omega_0 + \alpha t$ $\qquad S = r\theta$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\qquad v = r\omega$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $\qquad a = r\alpha$

$\theta - \theta_0 = \frac{1}{2}(\omega + \omega_0) t$

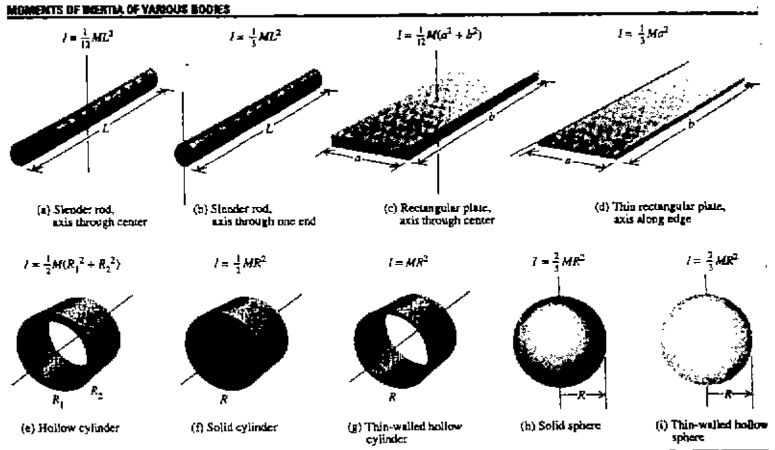
$X_{C.M} = \frac{\sum_i x_i M_i}{\sum M_i}$ $\qquad I = \sum_i m_i r_i^2$

$\vec{\tau}_{Net} = I\alpha$ $\qquad KE_{ROT} = \frac{1}{2} I \omega^2$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta$

$\vec{\tau} = \vec{r} \times \vec{F}$ $\qquad \vec{L} = \vec{r} \times \vec{P}$ $\qquad \vec{\tau} = \frac{d\vec{L}}{dt}$

TABLE 9-2
MOMENTS OF INERTIA OF VARIOUS BODIES



$A_1 V_1 = A_2 V_2$ $\qquad T = 2\pi\sqrt{\frac{L}{g}}$

$P + \frac{1}{2}\rho v^2 + \rho g y = \text{CONSTANT}$

$\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$ $\qquad T = 2\pi\sqrt{\frac{m}{k}}$

$\omega = \sqrt{\frac{k}{M}}$

$f(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + \cdots$