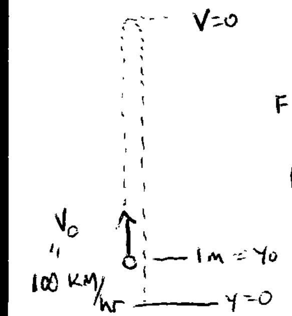
P121 Final Exam (May 10, 2001)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (10 pts):

A baseball player hits a ball straight up at 100 km per hour. How much time elapses between the moment the ball leaves the bat and the time it hits the ground. Assume the ball is one meter above the ground when it leaves the bat. Ignore air resistance and any rotational motion of the ball.



$100\ \text{KM/hr} = 100\ \text{KM/hr} \times \frac{1\ \text{hr}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{KM}} = 27.8\ \text{m/s}$

Find position at top

$v^2 = v_0^2 + 2a(Y_{top} - Y_0)$

$0 = (27.8)^2 - (2)(9.8)d$

$d = 39.4 \quad Y_{top} = 40.4\ \text{m}$

Time to top

$Y_{top} - Y_0 = \left(\frac{V_0 + V}{2}\right)t$

$39.4 = \frac{27.8}{2}t$

$t_{\text{to top}} = 2.83\ \text{s}$

Velocity at bottom

$V^2 = V_0^2 + 2a(0 - Y_{top})$

$V^2 = 2(9.8)40.4$

$V = 28.1\ \text{m/s}$ down

Time from top to bottom

$0 - Y_{top} = \left(\frac{V_{top} + V_{bott}}{2}\right)t$

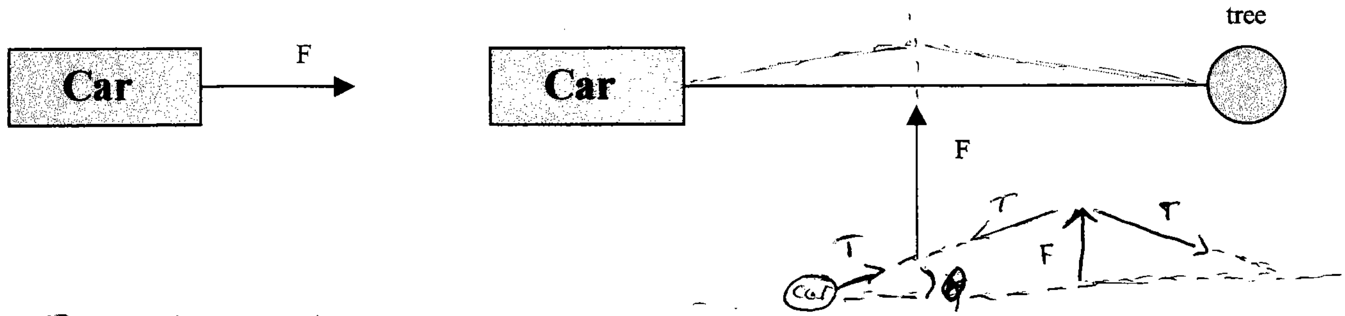
$40.4 = \frac{28.1}{2}t$

$t = 2.87\ \text{s}$ top to bott

Problem 2 (10 pts):

TOTAL Time $= 2.83 + 2.87 = 5.7\ \text{s}$

Your mother's car is stuck in the snow. She has to get to work and has asked for your help. You try all the usual tricks such as shoveling around the tires and putting sand around the tires. Nothing seems to work. Then you tie a rope to the car and pull with all the force you can muster (see the left hand figure). No luck Then you tie the rope to a tree and push as hard as you can in a direction perpendicular to the rope (see the right hand figure). The car moves and is eventually freed. Assume the magnitude of the force that you are able to exert on the rope is identical in both cases and that the rope can stretch slightly. Explain why the latter approach worked. Use diagrams and equations as needed.



In the 1ST case, the car is being pulled with a force $= |\vec{F}|$.

In the 2ND case, the car is being pulled with a Force $|T| > |\vec{F}|$ because $|\vec{F}|$ acts as a component of the TOTAL force T on the car.

If the rope makes an angle θ with the original direction of the rope, F and T are related by $F = 2T\sin\theta$ (see diagram) (comes from demanding $\sum F = 0$ in direction $\perp$ to rope. Let $\theta = 5°$ for example

Then $T \cong 6F$!

$T > F \Rightarrow$ The car moves

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Problem 3 (10 pts):

A cork bobbing on the surface of a lake has the vertical displacement as a function of time given by the graph below.

(a) What is the amplitude for the harmonic motion of the cork (look carefully!)? 5 m

(b) What is the period of the simple harmonic motion of the cork? 4 s

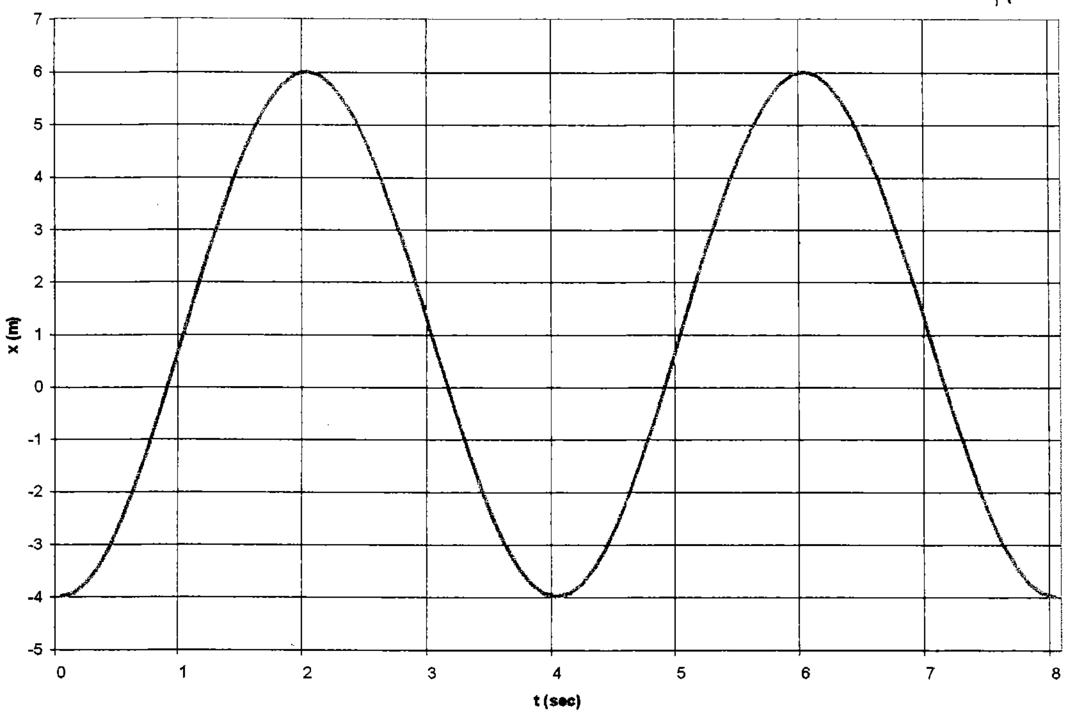
(c) What is the frequency of the water wave passing by the cork? $f = \frac{1}{T} = \frac{1}{4}\ s^{-1}$ or $.25\ Hz$ or $\omega = \frac{2\pi}{T} = 1.57\ s^{-1}$

(d) What is the wavelength of the water wave passing by the cork (assume the velocity of the wave is 1 m/s)?

$V = \lambda/T \qquad (1\ m/s)\, 4s = \lambda = 4\ m$

(e) Write a mathematical expression that describes the vertical motion of the cork with time.

$$x(t) = \overset{A}{5}\sin\left(\overset{\omega}{\frac{2\pi}{4}}t - \overset{\phi}{\frac{\pi}{2}}\right) \quad \text{or} \quad x(t) = \underset{A}{5}\cos\left(\overset{\omega}{\frac{2\pi}{4}}t - \overset{\phi}{\pi}\right)$$



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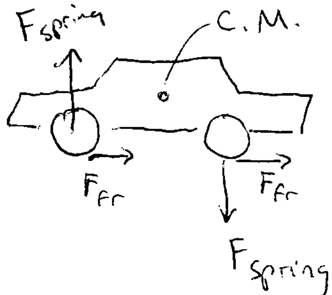
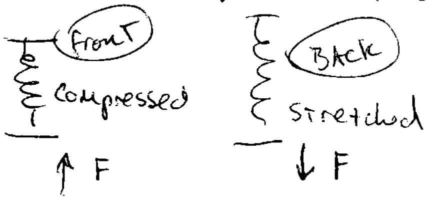
Problem 4 (10 pts):

(a) When you lock the brakes in a moving car, the front end of the car moves down while the back end rises. Explain why this is so.

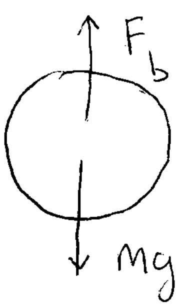


Refer to Drawing.

When brakes locked there is a force due to friction of road on tires that is directed backwards along the car. This causes a torque about the Center of Mass, ~~that~~ which in turn causes a rotation counterclockwise on the drawing (about the cm). This will continue until the springs that connect the car to the wheels are compressed (front) or stretched (back) enough that they provide a torque of equal magnitude and opposite in sign abt the C.M. which cancels the torque due to friction.



(b) Hydrogen or helium filled dirigibles and hot air balloons are often called "lighter than air" craft. Is this an accurate description? Why or why not.



According to Archimede's Principle the bouyant force (F_B) on an object is equal to the weight of the displaced fluid, which is air in the case of a dirigible or hot air balloon. If $F_b > Mg$ then the net force is up and the object will "fly".

$F_b > Mg$

WT of displaced Air > WT of object

∴ The weight of the object is less than the weight of the displaced air and the term "lighter than Air" makes sense.

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Problem 5 (10 pts):

A body of mass m is whirled at a constant angular velocity on the end of a string of length R. Calculate the length of string (in terms of R) needed to double the kinetic energy of the body while maintaining the angular velocity.

$KE_1 = \frac{1}{2} m v_1^2$

$a_c = \frac{m v_1^2}{R}$

$\omega_1 = \frac{v_1}{R}$

$KE_1 = \frac{1}{2} m \omega_1^2 R^2$

$KE_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} m \omega_2^2 R_2^2 = \frac{1}{2} m \omega_1^2 R_2^2$

$\omega_2 = \omega_1$

Let $KE_2 = 2 KE_1$

$\frac{1}{2} m \omega_1^2 R_2^2 = 2 \frac{1}{2} m \omega_1^2 R_1^2$

$\therefore R_2^2 = 2 R_1^2 \qquad R_2 = \sqrt{2} R$

length Required to double KE w/ ω const.

Problem 6 (10 pts):

A rod of an unknown material has a length of 20.05 cm as measured on a steel ruler (steel has a linear coefficient of thermal expansion of 1.2×10^{-5} per degree centigrade). Both the rod and the steel ruler are placed in an oven at 270 degrees centigrade. At the new temperature the rod measures 20.11 cm on the same ruler. What is the linear coefficient of thermal expansion for the unknown material?

Rod 20.05 cm

Steel

270°

20.11 ΔL

~~$\Delta L_{Rod} = \alpha L_{Rod} \Delta T$~~

~~$\Delta L_{steel} = \alpha_{steel} L_{steel} \Delta T$~~

~~For steel~~

~~$\Delta L = L \alpha \Delta T = (20.05)(1.2 \times 10^{-5})(250)$~~

~~$\Delta L = 0.06$~~

~~New steel length at 270~~

~~$20.05 + 0.06 = 20.11$~~

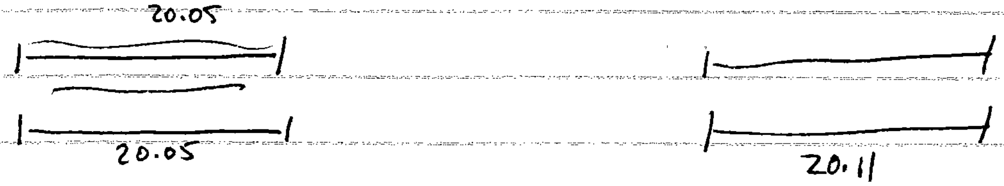
~~Rod measures 20.11 cm~~

~~∴ Rod and steel expand the same amount~~

~~$\therefore \alpha_{Rod} = \alpha_{steel} = 1.2 \times 10^{-5}\ C^{\circ -1}$~~

Incorrect soln

SEE NEXT PAGE



$$\frac{\left(\frac{\Delta L}{L}\right)_{rod}}{\left(\frac{\Delta L}{L}\right)_{Ruler}} = \frac{20.11}{20.05}$$

$$\Delta L = \alpha \, \Delta T \, L$$

$$\frac{\Delta L}{L} = \alpha \, \Delta T$$

$$\frac{\alpha_{rod}}{\alpha_{steel}} = \frac{20.11}{20.05}$$

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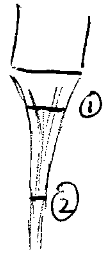
Problem 8 (10 pts):

(a) Courageous physics students pour non-alcoholic beer in a funnel and watch it pour on the floor as they prepare for a physics project. They notice the stream narrows as the beer falls from the funnel. Washing up later, they notice as water emerges from a faucet, the stream narrows as the water falls. Being unrepentant physics geeks, they hypothesize the narrowing is due to the same physical effect in both cases. Explain the cause for the narrowing of the fluid stream in these examples. (The cause is the same in both cases and has nothing to do with the specific nature of the fluid or the state of mind of the observers.)

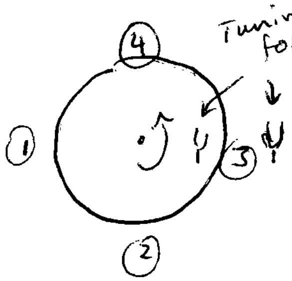


Bernoulli's eqn $\text{const} = P + \frac{1}{2}\rho v^2 + \rho g h$ implies that the velocity of the fluid in the stream at 2 is greater than that at 1. We expect this due to the fact that as the fluid falls, gravitational potential energy is converted into KE of motion.

So, $V_2 > V_1$

Mass conservation implies $A_1 V_1 = A_2 V_2$, where A_1 is the cross sectional area of the stream at 1 and A_2 is the cross sectional area at 2. Since $V_2 > V_1$ we must have $A_2 < A_1$ to preserve the equality above. Thus the stream narrows as it falls

(b) Two tuning forks have identical frequencies, but one is stationary and the other is mounted at the rim of a rotating platform. What does a listener hear and why?



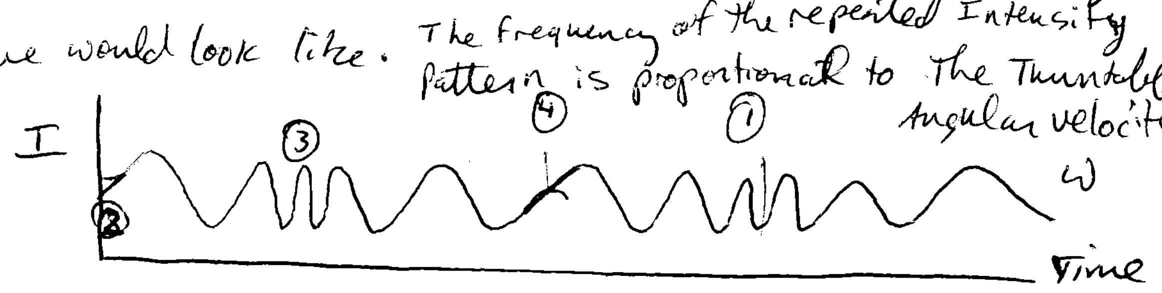
Tuning forks

at 4 and 2 The frequencies are identical. One hears that frequency at a given intensity

at 1 The frequency of the rotating fork is ~~blueshifted~~ Doppler shifted to a slightly higher frequency than the stationary fork. So one hears the intensity modulated with a frequency that depends on ~~the~~ ω for the turntable (or the degree of the doppler shift). In other words one hears beats. Similar for position 3 but frequency of moving fork is lowered due to Doppler shift.

ear

So as the fork goes around one hears beats that vary in frequency. The intensity curve would look like. The frequency of the repeated Intensity pattern is proportional to the turntable angular velocity ω



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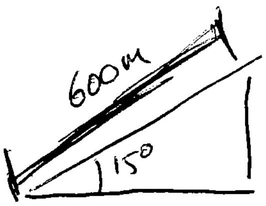
Problem 9 (10 pts):

A T-bar tow pulls skiers up a 600-m slope inclined at 15 degrees with respect to the horizontal. (The slope is 600-m in length, not height.) Assume the mass of the average skier is 75 kg. Assume the rope and T-bars are massless.

(a) Let the slope be frictionless. How much work is expended by the rope tow pulling one skier up the slope?



$h = 600 \sin 15 = 155.3$ M

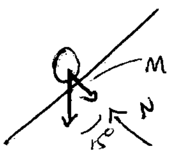
Energy Conservation $\Rightarrow$ Work done $= mgh = (75)(9.8)(155.3)$

$= 1.14 \times 10^5$ J

(b) Suppose μ_k=0.06 (the slope is no longer frictionless). How much work is expended by the rope tow pulling one skier up the slope?

Energy cons $\Rightarrow$ Work done $= Mgh + E_{\text{lost due to friction}}$

$= 1.14 \times 10^5 \text{ J} + \mu_k (Mg \cos 15)(600)$



$Mg \cos 15 = N$

$(0.06)(75)(9.8)(\cos 15)\ 600$

2.55×10^4 J

work done $= 1.39 \times 10^5$ J

(c) This rope tow is designed to pull 80 skiers up the slope at 2.5 m/s. Assuming μ_k=0.06, what is the power required for the rope tow motor?

TOTAL work $= (80)(1.39 \times 10^5)$ J

in Time

$d = vT$

$\frac{600 \text{ M}}{2.5 \text{ M/s}} = 240$ s

Power $= \frac{\text{TOT wk}}{\text{Time}} = 4.6 \times 10^4$ WATTS

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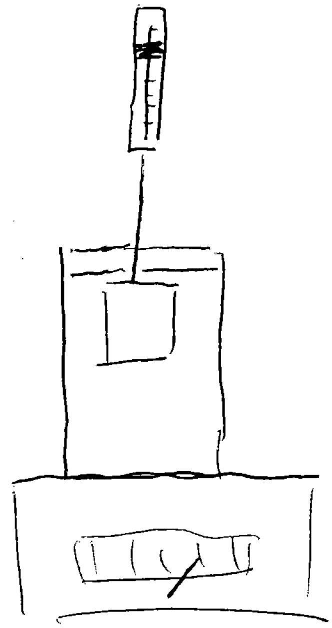
Problem 10 (10 pts):

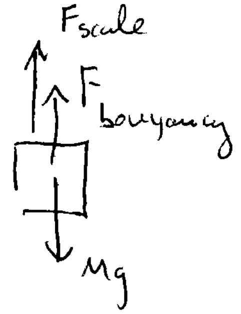
A beaker of mass 1 kg containing 2 kg of water rests on a scale. A 2-kg block of aluminum (specific gravity 2.7) suspended from a spring scale is submerged in the water as in the figure below. Find the readings on both scales.



$$\text{Specific gravity} = \frac{\rho_{\text{material}}}{\rho_{\text{water at } 4^\circ C}} \qquad \text{Mass} = \rho_{\text{Block}} V_{\text{Block}}$$

$$V_{\text{block}} = \frac{m_{\text{block}}}{\rho_{\text{block}}}$$

upper scale



$\Sigma F_y = 0$

$$F_{\text{scale}} = Mg - F_{\text{bouyant}}$$

volume of block

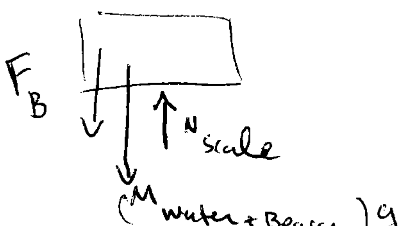
$$= Mg - \rho_w V_{\text{block}} g$$

$$= M_{\text{block}} g - \rho_w \frac{M_{\text{block}}}{\rho_{\text{block}}} g$$

$$F_{\text{scale}} = M_{\text{block}}\left(g - \frac{g}{\text{s.g.}_{\text{Al}}}\right) = 2\left(9.8 - \frac{9.8}{2.7}\right)$$

$$\boxed{F_{\substack{\text{scale}\\(\text{Top})}} = 12.3\text{ N}}$$

Bottom Scale



IF Water exerts bouyant force up on Al block, By Newton's 3rd law the block exerts a Force of equal Magnitude down on the water.

$$N_{\text{scale}} = (M_{\text{water+beaker}})g + M_{\text{block}} \frac{\rho_w g}{\rho_{\text{block}}} = (3)\,9.8 + \frac{2(9.8)}{2.7} = \cancel{30.1}\ \cancel{13.5\text{ N}}$$

$$\boxed{F_{\substack{\text{Scale}\\\text{lower}}} = 36.6\text{ N}}$$

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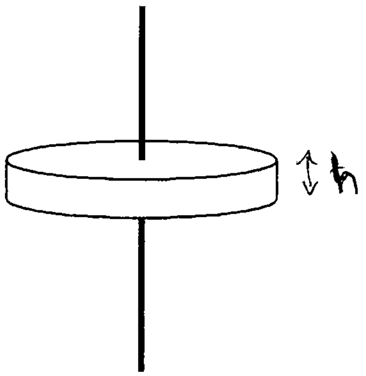
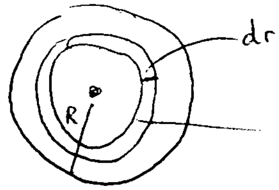
Problem 11 (10 pts):

of radius R, mass M, and thickness h

Use integration to determine the moment of inertia of a thin, uniform disk rotating about an axis perpendicular to the plane of the disk and passing through the center of the disk. **Hint:** The differential volume of an annular ring of width dr is dv=(2πr)dr

h





$$\rho = \frac{M_{disk}}{\pi R^2 h}$$

$$I = \int r^2\,dm = \int r^2 \rho\,dv$$

$$I = \int_0^R r^2 \rho\, 2\pi r h\,dr = \rho 2\pi h \int_0^R r^3\,dr = \rho 2\pi h \frac{R^4}{4}$$

$$I = \frac{M}{\pi R^2 h}\, 2\pi h \frac{R^4}{4} = \frac{1}{2} M R^2$$

1)	/10
2)	/10
3)	/10
4)	/10
5)	/10
6)	/10
7)	/10
8)	/10
9)	/10
10)	/10
11)	/10
tot	/110

It'll be a few days before I'll be able to pull together the final grades in the course. Once I have the final grades, I plan to email you individually with the information that I have for you in my spreadsheet. You'll want to check it for accuracy. This will only happen if you have given me your xxxx.mail.rochester.edu address. You can look on the project listings (on the class website) to see if I have the correct address for you in my spreadsheet.

Have a great summer! Don't bore your parents with too much discussion of interference in the water waves on the lake!