P113 University of Rochester NAME Solution Key - SLM
S. Manly Fall 2006

Exam 2 (October 26, 2006)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (14 pts, justify your answers):

Jill runs up a flight of stairs. The gain in her potential energy is U. If she runs up the same stairs with twice the speed, what is her gain in potential energy?

a) **U**
b) **2U**
c) **(1/4)U**
d) **4U**
e) **(1/2) U**

$U \sim mgh$

No dependence on rate or speed

Jill runs up a flight of stairs. She exerts power P while doing this. If she runs up the same stairs with twice the speed, what power does she expend?

a) **P**
b) **2P**
c) **(1/4)P**
d) **4P**
e) **(1/2) P**

twice the speed $\rightarrow \frac{1}{2}$ the time

Same work done

$$\text{power} \sim \frac{\text{work}}{\text{time}} \rightarrow 2P$$

Problem 2 (11 pts, show your work):

Evaluate the dot product (scalar product) between the two vectors $\vec{A} \cdot \vec{B}$
$\vec{A}=4\hat{i}-2\hat{j}+3\hat{k}$ **and** $\vec{B}=2\hat{i}+5\hat{j}-2\hat{k}$

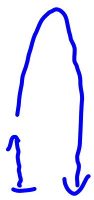
$$\vec{A} \cdot \vec{B} = (4)(2) + (-2)(+5) + (3)(-2)$$

$$\vec{A} \cdot \vec{B} = 8 - 10 - 6 = -8$$

1)	14 /14
2)	11 /11
3)	15 /15
4)	15 /15
5)	15 /15
6)	15 /15
7)	15 /15
tot	100/100

Problem 3 (15 pts):

In some parts of the world people celebrate by shooting guns into the air. Briefly explain, using concepts we have discussed in class, why this is a bad idea.

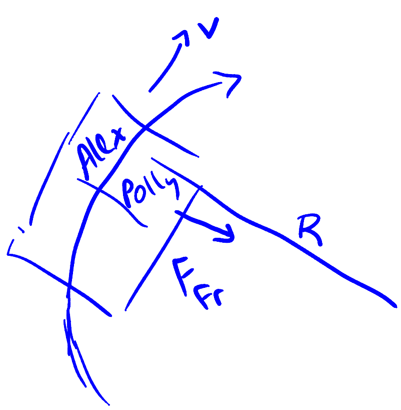


From energy conservation the bullet comes down with the same speed that it leaves the gun going up in the absence of air friction. Air friction slows the bullet down somewhat — but it still comes down with a large speed that is hazardous to all the people and dogs in the vicinity.

Killing someone on roof, inviting ATTAck by overflying jet, etc. would get a little credit.

Problem 4 (15 pts, show your work):

Alex McStudley offers Polly Puritan a ride to the big game. He's very happy when he sees that Polly forgets to buckle her seatbelt. You see, Alex would like to be closer to Polly, if you know what I mean. Fortunately, Alex has taken physics and decides to use his newfound knowledge to help him accomplish his aims with respect to Polly. Alex makes a quick turn in order to make Polly slide closer to him. If Alex maintains a constant speed of 25 m/s, what is the maximum turn radius Alex could make in the car and still have Polly slide toward him? ***Assume the coefficient of static friction between Polly and the car seat is 0.4 and the coefficient of kinetic friction between Polly and the car seat is 0.2.***



Polly begins to slide when the centripetal force necessary to keep Polly on the circle is greater than the maximum frictional force. For any radius less than that satisfying

$$\frac{mv^2}{R} = F_{fr} = \mu_s mg \Rightarrow R = v^2/\mu_s g$$

Polly begins to slide

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Problem 5 (15 pts, show your work):

Three masses lie in a line far away from all other masses, as shown in the sketch below. The middle mass lies at point P.

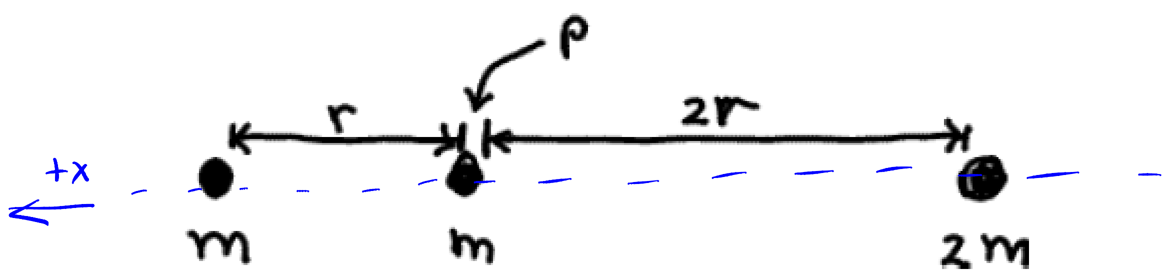
a) What is the net gravitational force on the mass located at point P due to the other masses?

$$\vec{F}(p) = \left[\frac{Gmm}{r^2}(+\hat{x}) + \frac{G(2m)m}{(2r)^2}(-\hat{x})\right] = \frac{Gmm}{2r^2}(+\hat{x})$$

toward the small mass

b) What is the net gravitational field at point P? (For simplicity, assume the mass at point P is not present for this part.)

$$\vec{g}(p) = \frac{\vec{F}(p)}{m} = \frac{Gm}{2r^2}(+\hat{x})$$



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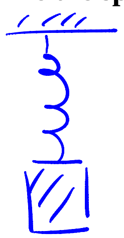
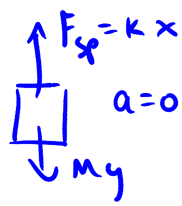
Problem 6 (15 ptsshow your work):

When a 0.5 kg block is hangs motionlessly on a spring, the spring is stretched 0.1 m beyond its natural length. Now, let this same block sits at rest on an inclined plane that makes an angle of 20 degrees with the horizontal as shown in the sketch below. The same spring is attached to the block (as shown in the sketch) and slowly pulled in the direction of the force shown on the diagram. ***The coefficient of static friction between the block and the inclined plane surface is 0.4. The coefficient of kinetic friction between the block and the inclined plane surface is 0.2.***

a) Determine the spring constant of the spring.

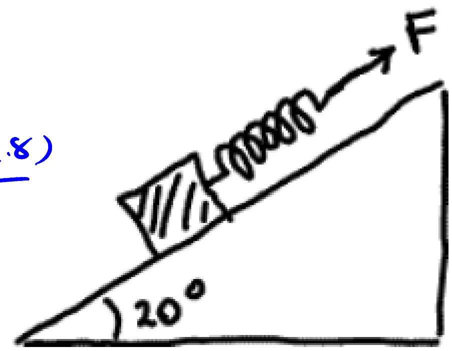




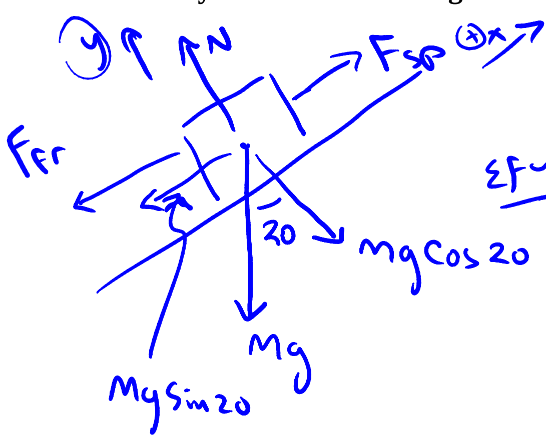
$kx = mg$

$k = \frac{mg}{x} = \frac{(0.5)(9.8)}{0.1}$

$k = 49\ N/m$



b) Calculate the minimum extension of the spring (beyond the natural length) necessary for the block to begin moving up the inclined plane.



$F_{fr} = \mu_s N$

$F_{sp} = kx$ minimum ext. of spring beyond Nat. length

$\Sigma F_y = 0$ $N = mg \cos 20$

$\Sigma F_x = 0$ at limit just before block moves due to Force of Spring

$$0 = kx - \mu_s mg \cos 20 - mg \sin 20$$

$$x = \frac{\mu_s mg \cos 20 + mg \sin 20}{k} = 0.07\ m$$

($\mu_s = .4$, $m = .5$, $k = 49$)

Problem 7 (15 pts, show your work):

The steam catapult on an aircraft carrier exerts a force on Airman Spiff's experimental jet of $\vec{F}(x) = (100x^2 + 200)\,\hat{x}$ Newtons where x is the distance the jet moves down the carrier deck from its starting point. Spiff and his aircraft have a combined mass of 2000 kg.

a) What is the work done by the catapult as the plane moves the first ten meters along the deck during launch?

$$W = \int \vec{F}\cdot d\vec{s} = \int_0^{10} (100x^2 + 200)\,dx = \left.\frac{100x^3}{3} + 200x\right|_0^{10}$$

$\vec{F}$, $d\vec{s}$ in same direction

so $\vec{F}\cdot d\vec{s} = |F||dx|$

$= 35333$ Joules

b) How fast is the plane moving at the ten meter point in the launch?

Work done = KE of plane (neglecting friction)

$$35333 = \frac{1}{2} m v^2$$

$$\frac{(2)(35333)}{2000} = v^2 \qquad v = 6 \text{ m/s}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$v = v_o + at$$

$$x = x_o + v_o t + \frac{1}{2}at^2$$

$$x = x_o + \left(\frac{v_o + v}{2}\right)t$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$x - x_o = \int_{t_o}^{t} v\,dt$$

$$v - v_o = \int_{t_o}^{t} a\,dt$$

$$\sum \vec{F} = m\vec{a}$$

$$F_{friction} = \mu_k N$$

$$F_{friction} \le \mu_s N$$

$$F_{centripetal} = \frac{mv^2}{r}$$

$$\vec{F}_{spring} = -k(\vec{x} - \vec{x}_o)$$

$$work = \int F \cdot ds$$

$$power = \frac{dw}{dt}$$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{F} = \frac{Gm_1 m_2 \hat{r}}{r^2}$$

$$G = 6.67x10^{-11}\ \text{Nm}^2/\text{kg}^2$$

$$PE_{grav} = -\frac{GM}{r}$$

$$PE_{spring} = \frac{1}{2}kx^2$$

circumference of circle $= 2\pi r$

area of circle $= \pi r^2$

quadratic equation $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$