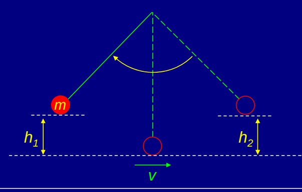
Example: The simple pendulum

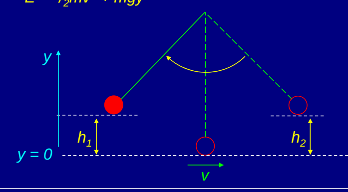
- Suppose we release a mass m from rest a distance h_1 above its lowest possible point.
  - What is the maximum speed of the mass and where does this happen?
  - To what height h_2 does it rise on the other side?



Example: The simple pendulum

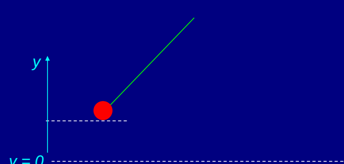
- Kinetic+potential energy is conserved since gravity is a conservative force ($E = K + U$ is constant)
- Choose $y = 0$ at the bottom of the swing, and $U = 0$ at $y = 0$ (arbitrary choice)

$E = \frac{1}{2}mv^2 + mgy$



Example: The simple pendulum

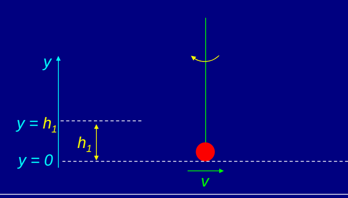
- $E = \frac{1}{2}mv^2 + mgy$.
  - Initially, $y = h_1$ and $v = 0$, so $E = mgh_1$.
  - Since $E = mgh_1$ initially, $E = mgh_1$ always since energy is conserved.



Example: The simple pendulum

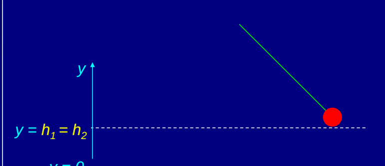
- $\frac{1}{2}mv^2$ will be maximum at the bottom of the swing.
- So at $y = 0$ ⇨ $\frac{1}{2}mv^2 = mgh_1$ ⇨ $v^2 = 2gh_1$

$$v = \sqrt{2gh_1}$$



Example: The simple pendulum

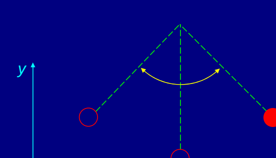
- Since $E = mgh_1 = \frac{1}{2}mv^2 + mgy$ it is clear that the maximum height on the other side will be at $y = h_1 = h_2$ and $v = 0$.
- The ball returns to its original height.



Example: The simple pendulum

- The ball will oscillate back and forth. The limits on its height and speed are a consequence of the sharing of energy between K and U.

$E = \frac{1}{2}mv^2 + mgy = K + U = constant.$



Generalized Work/Energy Theorem:

$W_{NC} = \Delta K + \Delta U = \Delta E_{mechanical}$

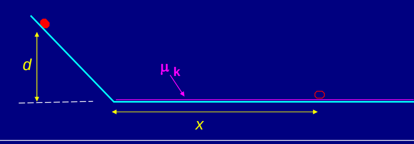
- The change in kinetic+potential energy of a system is equal to the work done on it by non-conservative forces. $E_{mechanical}$=K+U of system not conserved!

  - If all the forces are conservative, we know that K+U energy is conserved: $\Delta K + \Delta U = \Delta E_{mechanical} = 0$ which says that $W_{NC} = 0$.

  - If some non-conservative force (like friction) does work, K+U energy will not be conserved and $W_{NC} = \Delta E$.

Problem: Block Sliding with Friction

- A block slides down a frictionless ramp. Suppose the horizontal (bottom) portion of the track is rough, such that the coefficient of kinetic friction between the block and the track is μ_k.
  - How far, x, does the block go along the bottom portion of the track before stopping?



Problem: Block Sliding with Friction...

- Using $W_{NC} = \Delta K + \Delta U$
- As before, $\Delta U = -mgd$
- W_{NC} = work done by friction = $-\mu_k mgx$.
- $\Delta K = 0$ since the block starts out and ends up at rest.
- $W_{NC} = \Delta U \Rightarrow -\mu_k mgx = -mgd$

$x = d / \mu_k$