Physics 113 - September 12, 2006

1-d Motion, Const accel. eqns

organizational issues

Workshops begin today

Workshop Section list is on the web

Workshop migration

Labs start this week

A-B Assignments made

Review of last class

Kinematic variables

x (or y or z) $\equiv$ position

$v \equiv$ velocity

$a \equiv$ Acceleration

$t \equiv$ time

in 1-d motion direction given by Algebraic sign

Have $x(t), v(t), a(t)$

Not Independent

Average Speed $= \frac{\Delta x}{\Delta t}$

Distance Traveled

Average velocity $= \frac{\Delta x}{\Delta t}$

$\bar{v}$

Displacement over time interval

in limit $\Delta t \to 0$

Average Velocity $\to$ instantaneous Velocity

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \equiv v$$

Also known as "velocity"

Similarly,

$$\text{Average Acceleration} = \frac{\Delta v}{\Delta t}$$

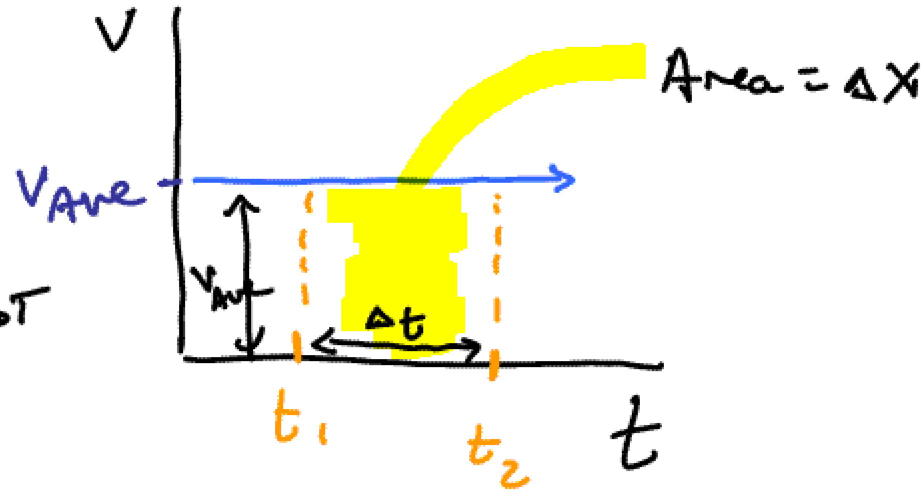
$$\text{instantaneous Acceleration or acceleration} = \frac{dv}{dt}$$

$$= \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$$

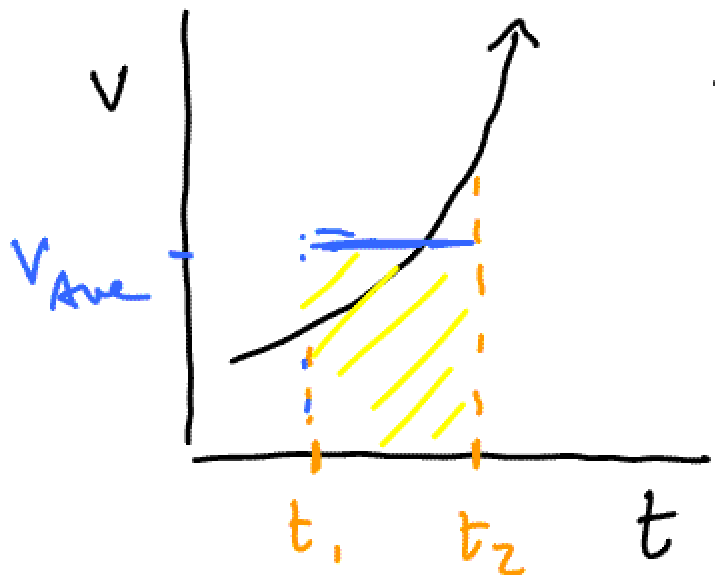
do x-t, v-t PRS exercises

$$\frac{\Delta x}{\Delta t} = V_{Ave}$$

$$V(t) = \text{const}$$



$$\Delta x = V_{Ave} \Delta t$$



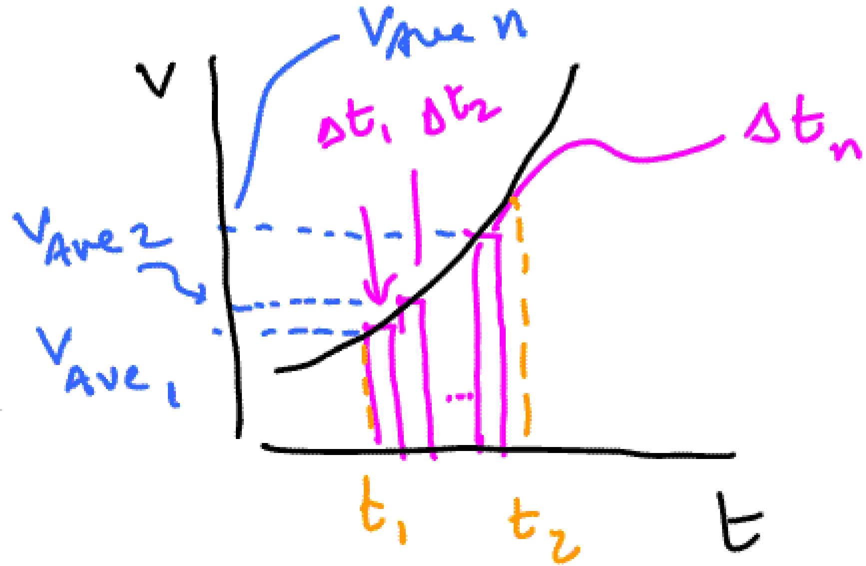
$$\Delta x = V_{Ave} \Delta t$$

interpret.

not so clean

...

$$\Delta x = \sum_i V_{Ave\,i}\,\Delta t_i$$



go to $\Delta t \to 0$ limit

$$\Delta x = \lim_{\Delta t \to 0} \sum_i V_{Ave\,i}\,\Delta t_i$$

$$\Delta x = \int_{t_1}^{t_2} V\,dt$$

$$\Delta t \longrightarrow dt$$

$V_{Ave} \longrightarrow V$ (instantaneous) if $g = \int f\,dt$, then $f = \frac{dg}{dt}$

$$\boxed{\underbrace{\Delta x}_{x - x_0} = \int_{t_0}^{t} v\,dt}$$

$v(t)$

very General
Always true

$$\frac{\Delta v}{\Delta t} = a_{\text{ave}} \longrightarrow \frac{dv}{dt} = a$$

$$\underbrace{v - v_0}_{\Delta v} = \int_{v_0}^{v} dv = \int_{t_0}^{t} a\,dt$$

$a(t)$

$$\boxed{v - v_0 = \int_{t_0}^{t} a\,dt}$$

very general
Always
True

Special case

$a \equiv \text{CONSTANT}$

$$v - v_0 = \int_{t_0}^{t} a\,dt = a\int_{t_0}^{t} dt = a\; t\Big|_{t_0}^{t}$$

$$v - v_0 = a(t - t_0)$$

$$\boxed{v = v_0 + at}$$

$t_0 = 0$ for simplicity

v, a, t

NO x

$$x - x_0 = \int_{t_0}^{t} v\,dt$$

$$x - x_0 = \int_{t_0}^{t} (v_0 + at)\,dt$$

$$\int t = t^2/2$$

$$x - x_0 = \int_{t_0}^{t} v_0\,dt + \int_{t_0}^{t} at\,dt$$

$$x - x_0 = v_0(t - t_0) + \frac{a}{2}(t^2 - t_0^2)$$

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2} a t^2}$$

x, t, a

Not v

$$\Delta x = v_{Ave} \Delta t$$

Const a $\longrightarrow$ v changes at const rate

$$v_{Ave} = \frac{v + v_0}{2}$$

$$x - x_0 = \frac{v + v_0}{2} (t - \cancel{t_0}^{\,0})$$

$$\boxed{x = x_0 + \left(\frac{v + v_0}{2}\right) t}$$

x, v, t

No a

$v = v_0 + at$

Solve for t $\qquad \frac{v - v_0}{a} = t$

Subst. in above eqn

$$x = x_0 + \left(\frac{v+v_0}{2}\right)\left(\frac{v-v_0}{a}\right)$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

v, a, x

No t

Woodpecker problem

$v_0 = 0.6 \ m/s$

comes to stop in 2 mm

(a) what is accel. of head in units of g ?

$$g \equiv 9.8\, m/s^2$$

Know V_0

$x - x_0$

$V = 0$

want a

$$\underbrace{v^2}_{0} = \underbrace{v_0^2}_{+0.6\, m/s} + 2a(\overbrace{x - x_0}^{+2\,mm})$$

$2\,mm = .002\,m$

$$\frac{-(0.6)^2}{2(.002)} = a$$

$a = -90\, m/s^2$

$$\frac{|-90|}{9.8} \sim 9\ g\text{'s}$$

(b) how much time head to stop

$$v = v_0 + at$$

$0 \qquad .6 \, m/s \qquad -90$

$$t = \frac{-.6}{-90} = 6.7 \, ms$$

millisec

.0067

Tendon's stretch

Allow brain to stop in 4.5 mm

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$a = -40 \, m/s^2$$

or $\underline{4g}$