Physics 113 - Sept. 19, 2006

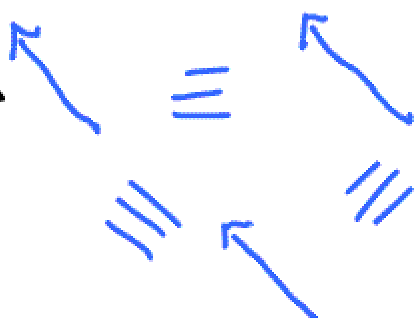
Vectors, multidimensional motion

Last time

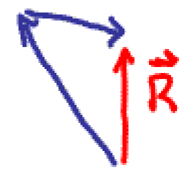
Scalar $\Rightarrow$ # — magnitude only

Vector $\Rightarrow$ 3 #s — Magnitude + Direction

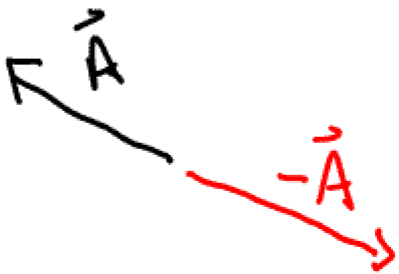
$\vec{A}$



graphical addition of vectors

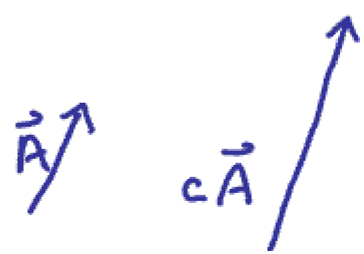


$$\vec{A} + \vec{C} = \vec{R}$$

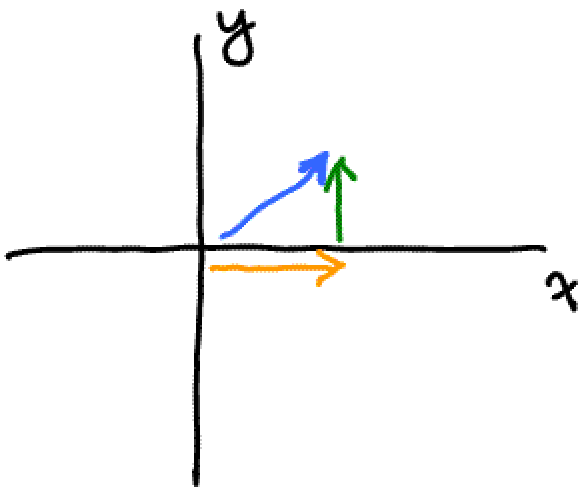


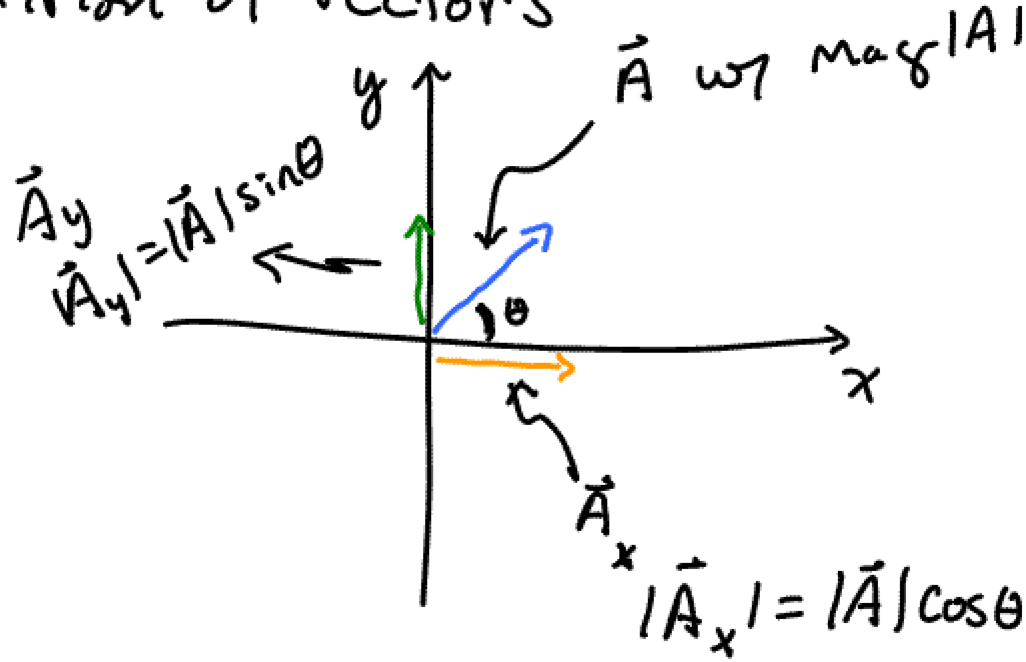
$$c\vec{A} = |c\vec{A}| \text{ direction of } \vec{A}$$

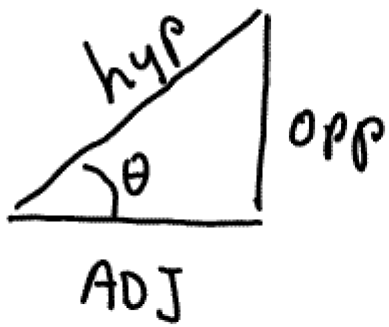
$c = 2$



Analytical Addition of vectors



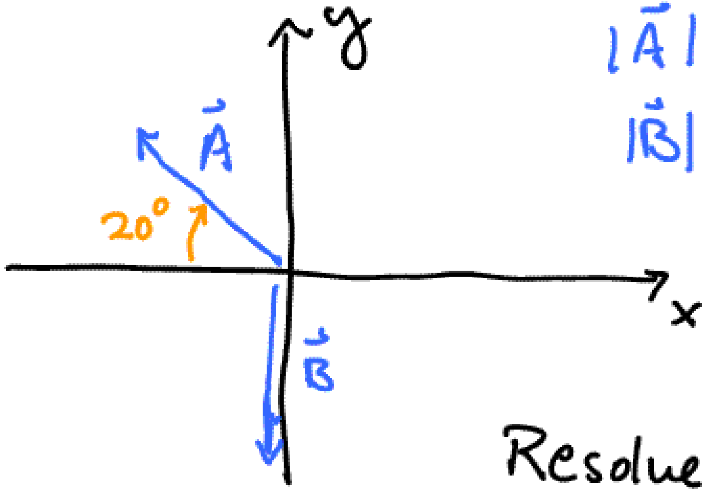




$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{ADJ}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{AdJ}}$$



$|\vec{A}| = 4\,m$

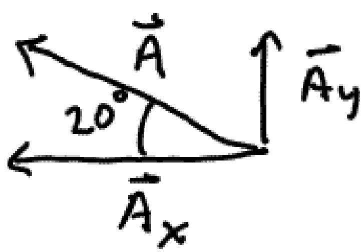
$|\vec{B}| = 5\,m$

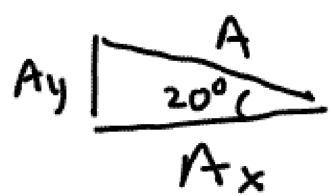
Find $\vec{A} + \vec{B} = \vec{R}$

Resolve vectors

$\vec{B}$ along y $\qquad |\vec{B}_y| \equiv B_y = |\vec{B}|$

$B_x = 0$





$$\left.\begin{aligned} A_y &= A \sin 20 \\ A_x &= A \cos 20 \end{aligned}\right\} \text{magnitudes only}$$

$$R_x = -A_x + \cancel{B_x}$$

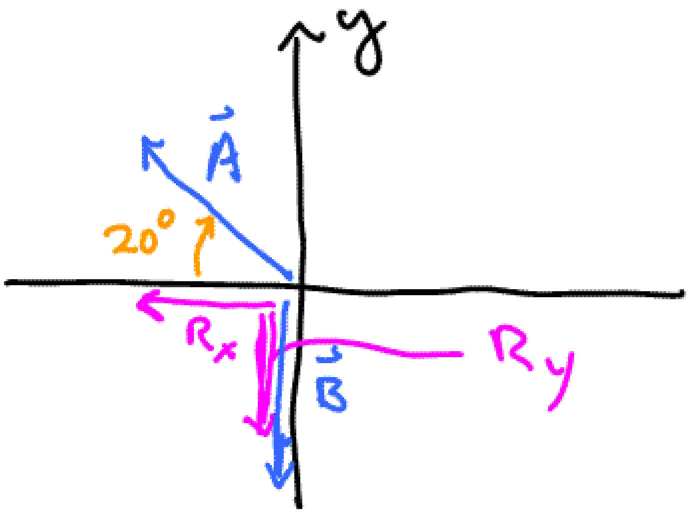
0

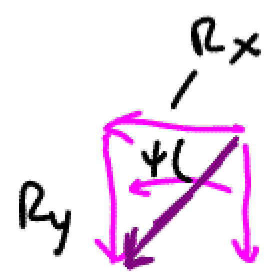
$$R_y = A_y - B_y$$

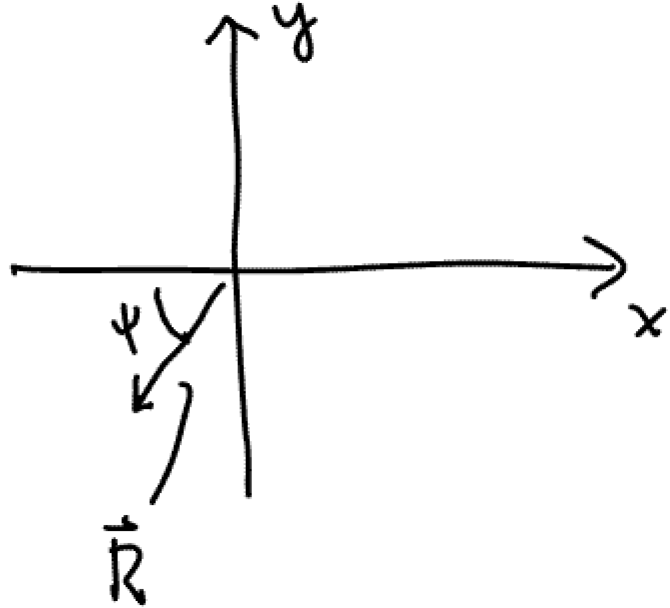
provide appropriate 1-d directions for these components

$$R_x = -|\vec{A}| \cos 20^\circ = -3.7\,m$$

$$R_y = +|\vec{A}| \sin 20 - |\vec{B}| = +4 \sin 20 - 5 = -3.6\,m$$







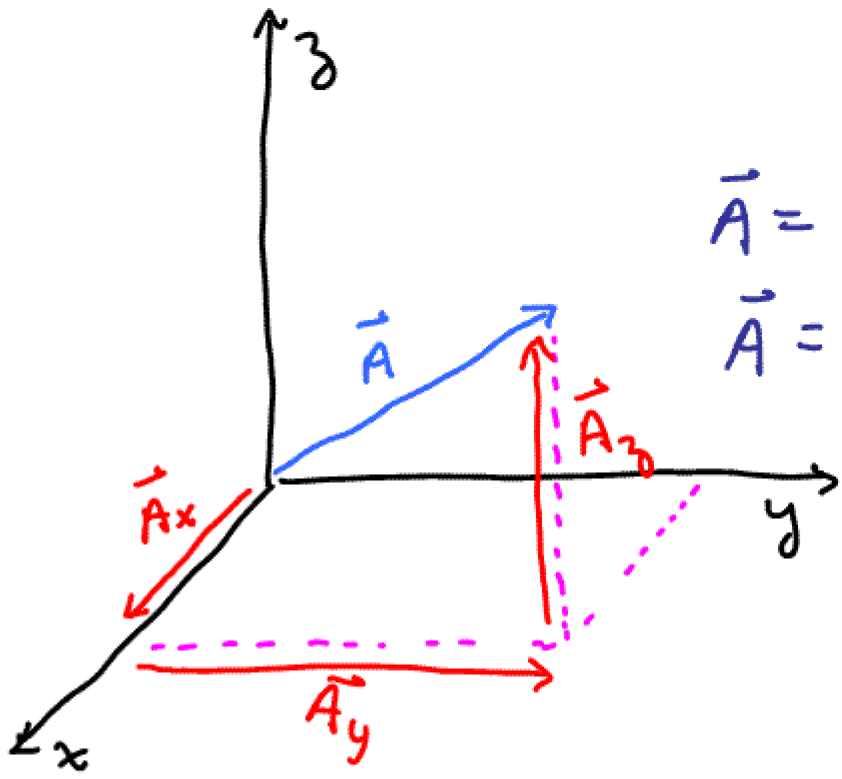
$$\tan\psi = \frac{R_y}{R_x}$$

$$= \frac{3.6}{3.7}$$

$$\psi = 44^\circ$$

$$|\vec{R}|^2 = R_x^2 + R_y^2 =$$

$$|\vec{R}| = 5.2 \text{ m}$$



$$\vec{A} = (A_x, A_y, A_z)$$

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

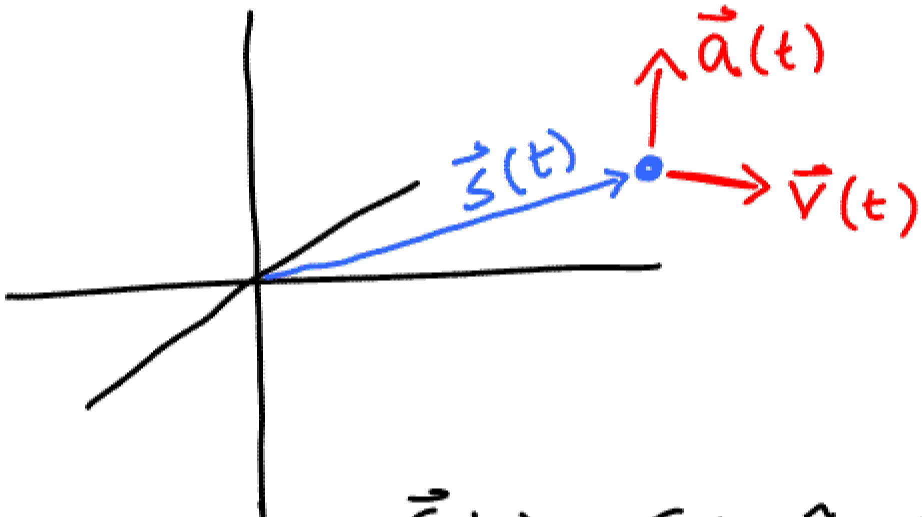
$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{A} = |\vec{A}|\, \hat{A}$$

$\vec{A}$ --- Mag. and direction

$\hat{A}$ ≡ A-hat ≡ unit vector in $\vec{A}$ direction

$$\hat{A} \equiv \frac{\vec{A}}{|\vec{A}|}$$



$$\vec{S}(t) = S_x(t)\,\hat{x} + S_y(t)\,\hat{y} + S_z(t)\,\hat{z}$$

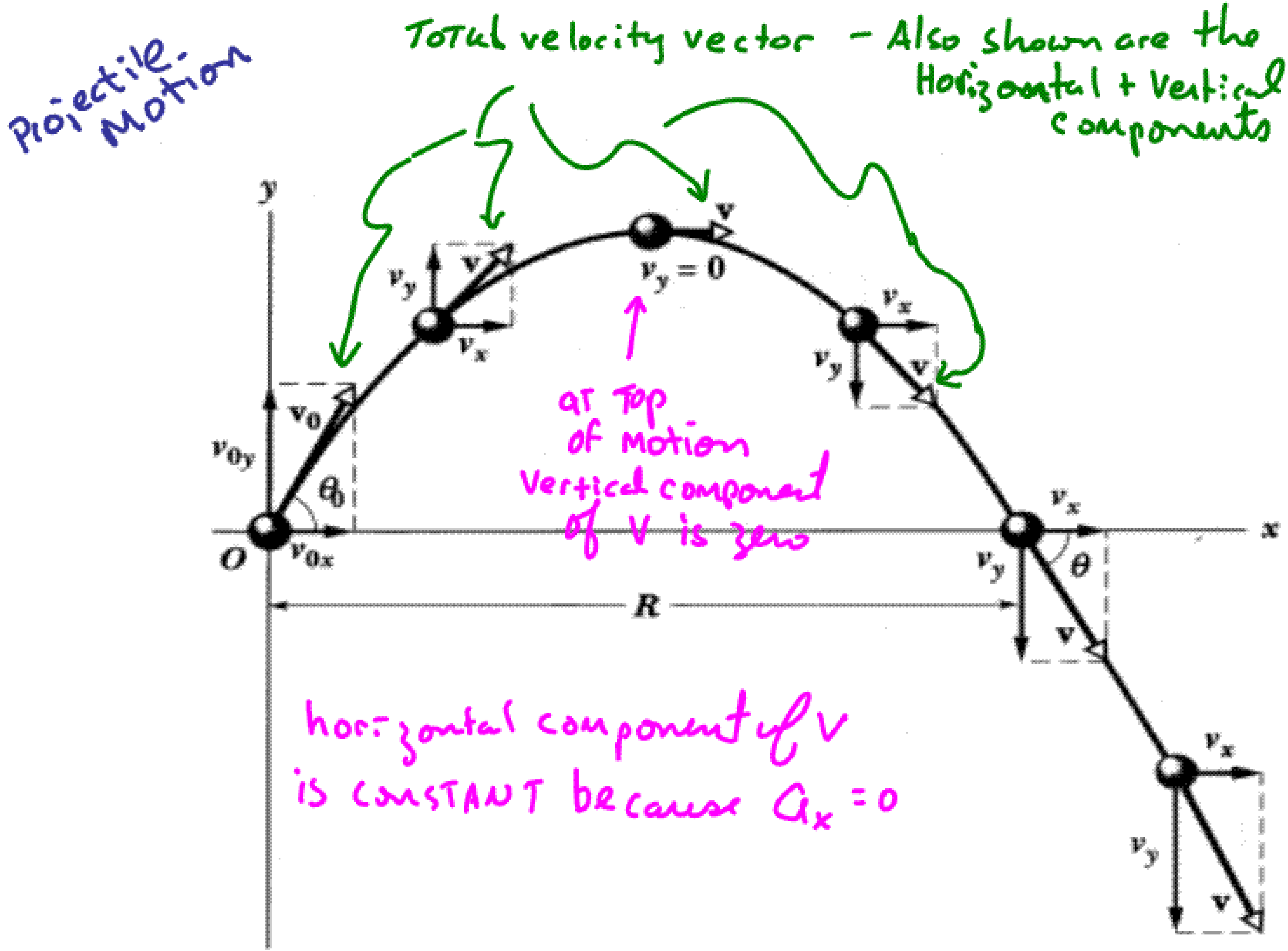
$$\frac{d\vec{s}(t)}{dt} = \vec{v}(t) = v_x(t)\,\hat{x} + v_y(t)\,\hat{y} + v_z(t)\,\hat{z}$$

$$= \frac{ds_x}{dt}\,\hat{x} + \frac{ds_y}{dt}\,\hat{y} + \frac{ds_z}{dt}\,\hat{z}$$

$$\frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{s}(t)}{dt^2} = \vec{a}(t) = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$$

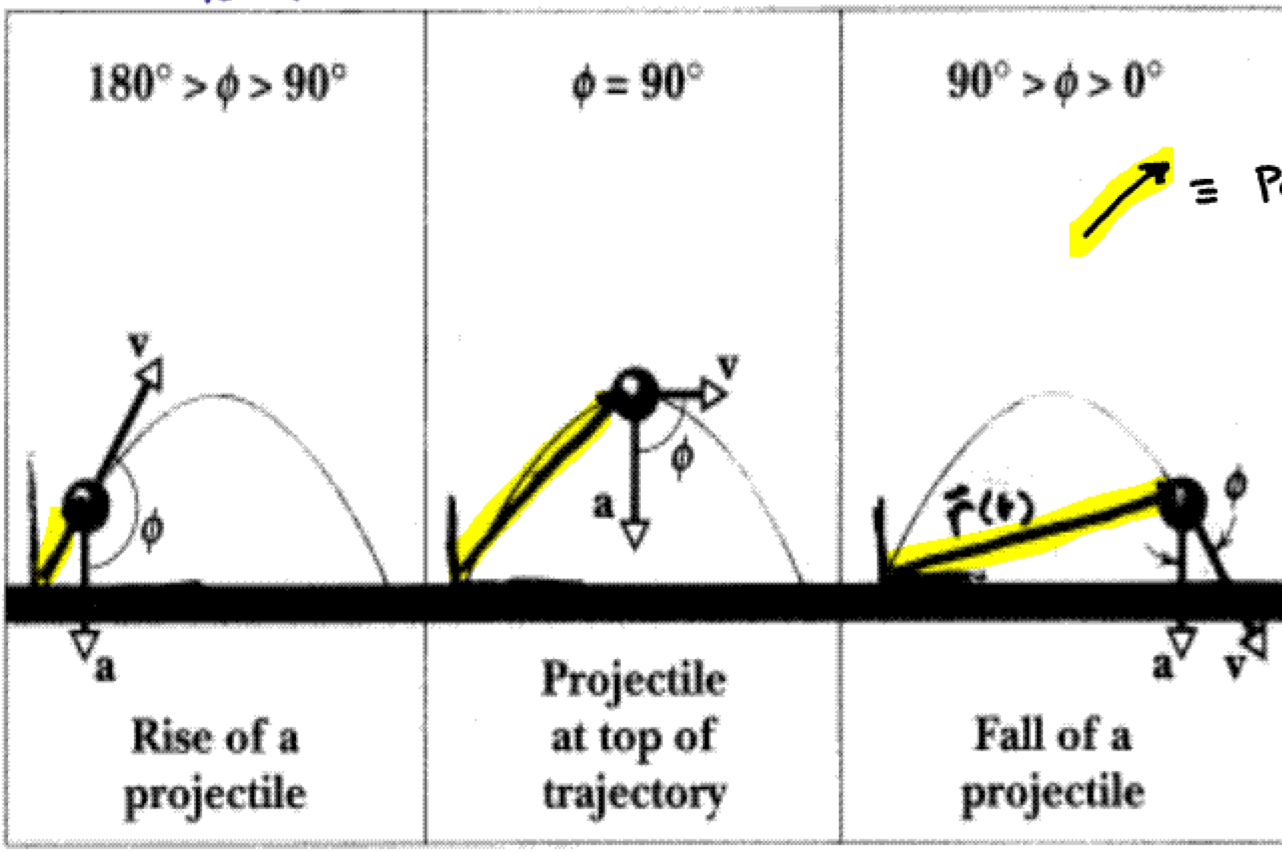
$$= \frac{dv_x}{dt}\hat{x} + \frac{dv_y}{dt}\hat{y} + \frac{dV_z}{dt}\hat{z}$$

$$= \frac{d^2S_x}{dt^2}\hat{x} + \frac{d^2S_y}{dt^2}\hat{y} + \frac{d^2S_z}{dt^2}\hat{z}$$

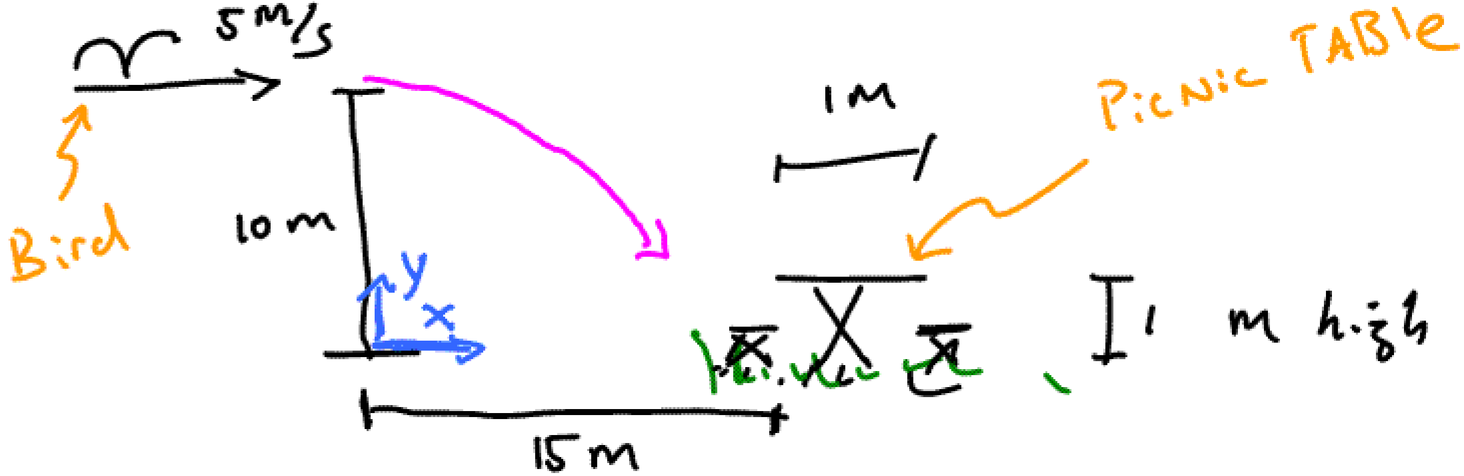

Projectile Motion
Total velocity vector - Also shown are the Horizontal + Vertical components
y
v
vy = 0
vy
vx
v0
v0y
θ0
O
v0x
x
R
θ
at top of motion vertical component of v is zero
horizontal component of v is constant because ax = 0

$\vec{a}$ = constant and down

$\vec{v}$ = changes in magnitude + direction — Always tangent to direction of motion



= Position vector $\vec{r}(t)$



does bird decorate the picnic table ??

y

$a_y = -9.8\ m/s^2$

$V_{oy} = 0$

$y_o = +10\ m$

$y = 1\ m$

x

$a_x = 0$

$V_{ox} = 5\ m/s$

$X_o = 0$

X ?

if x bet. 15 and 16 m when y = 1 m

then Table gets hit

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$1\,m = 10m + 0 - \frac{1}{2}(9.8)\,t^2\,m$$

$$9 = \frac{1}{2}\,9.8\,t^2$$

$$t = \sqrt{\frac{18}{9.8}}$$

$$t = 1.4\,s$$

$$x = \underbrace{\cancel{x_0}}_{0} + v_{0x}t + \frac{1}{2}\underbrace{\cancel{a_x}}_{0}t^2$$

$$x = v_{0x}\,t$$

$$x = (5\,m/s)(1.4\,s)$$

$$x = 7\,m$$

Whew!

Picnic Table is Not hit!