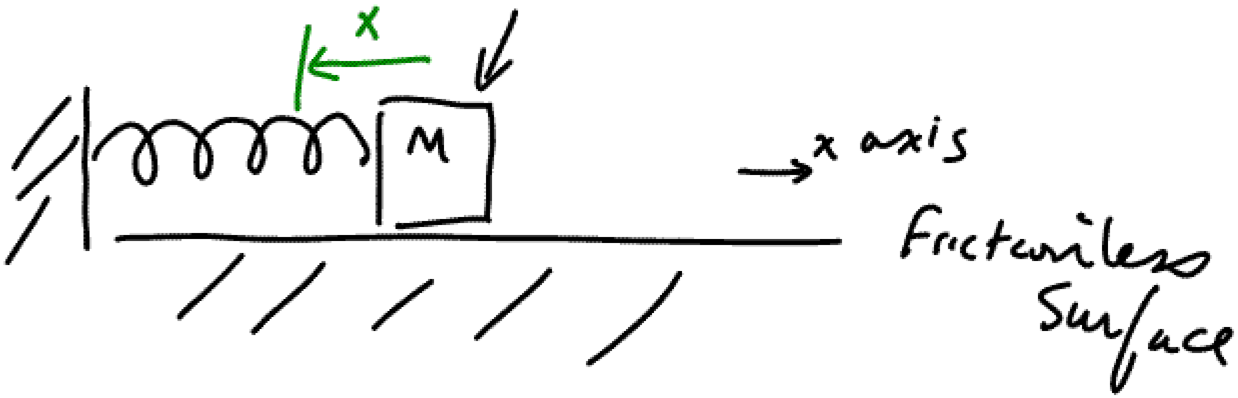
Physics 113 – Dec. 5, 2006

Simple Harmonic Motion



$$F = -kx$$

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

2ND order ordinary differential equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

any system that satisfies an equation of this form exhibits Simple Harmonic Motion

Assume a solution of form

$$x = A\cos(\omega t + \varphi)$$

ω, φ, A constants

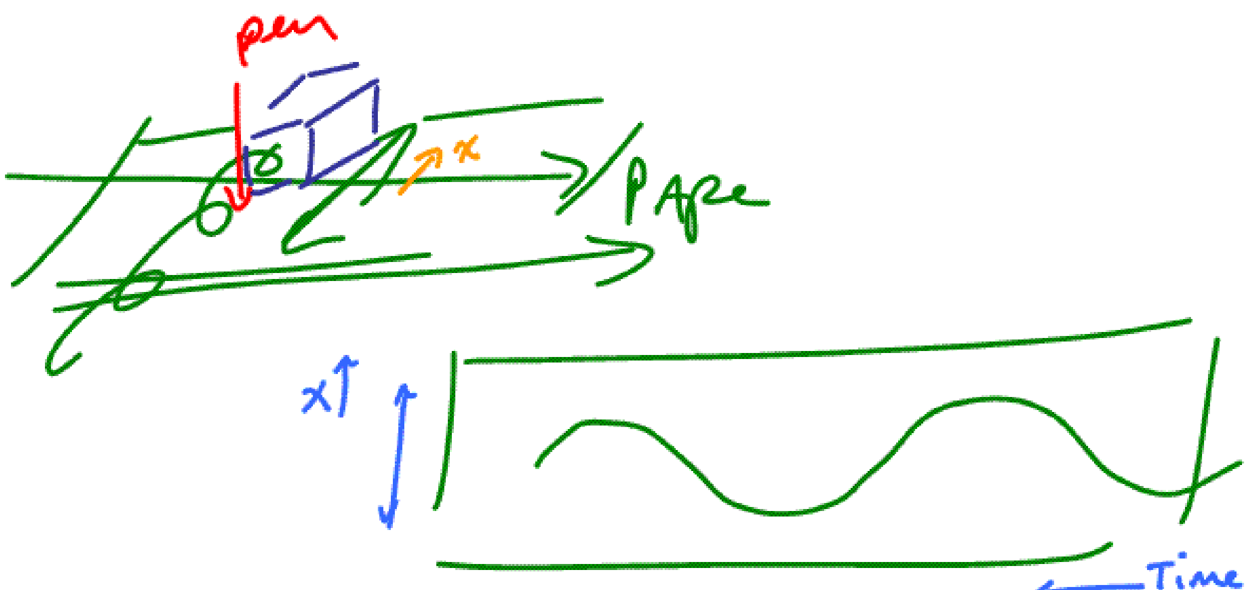
$$\frac{dx}{dt} = A\omega \sin(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \varphi)$$

you can plug soln into eqn and see that soln works so long as

Solves. eqn

$$\omega^2 = k/m \qquad \omega = \pm\sqrt{k/m}$$

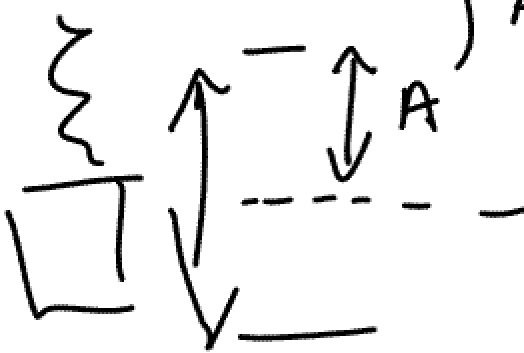


$$x(t) = A\cos(\omega t + \cancel{\phi})$$

phase angle – used to set conditions at $t=0$ (initial conditions)

forget for the moment

Amplitude of Motion



$$\cos(\omega t)$$

$n 2\pi$ motion Repeats

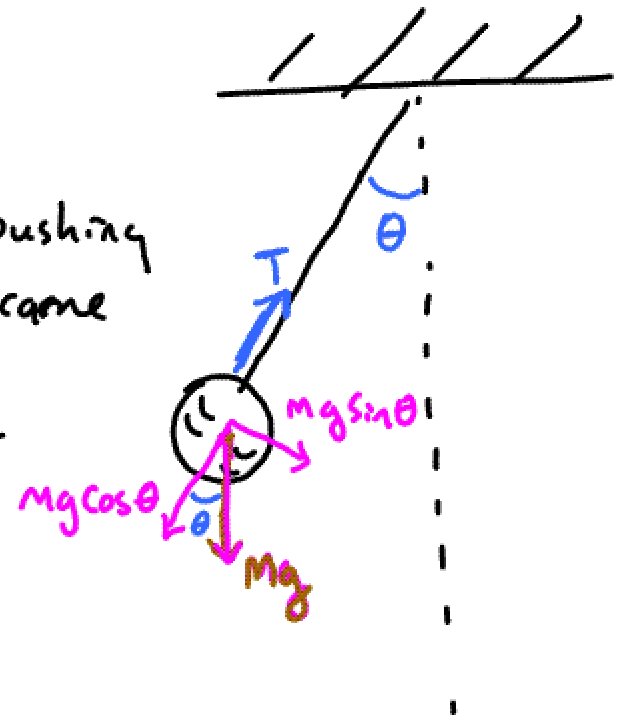
$$\omega = \frac{2\pi}{T}$$

Period

$$\omega t = \frac{2\pi}{T} t$$

The following was not done in lecture on 12/5 but may be helpful for simple harmonic motion:

Simple Pendulum

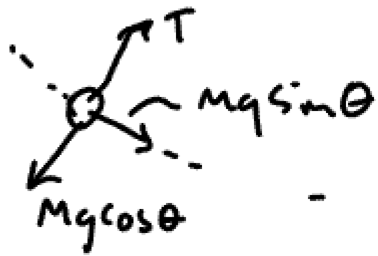


note There is a restoring force pushing Pendulum back from where it came of magnitude $Mg\sin\theta$

cord is L in length

$S = L\theta$

Evaluate Newtons second law for motion along the arclength S



$$m \frac{d^2 s}{dt^2} = -mg\sin\theta$$

$$m \frac{d^2 s}{dt^2} = -mg\sin\theta \approx -mg\theta = -mg\frac{s}{L}$$

True for small θ

using $s = L\theta$

$$\frac{d^2 s}{dt^2} + \frac{g}{L} s = 0$$

Eqn in SHM form

So, soln is

$S(t) = A\cos(\omega t + \phi)$

where $\omega^2 = g/L$