Exam 1 (October 4, 2012)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (5 pts, no partial credit):

A body moves at a constant speed in a straight line. Which one of the following statements must be true?

a) No force acts on the body.
b) A single constant force acts on the body in the direction of motion.
c) A single constant force acts on the body in a direction opposite that of the motion.
d) A net force of zero acts on the body.
e) A constant net force acts on the body in the direction of motion.

Problem 2 (15 pts):

Albert the mouse climbs down a ladder that is 6 m long and which makes a 60 degree angle with the floor (represented by the X-Z plane). Albert's ladder is in the X-Y plane at Z=0 and it leans up against the y-axis as shown in the sketch. After reaching the bottom of the ladder, Albert moves along the floor in the +Z direction for 4 m and then turns and walks 3 m in the +X direction.

a) On the sketch to the right, draw and label the vector that represents Albert’s displacement during his stroll about the house.

b) Determine the magnitude of Albert’s displacement during his walk.
**Problem 3 (10 pts):**

Consider the velocity-time graph to the right. Choose and circle the position-time graph below which is most consistent with the velocity-time graph. Assume both graphs describe the motion of the same body over the time shown.

![Graphs](image)

**Problem 4 (20 pts):**

A hockey puck of mass $M=1$ kg sits at rest at the center of an ice rink. The view from above is shown in the sketch. At time $t=0$, the puck is released (from rest) and it is under the influence of three constant forces in the directions shown. If the magnitude of $F_1$ is 2 Newtons, $F_2$ is 3 Newtons, $F_3$ is 6 Newtons, and the angle $\theta = 20$ degrees counterclockwise from the positive $x$ axis as shown in the sketch.

a) In what direction will the puck move initially?

\[
\begin{align*}
F_{3y} &= 6 \sin 20 = 2.1 \\
F_x &= F_{3x} - F_1 = 5.6 - 2 = 3.6 \text{ N} \\
\tan \gamma &= \frac{0.9}{3.6} \Rightarrow \gamma = 14^\circ
\end{align*}
\]

Puck moves at angle 14° clockwise from $+x$ axis.
Problem 4 continued:

b) How far will the puck travel in the first two seconds of its motion?

\[ \text{Force} = 3.6^2 + 0.9^2 \]
\[ F = 3.7 \text{ N} \]

Problem 5 (25 pts):

The brothers in Sigma Phi Epsilon fraternity decide to attack the students residing in Riverside with a catapult that launches large water balloons. Fortunately for the good folks in Riverside, an engineering major named Dudley hoping to design and build defensive missile systems for a living resides with them. When the brothers in Sig Ep fire a water balloon with an initial velocity of 89 m/s at an angle of 40 degrees to the horizontal toward Riverside, Dudley is ready for them. Dudley lays in wait a distance of 600 meters away along the line running between the Sig Ep catapult and Riverside. He launches a small missile straight upward that intercepts and destroys the water balloon in flight, saving the students in Riverside from inconvenient wetness. Dudley’s missile accelerated upward with a constant acceleration of 3g. (Ignore the height of the catapult. Ignore air friction. Assume Dudley launches his missile from the same elevation as the balloon is launched.) After the fact, the NY Times depicted the attack in Rochester using the sketch to the right.

a) Determine the maximum height of the balloon (assuming zero is the height of the launch site.)

\[ V_y^2 = V_{oy}^2 + 2a(y-y_0) \]
\[ V_y = 0, \quad a = -9.8 \text{ m/s}^2 \]
\[ \text{Height} = \frac{57.2^2}{(2)(9.8)} = 167 \text{ m} \]

b) If t=0 at the moment the balloon is launched, at what time does Dudley need to launch his missile for a successful interception?

1. Calculate height of balloon when it is above Dudley.
2. Calculate time to x = 600 m?
3. \[ x = x_0 + v_{0x}t + \frac{1}{2}a_0t^2 \]
4. \[ 600 = 68.1 \text{ t} \]
5. \[ t_{dud} = 8.8 \text{ s} \]
Problem 6 (25 pts):
Consider the physical situation shown in the sketch. A mass M=5kg is supported by two massless ropes inside an elevator. The rope on the left (rope 2) is attached to the roof of the elevator. The rope on the right (rope 1) passes over a massless and frictionless pulley and is attached to a mass (also 5 kg) resting on a frictionless inclined plane that makes an angle \( \theta \) with the floor of the elevator. The elevator accelerates upward with an acceleration \( a \). If the tensions in the two ropes are equal, what is the angle \( \theta \) in degrees?

Because mass is accelerating upward, it is easier to work in coordinates that are vertical and horizontal rather than along inclined plane and \( \theta \) to it.

\[
\begin{align*}
\Sigma F_x &= ma = 0 = T\cos\theta + N\sin\theta \\
\Sigma F_y &= ma = T\sin\theta + N\cos\theta - mg \\
2T &= T\sin\theta + T\cos^2\theta \\
2\sin\theta &= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta} \\
2\sin\theta &= 1 \\
\sin\theta &= \frac{1}{2} \\
\theta &= 30^\circ
\end{align*}
\]
\[
sin \theta = \frac{\text{opp}}{\text{hyp}} \\
\cos \theta = \frac{\text{adj}}{\text{hyp}} \\
\tan \theta = \frac{\text{opp}}{\text{adj}} \\
v = v_o + at \\
x = x_o + v_o t + \frac{1}{2} at^2 \\
v^2 = v_o^2 + 2a(x - x_o) \\
x - x_o = \int v dt \\
v - v_o = \int a dt \\
\sum \vec{F} = m\ddot{a} \\
F_{\text{centripetal}} = \frac{mv^2}{r} \\
\frac{d(x^n)}{dx} = nx^{n-1} \\
\int x^n \, dx = \frac{x^{n+1}}{n+1} \\
\text{circumference of circle} = 2\pi r \\
\text{area of circle} = \pi r^2 \\
\text{quadratic equation} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

on Problem 6:
Many of you set \(a=0\).

Because a cancels in the final answer, if you set \(a=0\) you will get the correct \(\theta\). This is luck. Note that \(T\) does depend on \(a\) explicitly and \(b, c, d, \text{and } a\) so we took serious error, so we took off 10 points even though you ended up with a correct \(\theta\).

\[T = m \left(\frac{a+g}{2}\right)\]

\(a=0\) is an assumption and

\[m = \text{our aim is to grade according to what we see as your thinking}\]

In this case, it worked against some of you.