Physics 113 - October 18, 2012

- Exam 2 — 1 week from now — here
- No P.S. next week due to Exam
- No lecture next Tuesday → Slides, Audio } Expect you to go thru/listen
- Workshops meet as usual next week

Newton's second law - updated version

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{P}}{dt}$$

$$\vec{P} \equiv m\vec{v} \equiv \text{Momentum}$$

For isolated system, Momentum is Conserved

$$\sum \vec{P}_i = \sum \vec{P}_F$$

$$\sum P_{i_x} = \sum P_{f_x}$$

$$\sum P_{i_y} = \sum P_{f_y}$$

$$\sum P_{i_z} = \sum P_{f_z}$$

In a Collision Momentum conservation good
if system isolated

If KE conserved $\longleftrightarrow$ elastic collision
(Think billiards)

If KE NOT conserved $\longleftrightarrow$ inelastic collision

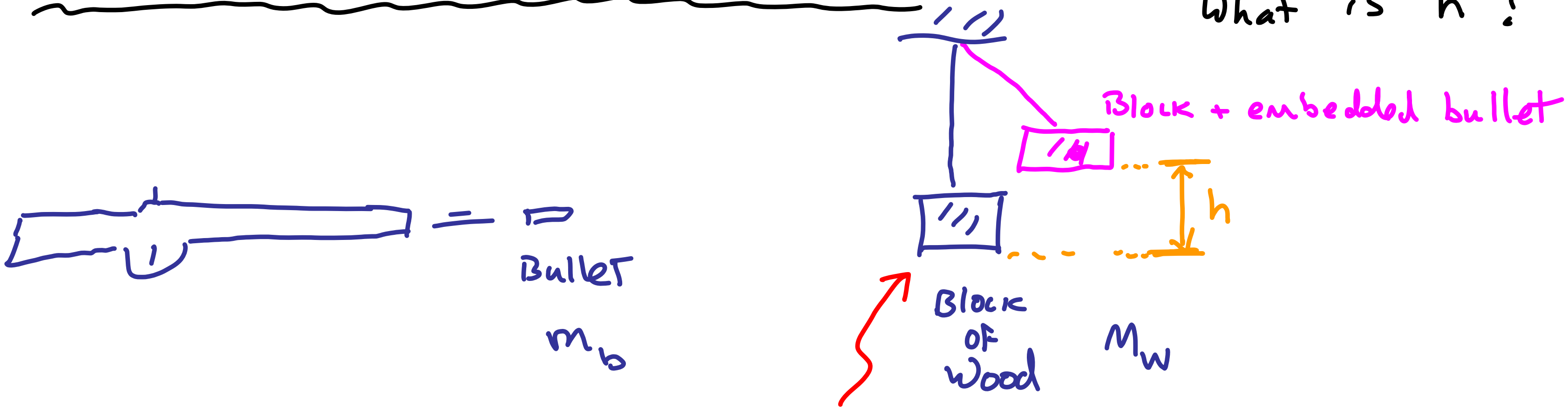
If KE NOT conserved

$$\sum KE_i \neq \sum KE_f$$

Ballistic Pendulum Example Problem

given V_b, m_b, M_w

What is h ?



If know V_{w+b} after impact

can use E cons to get h

How do we determine V_{w+b} after impact?

$KE_i = KE_f$

$$\frac{1}{2} m_b V_b^2 \cancel{= \frac{1}{2}} (m_b + M_w) V_{w+b}^2$$

inelastic collision

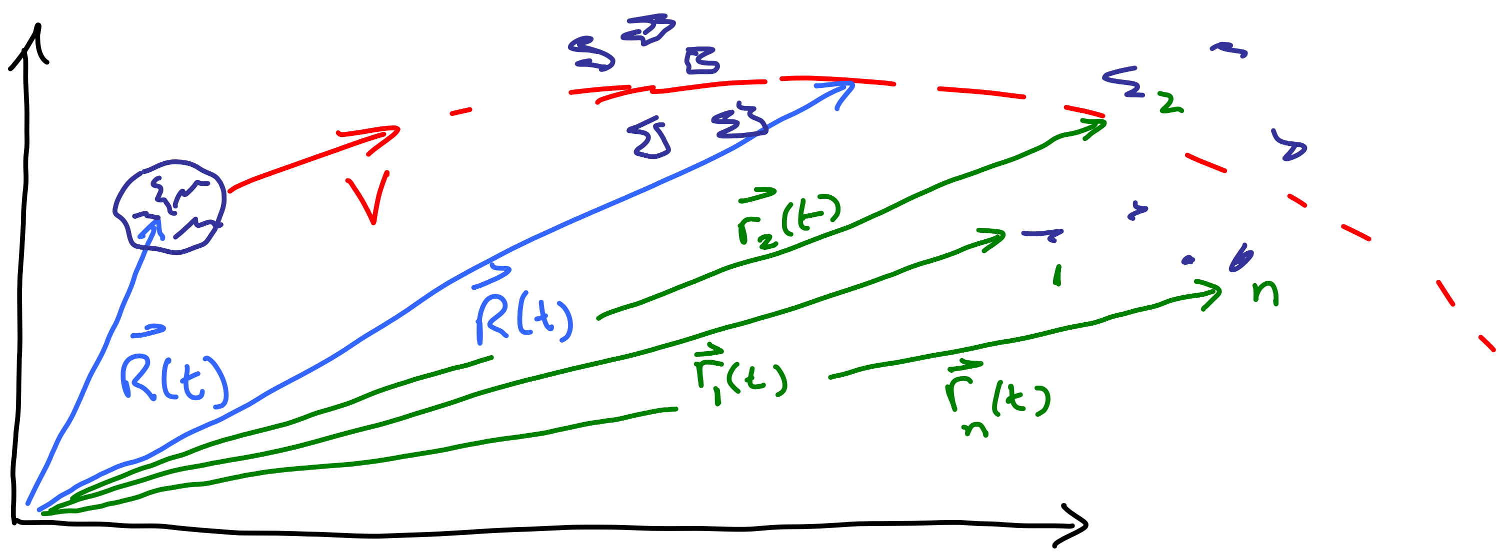
use p conservation

$$\underbrace{m_b V_b + M_w \cancel{V_w}^{\,0}}_{\Sigma P_i} = \underbrace{(M_b + M_w)\, V_{b+w}}_{\Sigma P_f}$$

get V_{b+w}

$$(m_b + M_w)\, g\, h = \frac{1}{2}(m_b + m_w)\, V_{b+w}^2$$

Center of Mass — Center of Mass coordinates



Momentum is conserved

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{V} + \cdots + m_n\vec{V}$$

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \cdots + m_n\frac{d\vec{r}_n}{dt}$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + M_n\vec{r}_n$$

$$\vec{R} = (X, Y, Z)$$

$$\vec{r}_1 = (x_1, y_1, z_1)$$

$$\vdots$$

$$MX = m_1x_1 + m_2x_2 + \cdots + m_nx_n$$

$$MY = m_1y_1 + m_2y_2 + \cdots + m_ny_n$$

$$MZ = m_1z_1 + m_2z_2 + \cdots + m_z z_n$$

center of mass coordinates

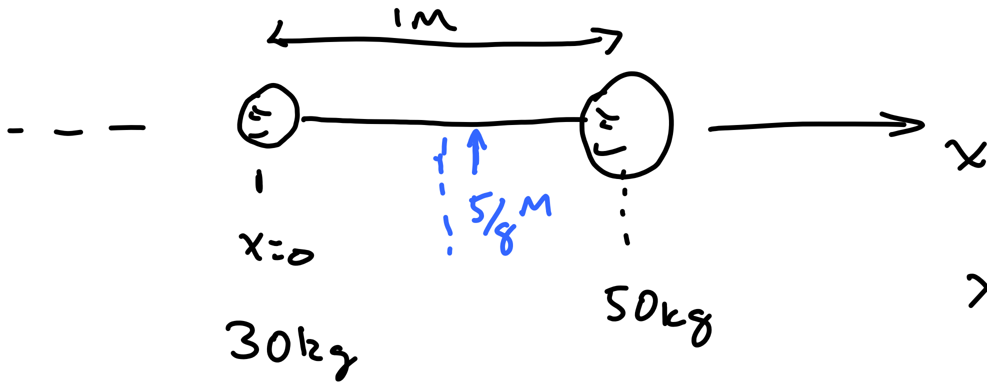
$$X = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{M} = \frac{\sum_i m_ix_i}{\sum m_i}$$

$$\phantom{X = \frac{m_1x_1}{M}} \qquad \qquad \qquad \qquad \qquad \qquad " \qquad M$$

mass weighted x coordinate of system

$$Y = \frac{\sum_i m_iy_i}{\sum m_i} = \frac{\sum_i m_iy_i}{M}$$

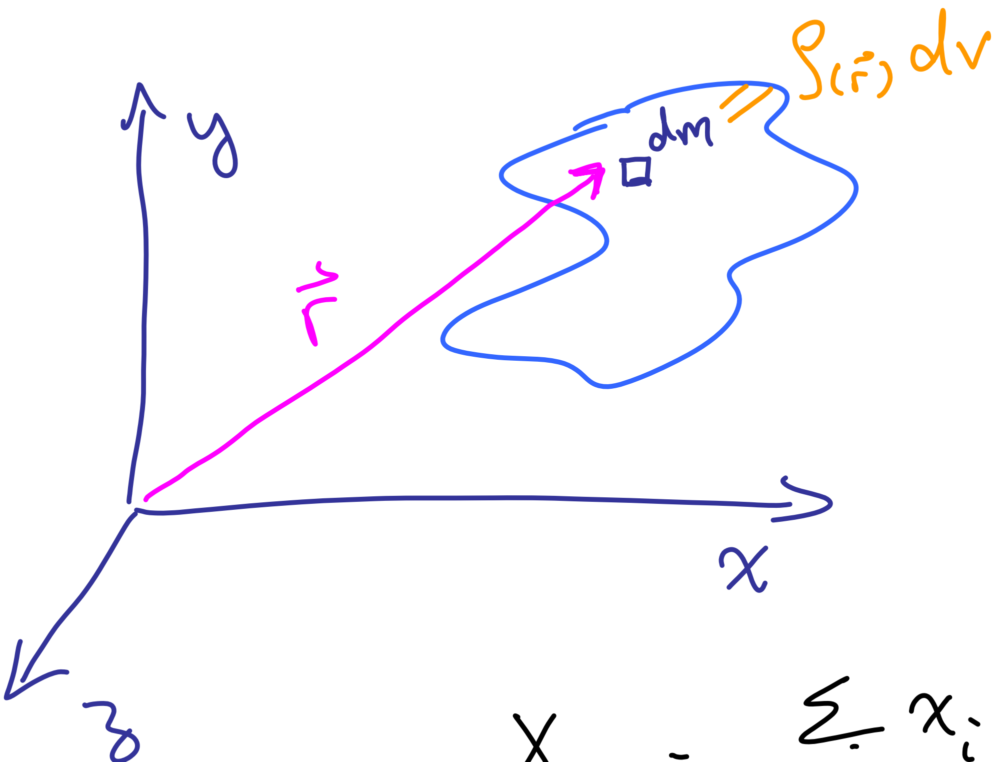
$$Z = \frac{\sum_i m_iz_i}{\sum m_i} = \frac{\sum_i m_iz_i}{M}$$



where is x_{cm}

$$x_{cm} = \frac{\sum m_i x_i}{M} = \frac{(0)(30) + (1)(50)}{30+50}$$

$$x_{cm} = 5/8 \text{ m}$$



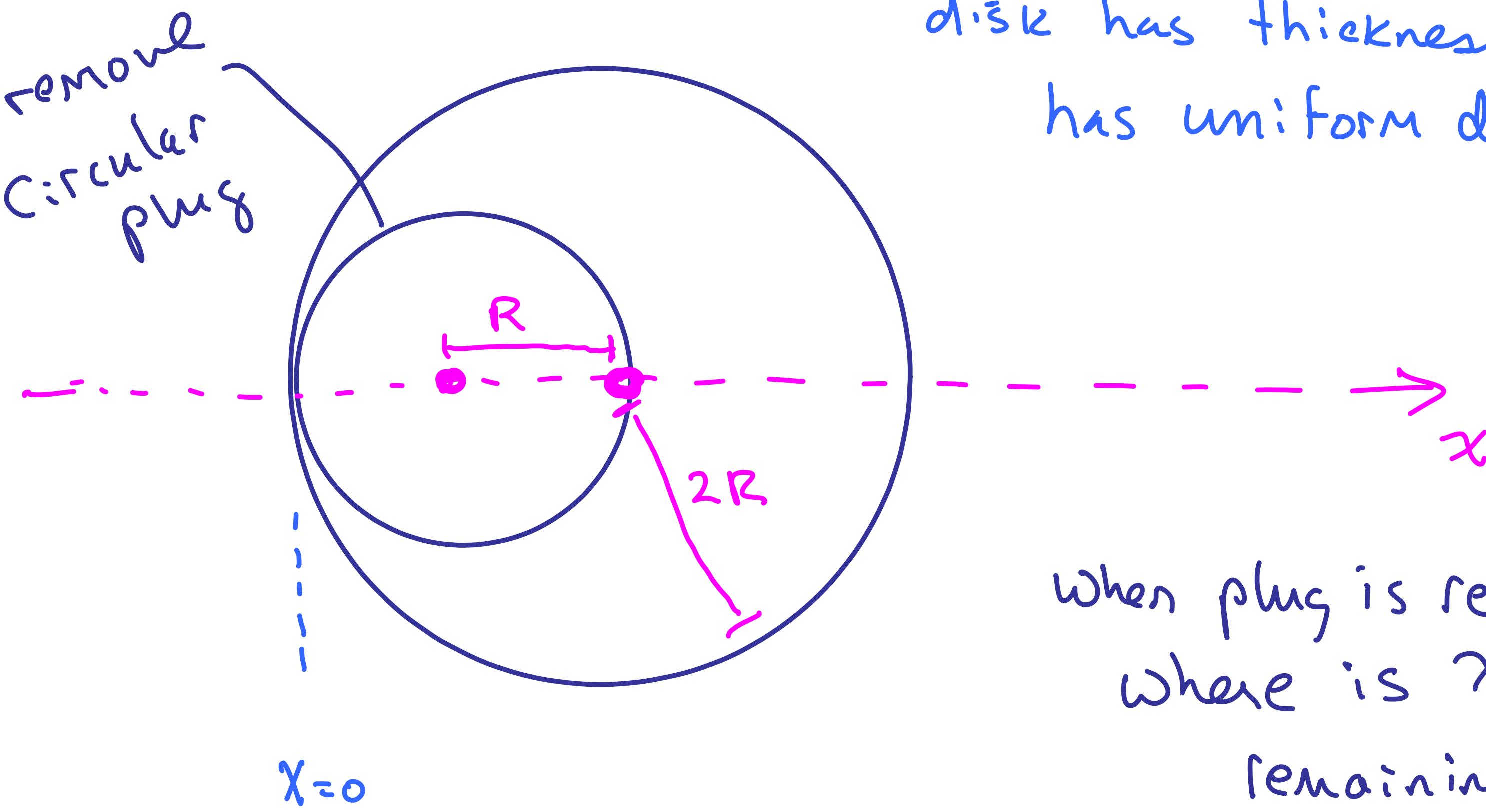
object has $\rho(\vec{r})$ mass density

3 d mass density

$$X_{cm} = \frac{\sum_i x_i (dm_i)}{\sum_i dm_i} \xrightarrow{\text{continuous limit}} \frac{\int_{vol} x\,dm}{\underbrace{\int_{vol} dm}_{=M}}$$

Similarly $Y_{cm} = \dfrac{\int_{vol} y\,dm}{\int_{vol} dm}$ $\qquad Z_{cm} = \dfrac{\int_{vol} z\,dm}{\int_{vol} dm}$

$$\vec{R} = \frac{\int_{vol} \vec{r}\,dm}{\int_{vol} dm} = \frac{\int_{vol} \rho(\vec{r})\,dV}{\int_{vol} dm}$$



disk has thickness t
has uniform density ρ

when plug is removed,
where is x_{cm} of
remaining part of Disk ?

$$\chi_{\substack{\text{Total}\\ \text{disk}}} = \frac{M_{\text{plug}}\,\chi_{\text{plug}} + M_{\text{disk-plug}}\,\chi_{\text{disk-plug}}}{M_{\text{plug}} + M_{\text{Disk-plug}}}$$

Asked to find

$$2R = \frac{M_p R + M_{d-pl}\,\chi_{d-pl}}{M}$$

$R^2 \rho t \pi$

$(4R^2 - R^2)\,\rho\pi t$

$= 3R^2\rho\pi t$

$$\underbrace{\rho\,\pi(2R)^2 t}_{\text{vol of big disk}} = 4R^2\rho\pi t$$

$$2R = \frac{R^2 \rho \pi t \, R + 3R^2 \rho \pi t \, X_{d\text{-}p1}}{4R^2 \rho \pi t}$$

$$2R = \frac{R + 3X_{d\text{-}p1}}{4}$$

$$8R = R + 3X_{d\text{-}p1}$$

$$2\tfrac{1}{3}R = \frac{7}{3}R = X_{d\text{-}p1}$$