Physics 113 - October 18, 2012

- Exam 2 - 1 week from now - here
- No P.S. next week due to Exam
- No lecture next Tuesday → Slides, Audio
- Workshops meet as usual next week

Expect you to go thru/listen
Newton's second law - updated version

\[ \Sigma \vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{P}}{dt} \]

\[ \vec{P} = m\vec{v} = \text{momentum} \]

For isolated system, Momentum is conserved

\[ \Sigma \vec{P}_i = \Sigma \vec{P}_f \]

\[ \Sigma P_{i_x} = \Sigma P_{f_x} \]
\[ \Sigma P_{i_y} = \Sigma P_{f_y} \]
\[ \Sigma P_{i_z} = \Sigma P_{f_z} \]
In a Collision, momentum conservation good if system isolated.

If KE conserved $\iff$ elastic collision (think billiards).

If KE not conserved $\iff$ inelastic collision.

If KE not conserved:

$$\sum KE_i \neq \sum KE_f$$
Ballistic Pendulum Example Problem

Given $v_b$, $m_b$, $M_w$

What is $h$?

If know $v_{w+b}$ after impact

Can use $E_{cons}$ to get $h$

How do we determine $v_{w+b}$ after impact?

$KE_i = KE_f$

$\frac{1}{2} m_b v_b^2 = \frac{1}{2} (M_b + M_w) v_{w+b}^2$

\text{inelastic collision}
Use p conservation

\[ mV_b + M_0 V_w = (M_b + M_w) V_{b+w} \]

get \( V_{b+w} \)

\[ (m_b + M_w) g h = \frac{1}{2} (m_b + M_w) V_{b+w}^2 \]
Center of Mass — Center of Mass coordinates

Momentum is conserved
\[ \mathbf{M} \ddot{\mathbf{V}} = m_1 \ddot{\mathbf{v}}_1 + m_2 \ddot{\mathbf{v}}_2 + \cdots + m_n \ddot{\mathbf{v}}_n \]

\[ M \frac{d\ddot{\mathbf{R}}}{dt} = m_1 \frac{d\ddot{x}_1}{dt} + m_2 \frac{d\ddot{x}_2}{dt} + \cdots + m_n \frac{d\ddot{x}_n}{dt} \]

\[ \mathbf{M}\ddot{\mathbf{R}} = m_1 \ddot{x}_1 + m_2 \ddot{x}_2 + \cdots + m_n \ddot{x}_n \]

\[ \mathbf{R} = (X, Y, Z) \]

\[ \mathbf{r}_i = (x_i, y_i, z_i) \]

\[ m_1 x_1 + m_2 x_2 + \cdots + m_n x_n \]

\[ M_x = m_1 y_1 + m_2 y_2 + \cdots + m_n y_n \]

\[ M_z = m_1 z_1 + m_2 z_2 + \cdots + m_n z_n \]
Center of Mass Coordinates:

\[
X = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{M}
\]

\[
Y = \frac{\sum m_i y_i}{\sum m_i} = \frac{\sum m_i y_i}{M}
\]

\[
Z = \frac{\sum m_i z_i}{\sum m_i} = \frac{\sum m_i z_i}{M}
\]

mass weighted x-coordinate of system
where is $x_{cm}$

\[ x_{cm} = \frac{\sum M_i x_i}{M} = \frac{(0)(30) + (1)(50)}{30 + 50} \]

\[ x_{cm} = \frac{5}{8} \text{ m} \]
An object has a 3D mass density \( \rho(\mathbf{r}) \). The center of mass \( \mathbf{X}_{\text{cm}} \) is given by the continuous limit:

\[
\mathbf{X}_{\text{cm}} = \frac{\sum_{i} \mathbf{x}_i \rho(\mathbf{r}_i) \, dm_i}{\sum_{i} \rho(\mathbf{r}_i) \, dm_i} \xrightarrow{\text{continuous limit}} \frac{\int_{\text{vol}} \mathbf{x} \, dm}{\int_{\text{vol}} \, dm} = \mathbf{X}_{\text{cm}}.
\]
Similarly, \( Y_{cm} = \frac{\int y \, dm}{\int dm} \) \( Z_{cm} = \frac{\int z \, dm}{\int dm} \)

\[
\bar{R} = \frac{\int r \, dm}{\int dm} = \frac{\int s(r) \, dv}{\int dm}
\]
Remove circular plug

Disk has thickness $t$ and uniform density $\rho$.

When plug is removed, where is the center of mass of the remaining part of the disk?
\[
\chi = \frac{M_{\text{plug}} X_{\text{plug}} + M_{\text{disk}} X_{\text{disk}} + \chi_{\text{disk}}}{M_{\text{plug}} + M_{\text{disk}} + \chi_{\text{disk}}}
\]

\[
2R = M_{p} R + M_{d-\text{pl}} \chi_{d-\text{pl}}
\]

\[
\left(4R^2 - R^2\right)8\pi T = 3R^2 8\pi T
\]

\[
\int_{0}^{\pi} (2R)^2 T = 4R^2 8\pi T
\]

vol of big disk
\[ 2R = \frac{R^2 8\pi t R + 3R^2 8\pi t \chi_{d-p1}}{4R^2 8\pi t} \]

\[ 2R = \frac{R + 3 \chi_{d-p1}}{4} \]

\[ 8R = R + 3 \chi_{d-p1} \]

\[ 2^{\frac{1}{3}} R = \frac{7}{3} R = \chi_{d-p1} \]