Physics 113 - October 23, 2012

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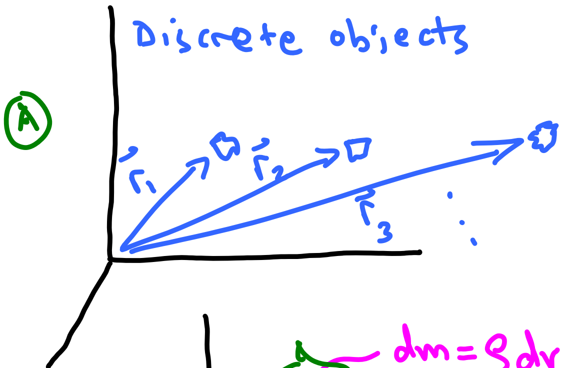
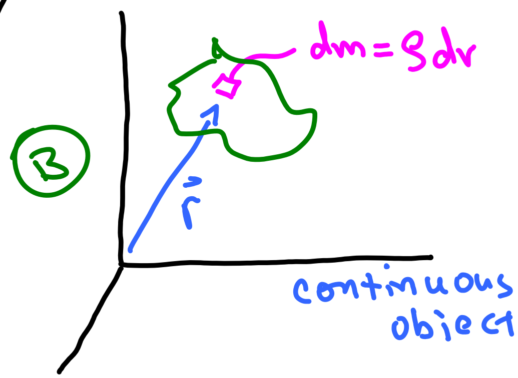
- Exam 2 - Hoyt, Oct. 25 during normal lecture slot
- P.S. 7 is posted ... due next week (Nov. 1)
- Workshops run normally this week

Last Time

Center-of-mass Coordinates $\rightarrow$ mass weighted average position





$$x_{cm} = \frac{\sum_i x_i m_i}{\sum m_i} \quad \text{- or -} \quad \frac{\int x\, dm}{\int dm}$$

usually

© $dm = \rho dv$

$$y_{cm} = \frac{\sum_i y_i m_i}{\sum m_i} \quad \text{- or -} \quad \frac{\int y\, dm}{\int dm}$$

$$z_{cm} = \frac{\sum_i z_i m_i}{\sum m_i} \quad \text{- or -} \quad \frac{\int z\, dm}{\int dm}$$

For discrete case

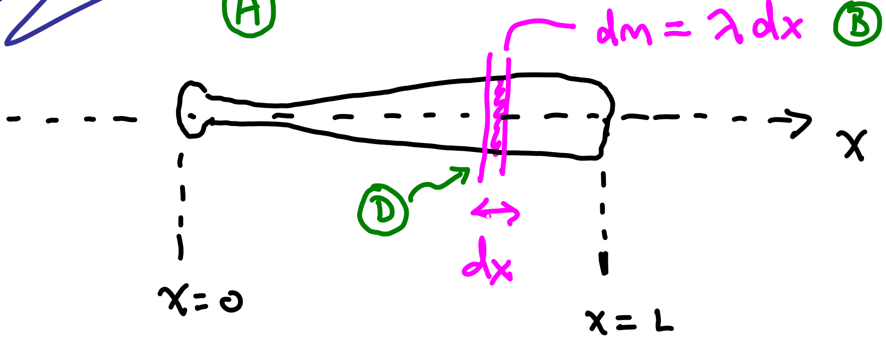
For continuous case

Example

Find Center of Mass along x

3

(A)



(C)

$\lambda \equiv$ linear
$\sigma \equiv$ area
$\rho \equiv$ volume

densities

(H) must have units of kg/m

you are given that bat has

"Linear mass density" $\equiv$ mass/length $= \lambda(x) = \lambda_0 \left(1 + \frac{x^2}{L^2}\right)$ where $0 \le x \le L$

(E) $$X_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^L x\, \lambda(x)\, dx}{\int_0^L \lambda(x)\, dx}$$ (F) (G)

4

Denominator

A $$M = \int dm = \int_0^L \lambda(x)\, dx = \int_0^L \lambda_0 \left(1 + \frac{x^2}{L^2}\right) dx = \int_0^L \lambda_0\, dx + \int_0^L \lambda_0 \frac{x^2}{L^2}\, dx$$ B

$$= \lambda_0 x \Big|_0^L + \lambda_0 \frac{x^3}{3L^2} \Big|_0^L = \lambda_0 L + \frac{\lambda_0 L}{3} = \frac{4}{3}\lambda_0 L$$ C

units are correct

$\frac{kg}{m}\, m = kg$

Numerator

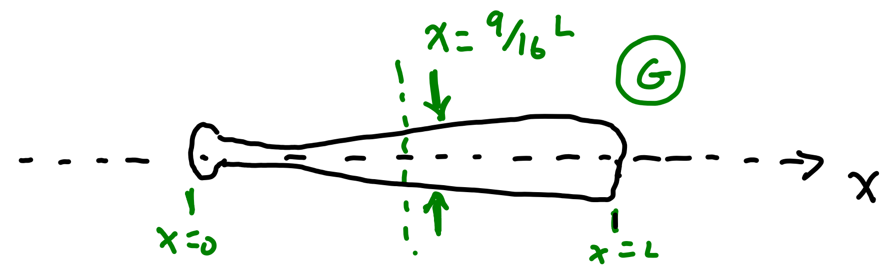
D $$\int x\, dm = \int_0^L \lambda_0 \left(1 + \frac{x^2}{L^2}\right) x\, dx = \int_0^L \lambda_0 x\, dx + \int_0^L \lambda_0 \frac{x^3}{L^2}\, dx$$ E

$$= \lambda_0 \frac{x^2}{2} \Big|_0^L + \frac{\lambda_0}{L^2} \frac{x^4}{4} \Big|_0^L = \frac{\lambda_0}{2} L^2 + \frac{\lambda_0 L^2}{4} = \frac{3}{4}\lambda_0 L^2$$ F

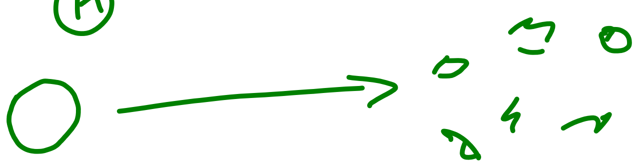
SO

$$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\frac{3}{4}\lambda_0 L^2}{\frac{4}{3}\lambda_0 L} = \boxed{\frac{9}{16} L}$$

G



A



B

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_n\vec{r}_n$$

C

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + \dots + m_n\frac{d\vec{r}_n}{dt}$$

D

$$M\vec{V}_{cm} = m_1\vec{v}_1 + \dots + m_n\vec{v}_n$$

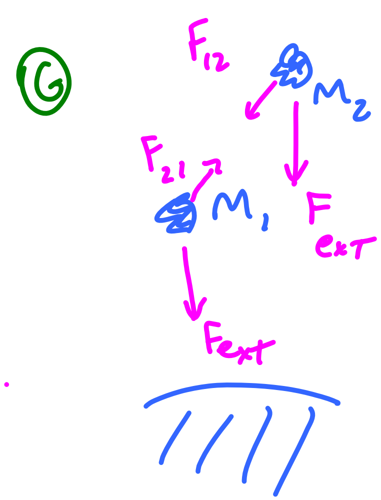
Momentum Conservation

E

$$M\vec{a}_{cm} = m_1\vec{a}_1 + \dots + m_n\vec{a}_n$$

$F_{21} = -F_{12}$

G



F

$$\Sigma\vec{F} = \underbrace{\Sigma\vec{F}_1}_{F_{1ext} + F_{21}} + \underbrace{\Sigma\vec{F}_2}_{F_{2ext} + F_{12}} + \dots + \Sigma\vec{F}_n = \Sigma\vec{F}_{external} + \underbrace{\cancel{\Sigma\vec{F}_{internal}}}_{0}$$

H

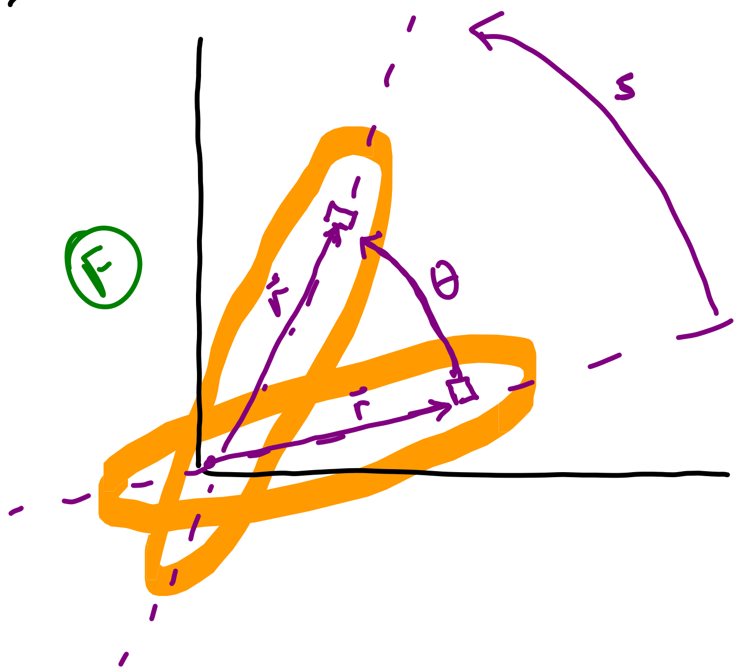
$F_{12} = -F_{21}$

I

Rotational Kinematics

6

F



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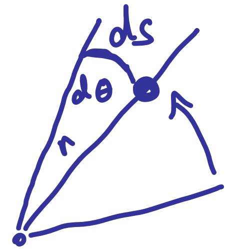

B

$$S = r\theta$$

Arclength = (radius)(Angle in radians)

$$ds = r\,d\theta$$ C

D

Tangential Linear velocity in m/s

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

Angular velocity $\equiv \omega$ in rad/s

E

Tangential Linear Acceleration in m/s^2

$$\frac{d^2s}{dt^2} = r\frac{d^2\theta}{dt^2} = r\frac{d\omega}{dt}$$

Angular Acceleration in rad/s^2 $\equiv \alpha$

If someone were to hit you w/ a bat ... would you rather be hit by the end of the bat or by part of the bat closer to handle?

(A)

$$s = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$

Recall (B)

$$\frac{dx}{dt} = v$$
$$dx = v\,dt$$
$$\int dx = \int v\,dt$$
$$x - x_0 = \int v\,dt$$

For rotational motion (C)

$$\frac{d\theta}{dt} = \omega$$
$$d\theta = \omega\,dt$$
$$\int d\theta = \int \omega\,dt$$
$$\boxed{\theta - \theta_0 = \int \omega\,dt}$$

True in general

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A
$$\frac{d\omega}{dt} = \alpha$$

$$d\omega = \alpha\, dt$$

True in general

$$\boxed{\omega - \omega_0 = \int \alpha\, dt}$$

Analogue to $v - v_0 = \int a\, dt$ for linear motion

Let us Assume $\alpha = $ constant

Constant angular acceleration

B
$$\omega - \omega_0 = \alpha \int dt$$

$$\omega - \omega_0 = \alpha (t - t_0)$$

$$\omega = \omega_0 + \alpha t \qquad (t_0 = 0)$$

Seen familiar? $v = v_0 + a t$

C
$$v = v_0 + at$$

$$r\omega = r\omega_0 + r\alpha t \rightarrow \omega = \omega_0 + \alpha t$$

A

Constant a eqns

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

B

Const α eqns

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \left(\frac{\omega + \omega_0}{2}\right)t$$

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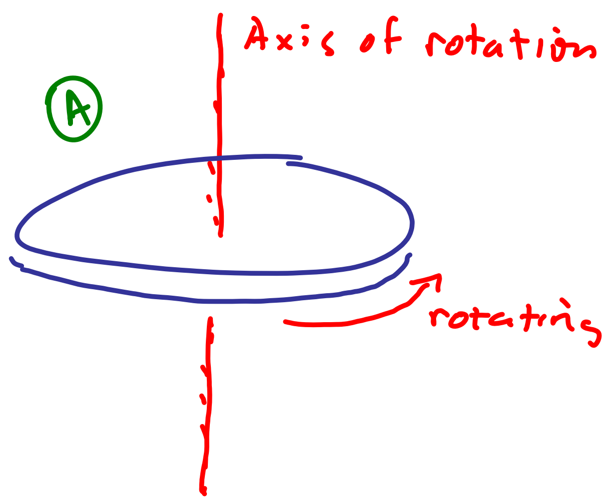


Disk initially rotating at 120 rad/s slows down with a constant angular accel. of 4 rad/s^2.

How much time elapses befor disk stops rotating?

B

$$\omega = \omega_0 + \alpha t$$

$$0 = 120 - (4) t$$

$$t = 30 \text{ Seconds}$$

11

$F = ma$ A

$\Rightarrow F = m r \alpha$ B

$\Rightarrow \underbrace{rF}_{\text{Angular Force}} = \underbrace{(mr^2)}_{\text{Angular Mass}} \underbrace{\alpha}_{\text{Angular Acceleration}}$ C

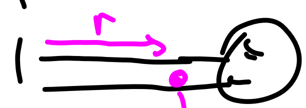
Angular Force

↳ gives Angular Acceleration

Angular Mass ⟶ moment of inertia

Angular Acceleration

D



pt of Application of force

The point of application of the force relative to the Axis of Rotation MATTERS

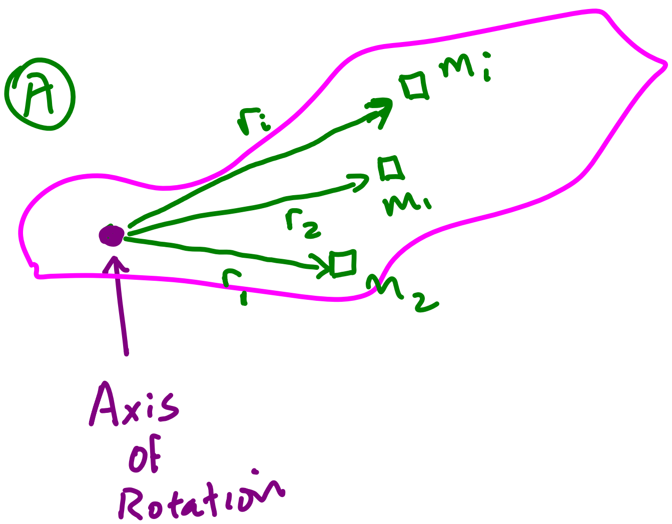
E

$$\boxed{\tau = I \alpha}$$

Torque

Moment of Inertia

Angular Acceleration



B

$$\tau = I\alpha$$

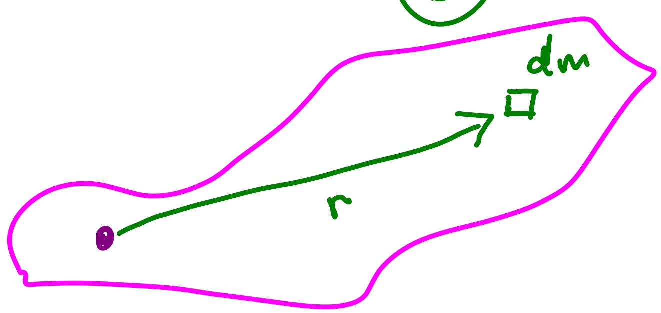
$$rF = (mr^2)\alpha$$

For i mass elements

C

$$\sum (rF)_i = \sum (mr^2)_i\, \alpha$$

D



E

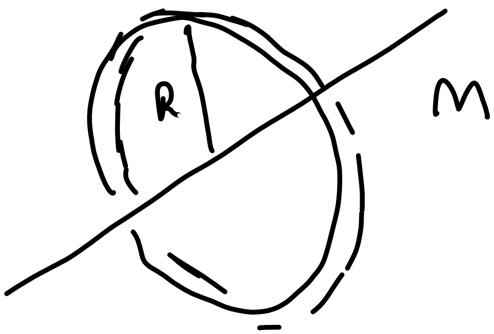
$$dI = r^2\, dm$$

$$\int_{vol} dI = \int_{vol} r^2\, dm$$

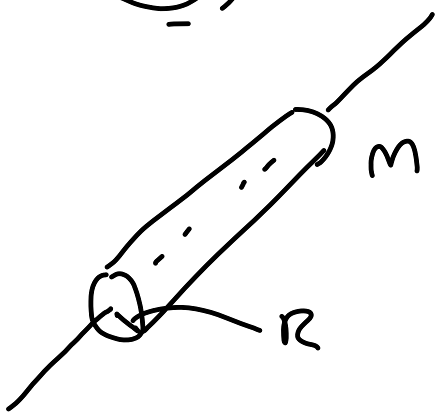
$$I = \int_{vol} r^2\, dm$$

F

$\rho\, dv$



$$I = MR^2$$



$$I = \frac{1}{2} MR^2$$

different shapes
different distributions of
mass about axis
of rotation

⇓

Have
Different
Moments
of
Inertia