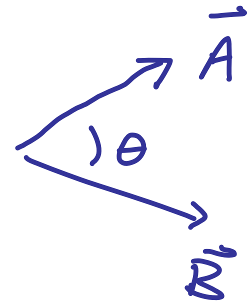
Physics 113 – November 8, 2012

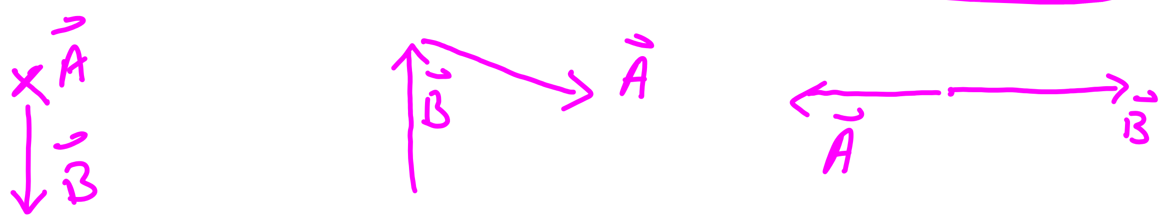
Last Time



$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta$

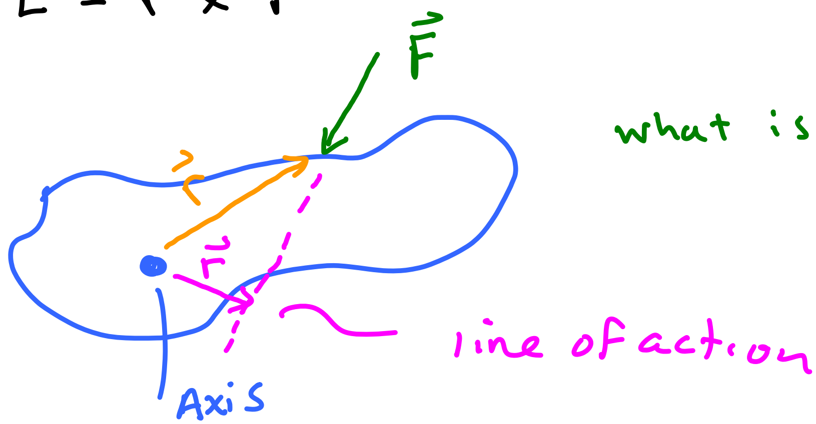
in direction into paper

(use Right hand rule)



A) Right
B) Left
C) up
D) down
E) in
F) out
G) zero

$$\vec{\tau} = \vec{r} \times \vec{F}$$



what is direction of the torque?

Angular Momentum $\equiv \vec{L} = I\vec{\omega}$ (recall linear momentum is $m\vec{v}$.)

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

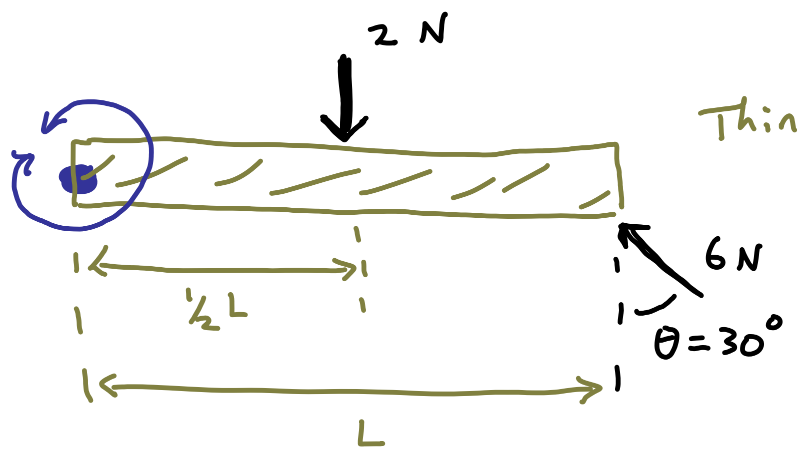
$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d(I\vec{\omega})}{dt} = I\frac{d\vec{\omega}}{dt}$$

Angular Momentum is Conserved

$$\vec{L}_{init} = \vec{L}_{final}$$

in an isolated system

Example



Thin rod of Length L
Free to rotate about an axis at one end

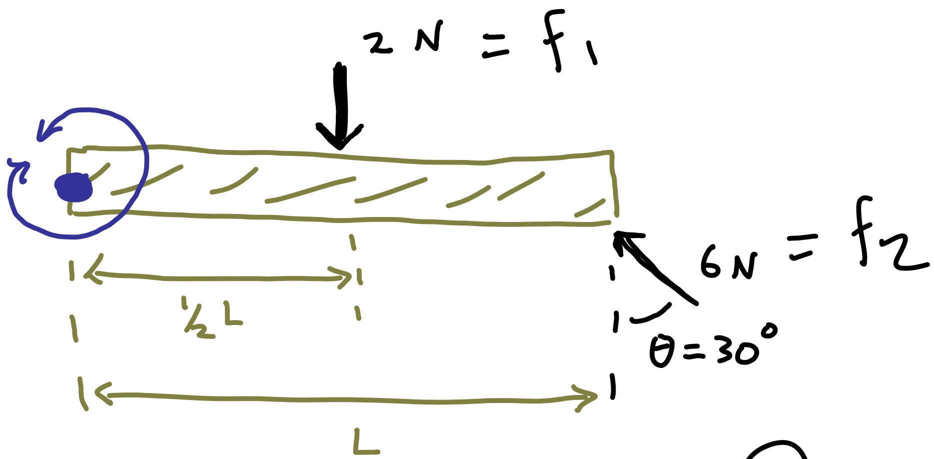
Mass rod = 3 kg

L = 2m

Find $\vec{\alpha}$ for rod at moment shown.

(given $I_{rod} = \frac{1}{3} M L^2$)

(ignore all other forces in this problem ... like gravity)

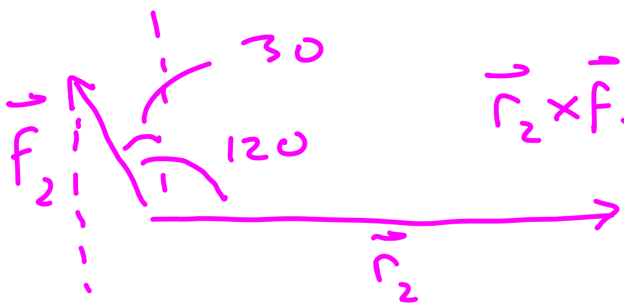


$$\Sigma \tau = I\alpha$$

$$I = \tfrac{1}{3} M L^2 \alpha$$

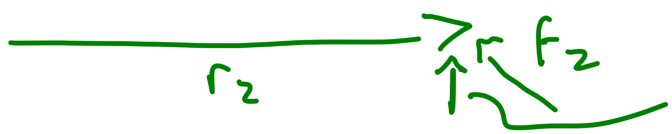
$$\Sigma \tau = \left(\tfrac{1}{2}L\right) f_1 - (L) f_2 \cos 30$$

↺ +



$$\vec{r}_2 \times \vec{f}_2 = |r_2||f_2| \sin 120$$

$$\tau = r_2 f_2 \cos 30$$



$$\Sigma \tau = I\alpha = \frac{1}{2} L F_1 - L F_2 \cos 30$$

$$\frac{1}{3} ML^2 \alpha = \frac{1}{3}(3) 2^2 \alpha = \frac{1}{2}(2)(2) - (2)(6) \cos 30$$

$$4\alpha = 2 - 12(.866)$$

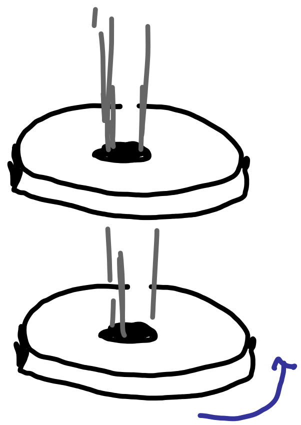
$$\alpha = -2.1 \text{ rad/s}^2$$

The negative sign tells you the net torque acts to rotate the rod counter-clockwise (opposite the direction we chose as positive)

$$\vec{\alpha} = 2.1 \text{ rad/s}^2 \ (\widehat{\text{out of screen}})$$

↑ unit vector

Example



$I_{disk} = \frac{1}{2} M R^2$

M

R

ω

Disk 1 w/ mass M, radius R rotates with ω.

a second (Identical) disk that is NOT rotating falls onto the first disk and "sticks" without slipping on the bottom disk

what is the final ω ?

KE cons

$$\left(\frac{1}{2} M \omega^2\right)_{KE_{init}} = \left(\frac{1}{2} (2M) \omega'^2\right)_{KE_{final}}$$

Use Angular Momentum conservation

$$I\,\omega_{init} = I'\,\omega_{final}$$

$$\frac{1}{2}MR^2\,\omega = \frac{1}{2}(2m)\,R^2\,\omega_{final}$$

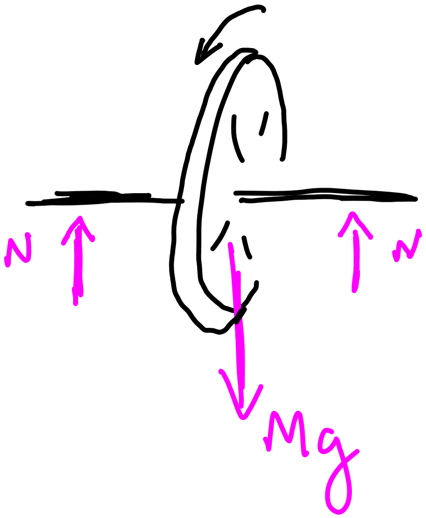
$$\omega = 2\,\omega_{final}$$

$$\omega_{final} = \frac{1}{2}\,\omega_{init}$$

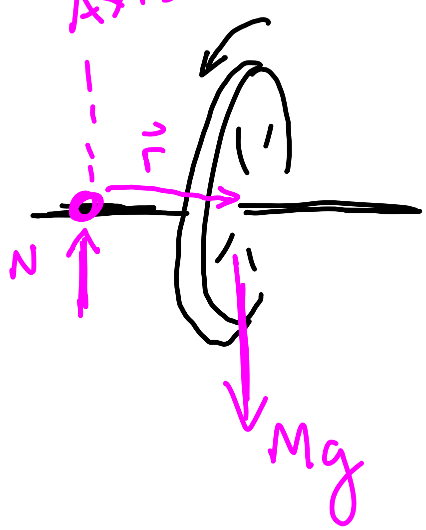
Spinning wheel demo

$$\vec{L} = I\vec{\omega}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$



Precession

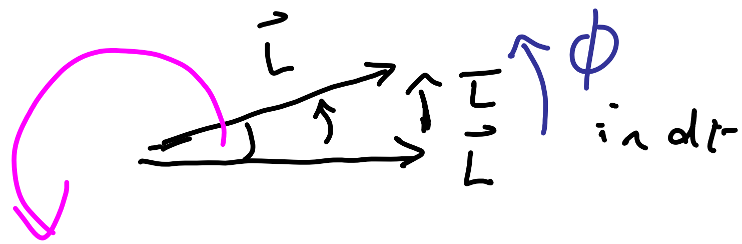


$\vec{\tau}$ into the page

wheel rotates
$\vec{L}$ rotates } into page

look from abov

Ω = Angular velocity of Precession



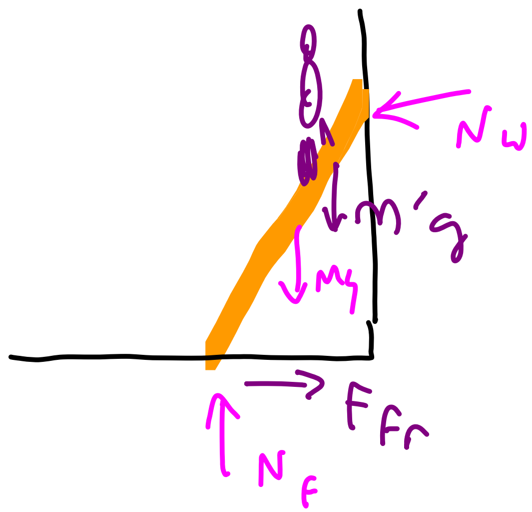
$$\tau = \frac{d\vec{L}}{dt}$$

$$mgr = \tau = \frac{d\vec{L}}{dt} = |\vec{L}|\frac{d\phi}{dt}$$

($\frac{d\phi}{dt} = \Omega$)

$$\Omega = \frac{mgr}{I\omega}$$

Static Equilibrium



$$\sum \vec{F} = m\vec{a} = 0$$

$$\sum \vec{\tau} = I\vec{\alpha} = 0$$