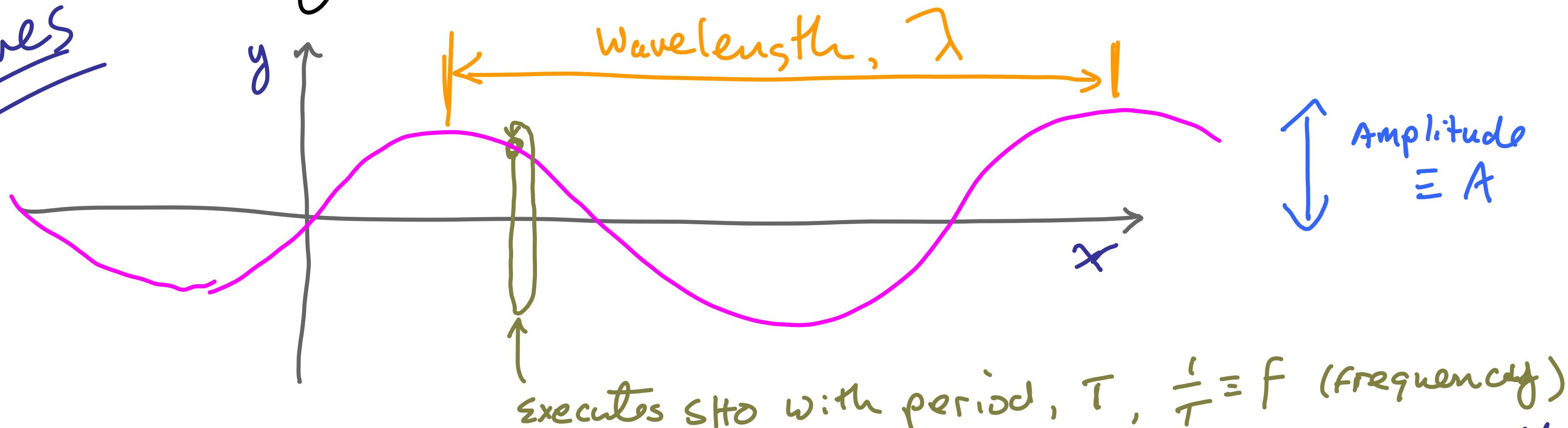
Physics 113 - December 6, 2012

Waves



$y(x,t) = A \sin(kx - \omega t + \varphi)$ ← initial phase — wave traveling to right, $+x$

$y(x,t) = A \sin(kx + \omega t + \varphi)$ — wave traveling to left, toward $-x$

$k = 2\pi/\lambda$ $\qquad \omega = \frac{2\pi}{T} = 2\pi f$

Look at constant phase $kx - \omega t + \varphi = \text{const}$

$$\frac{d}{dt}(kx - \omega t + \varphi) = 0$$

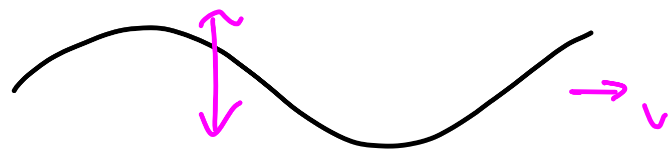
$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T} = v$$

for $kx + \omega t = \text{const}$ get $\frac{dx}{dt} = -\frac{\lambda}{T}$

$v = \lambda f$ or $v = \lambda \nu$

Transverse waves



string

Longitudinal waves

sound in air or slinky

v depends on what is vibrating

e.g. For Transverse wave on string

$$v=\sqrt{\frac{T}{\mu}}$$

T — Tension (NOT period in this case)

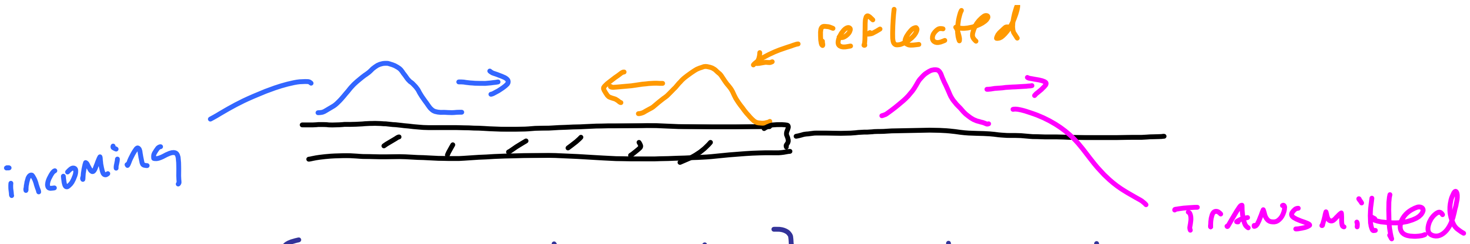
μ — Mass/length

longitudinal vibrations in Material (sound)

$$v=\sqrt{\frac{B}{\rho}}$$

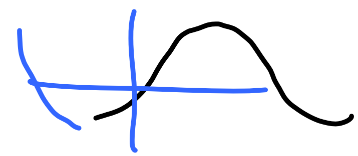
B — Bulk Modulus

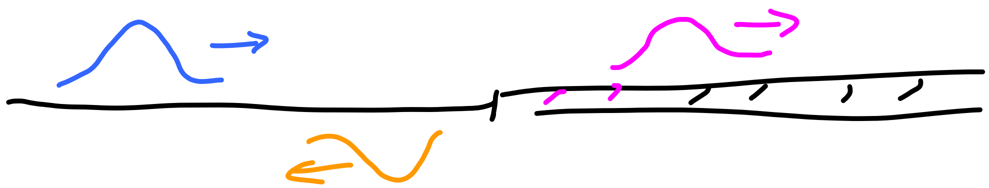
ρ — mass/volume



What happens at Boundary between Media?

[Heavy to light / Slow to fast] No phase change





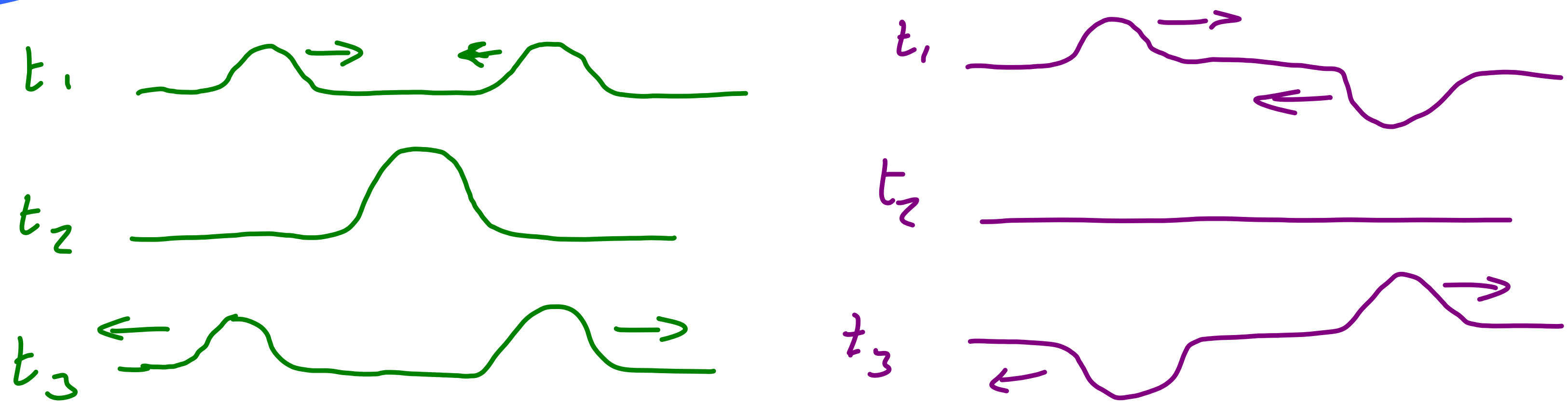
[light to heavy / fast to slow] No phase change for Transmitted wave

180° phase change for reflected wave

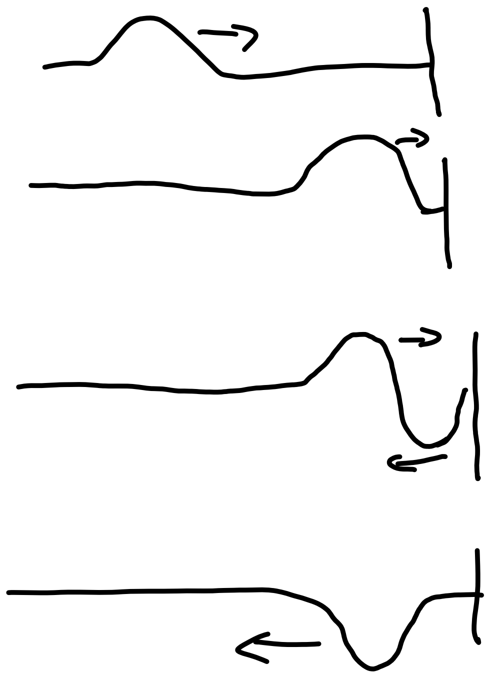
Waves exhibit superposition

... Total is the sum of the parts

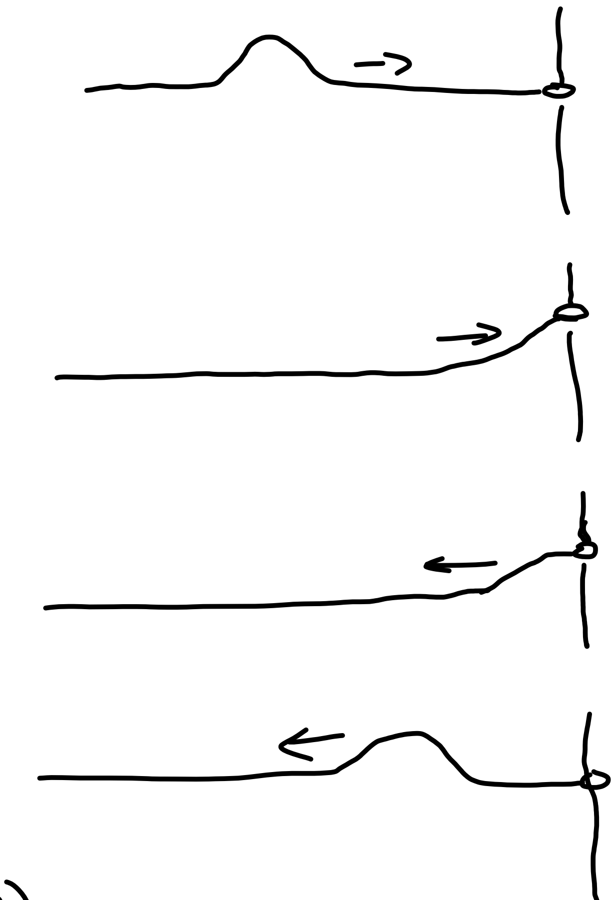
Interference



waves hitting obstacles:



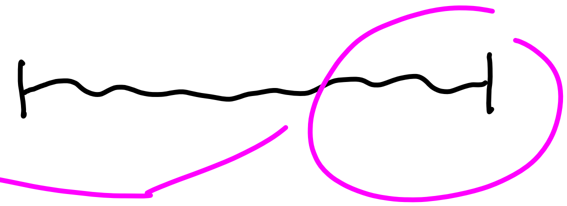
fixed end
(180° phase change at reflection)



loose end

No phase change

consider



string fixed at both ends

wave traveling to right

+

reflected wave traveling to left

both have same frequency and Amplitude

$$y_1(x,t) = A\sin(kx - \omega t)$$

$$y_2(x,t) = A\sin(kx + \omega t + \varphi)$$

We want this to be a reflected wave from fixed end so $\varphi = \pi$

$A\sin(x+\pi) = -A\sin(x)$

$$y_2(x,t) = -A\sin(kx + \omega t)$$

use Principle of Superposition

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\sin(kx-\omega t) - A\sin(kx+\omega t)$$

$$+A\sin(-kx-\omega t)$$

use Trig identity → $\sin C + \sin B = 2\sin\left[\frac{1}{2}(C+B)\right]\cos\left[\frac{1}{2}(C-B)\right]$

where $C \equiv kx-\omega t$ and $B = -kx-\omega t$

$$y(x,t) = (-2A)\sin(\omega t)\cos(kx)$$

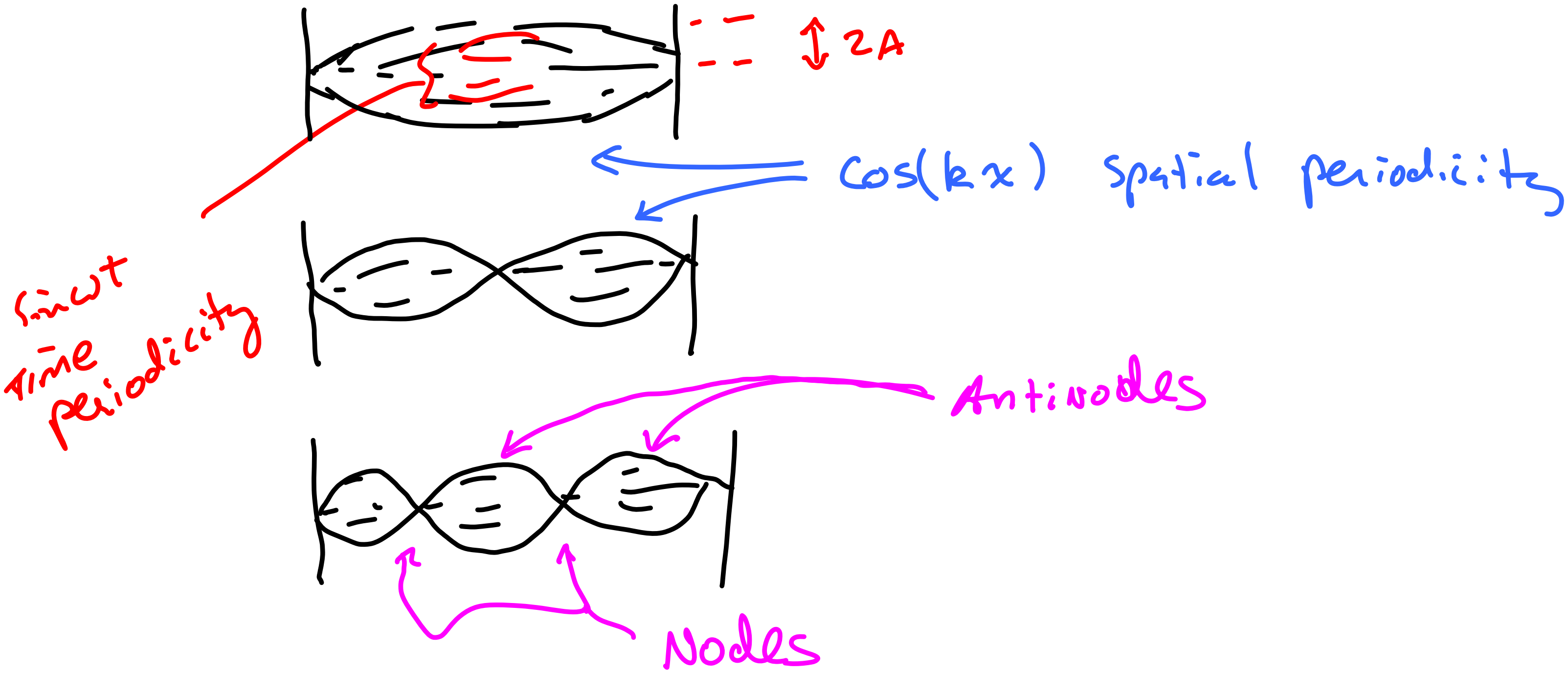
2A = Amplitude of Superposition

Time Variation

fixed form in space

Periodic in λ

Standing Waves



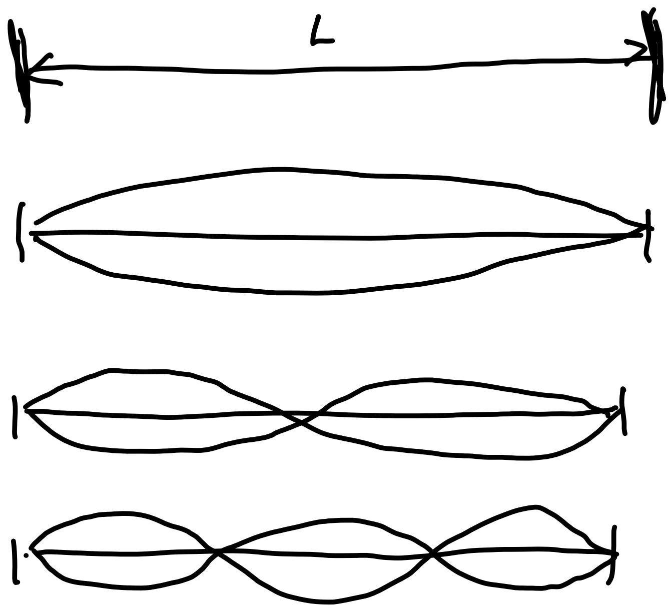
2A
cos(kx) spatial periodicity
sinωt
time periodicity
Antinodes
Nodes

String w/ mass/length μ and T and length L

What frequencies will it play well?
(resonate)

$$f \equiv \frac{1}{s} = s^{-1} \equiv Hz$$

Hertz



$$L = \frac{\lambda}{2}$$

$$L = \lambda$$

$$L = \frac{3\lambda}{2}$$

$$\vdots$$

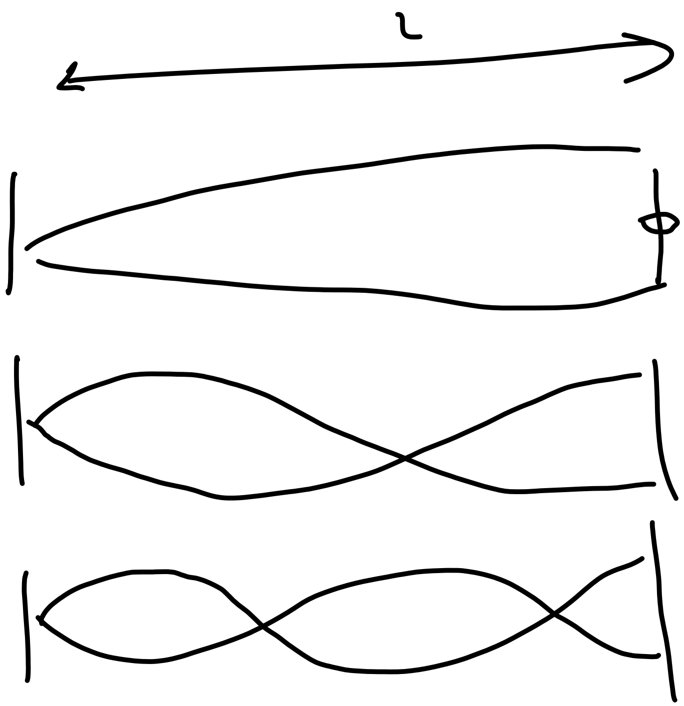
$$L = n\frac{\lambda_n}{2}$$

$$n = 1, 2, 3 \ldots$$

$$v = \lambda f$$

$$v = \sqrt{T/\mu}$$

$$L = \frac{n}{2}\frac{v}{f_n} = \frac{n}{2 f_n}\sqrt{T/\mu}$$



$$L = \frac{\lambda}{4}$$

$$L = \frac{3\lambda}{4} \quad \cdots$$

$$L = \frac{5\lambda}{4}$$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

Waves in Pipes