Waves

\[ y(x,t) = A \sin(kx - \omega \tau + \phi) \]

- \( k \) is the wave number (\( \frac{2\pi}{\lambda} \))
- \( \lambda \) is the wavelength
- \( \omega \) is the angular frequency
- \( \phi \) is the initial phase

\[ y(x,t) = A \sin\left(\frac{2\pi}{\lambda} x + \omega \tau + \phi\right) \]

- Wave traveling to \(+x\) right
- Wave traveling to \(-x\) left

\[ \frac{\lambda}{T} = \frac{2\pi}{\omega} = 2\pi f \]
Look at constant phase $kx - \omega t + \phi = \text{const}$

$$\frac{d}{dt}(kx - \omega t + \phi) = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi/\tau}{2\pi/\omega} = \frac{\omega}{\omega T} = \frac{1}{T} = \nu$$

For $kx + \omega t = \text{const}$

$$\frac{dx}{dt} = -\frac{\nu}{T}$$

$V = 2f$ or $V = \omega \nu$
Transverse Waves

Longitudinal Waves

\( v \) depends on what is vibrating

\( v \) for Transverse wave on String

\[ v = \sqrt{\frac{T}{\mu}} \]

For longitudinal vibrations in material (sound)

\[ v = \sqrt{\frac{B}{\rho}} \]
What happens at Boundary between Media?

- **Heavily to Light**
  - No phase change
  - Slow to Fast

- **Light to Heavy**
  - No phase change for Transmitted Wave
  - Fast to Slow
  - 180° phase change for reflected wave
Waves exhibit superposition... Total is the sum of the parts.
waves hitting obstacles:

fixed end (180° phase change at reflection)

loose end
no phase change
Consider a string fixed at both ends.

Wave traveling to right
+ reflected wave traveling to left

Both have some frequency and amplitude

\[ y_1(x,t) = A \sin(kx - \omega t) \]
\[ y_2(x,t) = A \sin(kx + \omega t + \phi) \]

\[ y_2(x,t) = -A \sin(kx + \omega t) \]

We want this to be a reflected wave from fixed end so \( \phi = \pi \)

\[ A \sin(x + \pi) = -A \sin(x) \]
Use Principle of Superposition

\[ y(x,t) = y_1(x,t) + y_2(x,t) = A \sin(kx - \omega t) - A \sin(kx + \omega t) + A \sin(-kx - \omega t) \]

Use Trig identity \( \sin C + \sin B = 2 \sin(\frac{1}{2}(C+B)) \cos(\frac{1}{2}(C-B)) \)

Where \( C \equiv kx - \omega t \) and \( B = -kx - \omega t \)

\[ y(x,t) = (-2A) \sin(\omega t) \cos(kx) \]

\( 2A \) = Amplitude of Superposition

Fixed form in space

Periodic in \( \lambda \)

Standing Waves
A string of mass/mass-length $m$ and $T$ and length $L$.

What frequencies will it play well? (resonate)

$L = \frac{n \pi}{2}$

$\eta = 1, 2, 3, \ldots$

$v = 2f$

$L = n \frac{v}{2f_m} = \frac{n \sqrt{T}}{2f_m}$
\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \]

\[ L = \frac{\lambda}{4} \]

\[ L = \frac{3\lambda}{4} \quad \ldots \]

\[ L = \frac{5\lambda}{4} \]
Waves in pipes

Pressure waves (sound)

High pressure

Displacement Antinode

L = λ

L = 3/4 λ