P113 University of Rochester NAME Soln key – SLM
S. Manly Fall 2013

Exam 2 (November 12, 2013)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (16 pts, 4 pts per part, show your work):

Thad Flintstone, like his very famous great-grandfather Fred, likes to move rocks. He has a job at the Slate Rock and Gravel Company. At the end of the day, Thad calculates how much work he did at work. Thad hasn't had physics and he needs you to help him as he determines the work he did during a number of tasks:

a) Thad lifts a 20 kg rock vertically upward at a constant speed of 0.5 m/s for a distance of 2 meters. How much work does Thad do on the rock as he performs this task?

$W = \vec{F}_{Thad}\cdot\vec{S} = |\vec{F}||\vec{S}| = mgs = (20)(9.8)(2) = 392\,J$ $\quad \uparrow \vec{F}_{Thad} \quad \uparrow \vec{S}$

b) Thad lowers a 20 kg rock vertically downward at a constant speed of 0.5 m/s for a distance of 2 meters. How much work does Thad do on the rock as he performs this task?

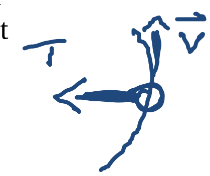
$W = -|\vec{F}||\vec{S}| = -mgs = -(20)(9.8)(2) = -392\,J$ $\quad \uparrow \vec{F}_{Thad} \quad \downarrow \vec{S}$

c) Thad carries a 20 kg rock horizontally at a constant speed of 0.5 m/s for a distance of 5 meters. How much work does Thad do on the rock as he performs this task?

$W = \vec{F}_{Thad}\cdot\vec{S}$ $\quad \vec{F}_{Thad} \perp$ to $\vec{S}$ so $W = 0$

d) After attaching a 0.3 kg rock to a (massless) string, Thad spins the rock in a horizontal circle in the air with a radius of 1.5 m. How much work does Thad do on the rock as it goes one time around the circle?

Force on rock $\perp$ to motion $\quad W = 0$



Problem 2 (8 pts, show your work):

Suppose the earth retains its current mass but is magically compressed in such a way that it has a radius that is one third of the current radius.

a) (4 pts) What is the new value of "g" on this new earth, i.e. if an object is dropped near the surface of this new, compressed planet, what would be the acceleration of the object toward the center of the planet (ignore air resistance)?

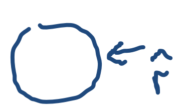
on Earth $F = \dfrac{GM_E m}{R_E^2}$ $\longrightarrow$ New planet $F = \dfrac{GM_E m}{(R_E/3)^2} = 9\underbrace{\boxed{\dfrac{GM_E}{R_E^2}}}_{g}m = 9mg$

So New "g" is $9g$

b) (4 pts) What is the gravitational field at a point near the surface of the new planet?

$\vec{g}$ = grav. field = $\dfrac{\vec{F}_{gravity}}{m}$

$\vec{g} = \dfrac{9GM_E}{R_E^2}$ radially toward center of planet



or $\vec{g} = 9g(-\hat{r})$

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Problem 3 (5 pts, no need to show work):

Two vectors lie in the xy plane. Vector A has an x component of +3 and a y component of -2. Vector B has an x component of +4 and a y component of +5. Compute the dot product, A·B, of the two vectors.

$$\vec{A}\cdot\vec{B} = (3)(4) + (-2)(5) = 12 - 10 = 2$$

Problem 4 (15 pts, show work):

You and your buddy Jimmy Joe Johnson attend a big NASCAR auto race. Jimmy Joe drank a bit too much coffee at breakfast and he starts muttering about life and circular motion and the frictional wear on tires. During Jimmy's stream of comments he asks you a question because he knows you have *awesome* physics skills. Jimmy's question follows:

A car of mass 1000 kg drives without slipping in a circle of radius 50 meters with a speed of 10 meters/second on a horizontal (flat) road. The coefficient of kinetic friction between the road and the tires is 0.2 and the coefficient of static friction between the road and the tires is 0.9. What is the magnitude frictional force between the road and the car?

Please answer Jimmy's question in the space below. Show your work.

$\vec{F}_{fr}$

$\vec{F}_{fr}$ Toward center of circle is centripetal force that keeps car on circle

$$|\vec{F}_{fr}| = \frac{mv^2}{R} = \frac{(1000)(10)^2}{50} = 2000\ N$$

Note

$F_{fr} = \mu_s N \equiv$ Maximum f_{fr} before car slips but actual Friction needed to keep car on circle in this case is $< \mu_s N$

if you say $F_{fr} = \mu_s N$ here and all else okay → score is 7/15

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Problem 5 (8 pts, show work):

A wheel rotates through 6.0 radians in 2.0 seconds as it is uniformly brought to rest. The initial angular velocity of the wheel before braking began was

a) 0.60 rad/s
b) 0.90 rad/s
c) 1.8 rad/s
d) 6.0 rad/s
e) 7.2 rad/s

$$\theta - \theta_0 = \frac{\omega + \omega_0}{2} t$$

$$6 = \frac{0 + \omega_0}{2}(2)$$

$$6 = \omega_0$$

Problem 6 (15 pts, show work and note that it is an elastic collision):

A ball of mass m = 0.2 kg is sliding (frictionlessly on ice) at 1.3 m/s along the x-axis in the positive direction. This ball collides elastically with another ball of mass M. After the collision, the first ball reverses direction and moves at 1.0 m/s in the negative x direction. What is the value of the mass, M, of the second ball and what is the velocity of the second ball after the collision? *By the way, I said that the collision is elastic. And, besides that, it is an elastic collision.*

Cons of Momentum

along x

$$m v_i = -m v_f + MV$$

$$(1.3 + 1.0)(.2) = \boxed{MV = 0.46} \quad \text{I}$$

along y or z, small ball has only v in the x at start and end. So large ball must have only momentum in x.

Cons. of Kinetic Energy

OK because collision is elastic

$$\frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} M V^2$$

From I

$$\frac{1}{2}(.2)(1.3^2 - 1^2) = \frac{1}{2} M \left(\frac{.46}{M}\right)^2$$

$$\boxed{M = \frac{.46^2}{0.14} = 1.5 \text{ kg}}$$

From I

$$\boxed{V = \frac{.46}{1.5} = 0.3 \text{ m/s}}$$

Problem 7 (15 pts, show your work):

A mass of 2 kg rests at the origin of the x axis (x=0). At t=0, a different mass of 5 kg is at rest at x=2 meters.

a) (5 pts) Where is the center of the mass of the system along the x-axis at t=0?

$$x_{cm} = \frac{\sum x_i m_i}{\sum m_i} = \frac{(2)(0)+(5)(2)}{7} = \frac{10}{7}\,m \cong 1.4\ m$$

b) (5 pts) Suppose the 5 kg mass is released at t=0 and is moved by an external force at a constant velocity of 1 m/s in the positive x direction. For t>0, what is the speed of the center of mass of the system along the positive x direction?
(Note that the external force only acts to move the 5 kg mass and does not factor in to the calculation of the center of mass position/velocity.)

$$x_{cm} = \frac{(0)m_1 + x_2 m_2}{m_1+m_2} \qquad \frac{dx_{cm}}{dt} = V_{cm} = \frac{\frac{dx_2}{dt} m_2}{m_1+m_2} = \frac{V_2 m_2}{m_1+m_2} = (1)\frac{5}{7}\ m/s$$

c) (5 pts) Forget part (b). Instead, assume that the 5 kg mass is released and moved by the external force in the positive x direction with a velocity that increases from zero at t=0 to $v = Bt^2$ at time t, where B is a constant equal to 3 m/s^3. Where is the center of mass of the system at t=4 seconds?

$a \neq$ const.

$$V = Bt^2$$

$$x_2(t) - \underbrace{x_2(0)}_{2m} = \int_0^4 Bt^2 = B\frac{t^3}{3}\Big|_0^4 = B\frac{4^3}{3} = \frac{(3)(64)}{3} = 64\,m$$

$$x_2 = 64 + 2 = 66\,m$$

So at $t=4$ $$x_{cm} = \frac{(0)m_1 + 66(m_2)}{m_1+m_2}$$

$$x_{cm}(t=4) = \frac{(66)(5)}{7} \cong 47 \text{ meters}$$

1)	/16
2)	/8
3)	/5
4)	/15
5)	/8
6)	/15
7)	/15
8)	/18
tot	/100

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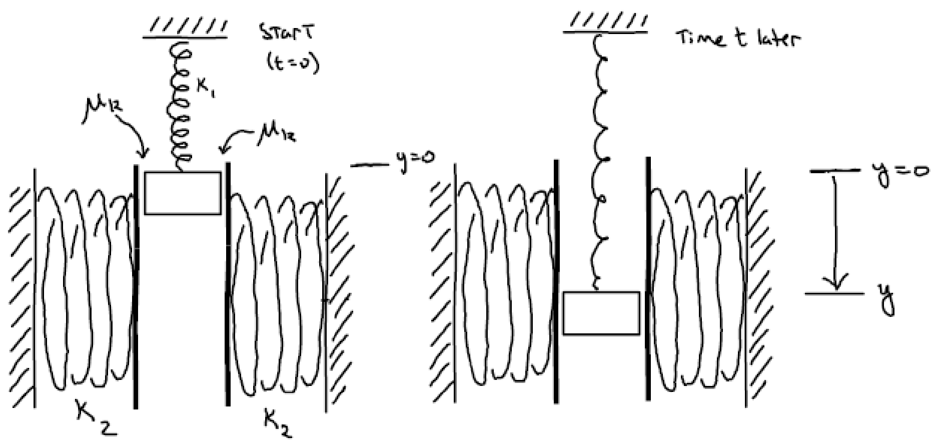
Problem 8 (18 pts):

Consider the situation sketched below. A block of mass M is attached to a spring hanging from the ceiling in a room that has spring constant K_1. Initially, as shown in the sketch on the left, when the mass is at a vertical postion defined as y=0, that spring is at its natural length (unstretched and uncompressed). The block is released and it falls. On the right, it is shown to be at some position y at a time t after it is released. The block is sandwiched between two vertically oriented plates as it falls. Each plate is pressed up against a vertical surface of the block (one on each side) as the block falls. The coefficient of kinetic friction, μ_k, between each plate and the falling block is the same on each side. The springs on each side press the plates up against the block. Each of these springs is compressed by an amount Δx and each has a spring constant of K_2. In terms of the parameters of the problem given here, solve for the velocity of the falling block when it reaches the position y. Ignore air resistance and assume the vertical plates remain parallel and vertical throughout the problem.



use energy conservation

$E_{start} = 0$ (vertical spring is unstretched, no motion at t=0)

$$E_{at\ position\ y} = \underbrace{\tfrac{1}{2}K_1 y^2}_{\text{PE spring}_1} - \underbrace{mgy}_{\text{grav PE}} + \underbrace{E_{Fr}}_{\text{energy lost to friction}} + \overbrace{\tfrac{1}{2}mv^2}^{KE}$$

$$= \underbrace{(2)}_{\text{each side}}\mu_k \underbrace{y}_{\text{distance}} \underbrace{K_2 \Delta x}_{\text{Normal force on each side}}$$

E const

$E_{start} = E_{at\ y}$

$$0 = \tfrac{1}{2}K_1 y^2 - mgy + 2\mu_k y K_2 \Delta x + \tfrac{1}{2}mv^2$$

$$v = \left[\frac{2}{M}\left(-2\mu_k y K_2 \Delta x + mgy - \tfrac{1}{2}K_1 y^2\right)\right]^{1/2}$$