P113 University of Rochester NAME Soln key – SM
S. Manly Fall 2013

# Exam 3 (December 5, 2013)

***Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.***

## Problem 1 (15 pts, 3 pts per part, no need to show work):

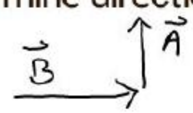
For each of the following parts, specify the direction of the vector in question as one of the following:

i. into the paper
ii. out of the paper
iii. up (in the plane of the paper)
iv. down (in the plane of the paper)
v. left (in the plane of the paper)
vi. right (in the plane of the paper)
vii. none of these

(a) determine direction of $\vec{A}x\vec{B}$


$\boxed{\text{i – into paper}}$

(b) determine direction of $\vec{B}x\vec{A}$




$\boxed{\text{ii – out of paper}}$

(c) determine direction of $\vec{A}x\vec{B}$


$\boxed{\text{iii – up}}$

(d) determine direction of $\vec{A}x\vec{B}$


$\boxed{\text{vii - None}}$

cross product is zero

(e) determine direction of $\vec{B}x\vec{A}$


$\boxed{\text{vii – none}}$

cross product is zero

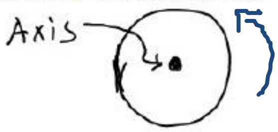
**Problem 2 (18 pts, 3 pts per part, no need to show work):**

| | |
|---|---|
| 1) | /15 |
| 2) | /18 |
| 3) | /6 |
| 4) | /15 |
| 5) | /15 |
| 6) | /15 |
| 7) | /16 |
| tot | /100 |

For each of the following parts, specify the direction of the vector in question as one of the following:

i. into the paper
ii. out of the paper
iii. up (in the plane of the paper)
iv. down (in the plane of the paper)
v. left (in the plane of the paper)
vi. right (in the plane of the paper)
vii. none of these

(a) determine direction of $\vec{\omega}$

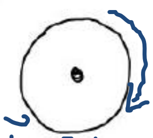



wheel rotates counter clockwise about axis

$\boxed{\text{ii - out of paper}}$

(b) determine direction of $\vec{\alpha}$

wheel spins clockwise about axis shown and is slowing down



$\boxed{\vec{\alpha} \text{ is ii - out of paper}}$

$\vec{\omega}$ into paper, $\vec{\alpha}$ opposite that due to slowing down

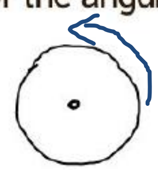
(c) determine the direction of the torque in (b)

$\vec{\tau} = I\vec{\alpha}$ - same direction as $\vec{\alpha}$

$\boxed{\vec{\tau} \text{ is ii - out of paper}}$

(d) determine the direction of the angular momentum, $\vec{L}$

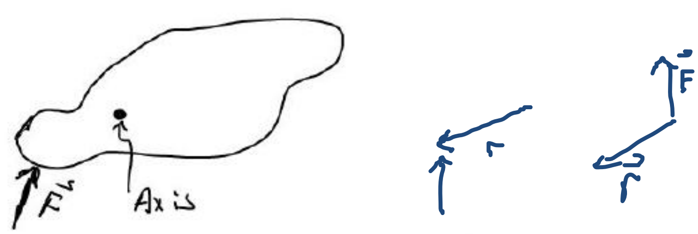
Wheel rotates counterclockwise about axis



$\vec{L}$ in direction of $\vec{\omega}$

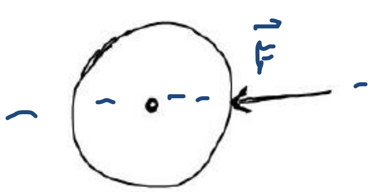
$\boxed{\vec{L} \text{ is ii - out of paper}}$

(e) determine the direction of the torque, $\vec{\tau}$




$\boxed{\vec{\tau} \text{ is i - into paper}}$

(f) determine the direction of the torque, $\vec{\tau}$

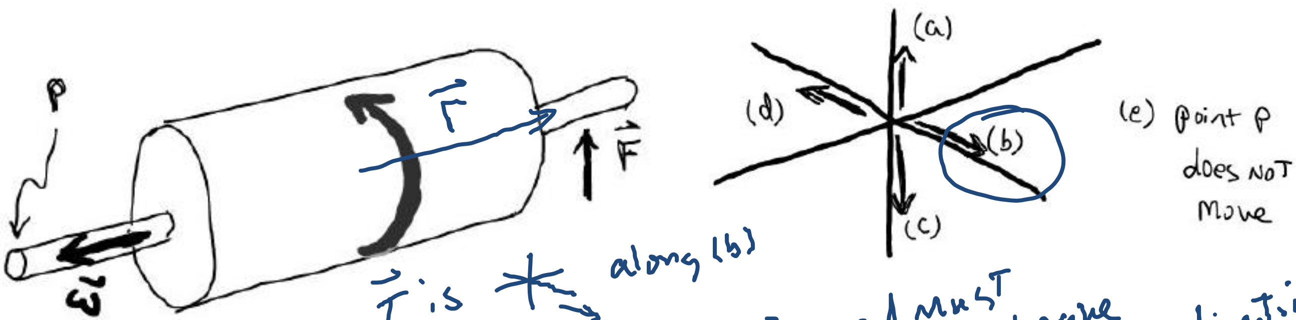



$\vec{r} = 0$ so $\vec{\tau} = 0$

$\boxed{\text{vii - none}}$

**Problem 3 (6 pts, indicate your reasoning):**

Studly McDaniel is a spacecraft repair technician at Al's Interstellar Spaceport and Casino. While patrons gamble and eat "the best garbage plates in the galaxy", Studly services their spaceships. One day Studly observes a cylinder spinning as sketched below. He pushes upward lightly on one axis of the spinning cylinder with a force F. He observes the other end of the cylinder, marked as point P in the sketch, to move in what direction? Make your choice from (a)-(e) shown below.

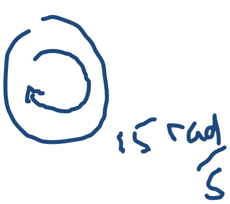



$\vec{\tau}$ is along (b)

$\vec{\tau} = \frac{d\vec{L}}{dt}$, $\vec{L}$ is along $\vec{\omega}$ and must shift in direction of torque so, point P will move in direction of (b)

**Problem 4 (15 pts, show work):**

Two hockey pucks slide frictionlessly across the ice during a practice shooting session for a hockey team. As it slides along, puck 1 spins clockwise with an angular velocity of 15 rad/s. The other puck, puck 2, spins counter-clockwise with an angular velocity of 4 rad/s. The two pucks collide in the middle of the ice and move apart. After the collision, puck 1 spins with an angular velocity of 7 rad/s clockwise. What is the magnitude and sense of spin direction of the angular velocity of puck 2 after the collision?
(*Assume the pucks are identical, of uniform density, and shaped like shot cylinders. Assume each has a moment of inertial of $I=(1/2)MR^2$. Let M=0.16 kg and R=0.038 m, if you need numbers.)*





Initial state $\vec{L}$

$= I_1\omega_1 + I_2\omega_2$

$= I_1(15) - I_2(4)$

def: clockwise rotation as (+)

Final state $\vec{L}$

$= I_1(7) + I_2\omega_2$

$I_1 = I_2$ ← given

$L_{initial} = L_{final}$ ← Ang Momentum conservation

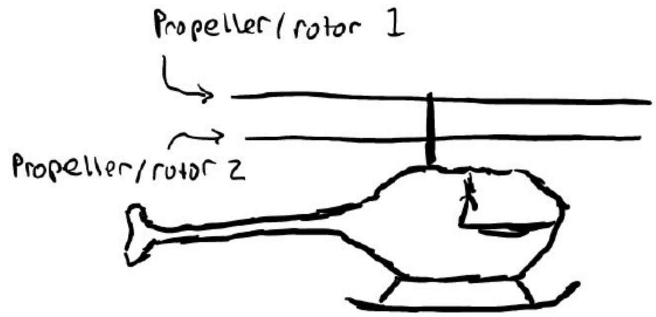
$15 - 4 = 7 + \omega_2$

$11 = 7 + \omega_2$

$\omega_2 = 4$ rad/s rotating in a clockwise sense

**Problem 5 (15 pts):**

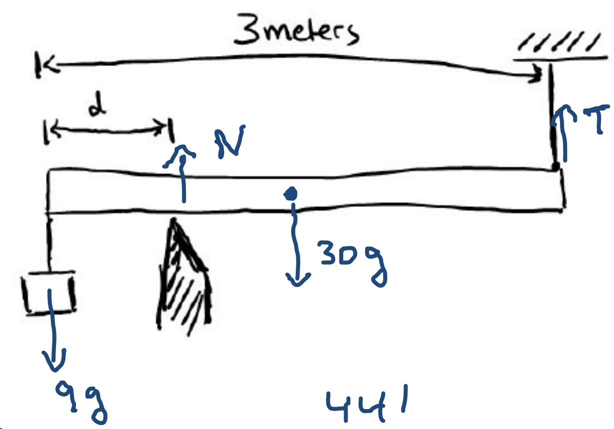
During the holiday season you may notice that small, radio controlled helicopters are very popular toys these days. When you look at one of these helicopters carefully, you may notice that they have two propellers (or rotors) that rotate in the horizontal plane as shown in the sketch to the side. (*The rotors are shown from the side in the sketch and rotate in the plane perpendicular to the paper*.) If you view the spinning rotors from above, and you see rotor 1 spinning clockwise, briefly describe the direction rotor 2 is spinning and how you know it should be spinning that direction.




If rotor 1 spins clockwise, rotor 2 must spin counterclockwise. With that the case, both rotors can speed up and slow down simultaneously. this way the rotors can change speed and maintain the conservation of angular momentum without the helicopter body spinning. If both rotors spin in the same direction, the helicopter body would spin around to maintain the conservation of angular momentum whenever the rotors spin. ... not a great way to fly

**Problem 6 (15 pts, show work):**

A uniform steel beam of length 3 meters and mass 30 kg has a mass of 9 kg hanging from one end (the left end in the sketch) and is attached to a massless rope attached to the ceiling on the other end (the right side in the sketch). It is supported by a wall at a distance d from the end of the beam, as shown in the sketch. If the normal force exerted by the wall on the beam is ~~215~~ 250 Newtons, determine the distance d and tension in the rope attached to the ceiling.




Translational equilibrium $\Sigma F_y = 0$

$$0 = T + N - 30g - 9g$$

$$T = 39g - N = (39)(9.8) - 250 = 132 \text{ N} = T$$

Rotational equilibrium $\Sigma \tau = 0$

Take Torques about left end

$$0 = -\underset{215}{N}d + (30)\underset{9.8}{g}(1.5) - \underset{132}{T}3$$

$$d = \frac{\overbrace{(30)(9.8)(1.5)}^{441} - (132)(3)}{215}$$

$$\boxed{d = \frac{45}{215} = 0.21 \text{ m}}$$

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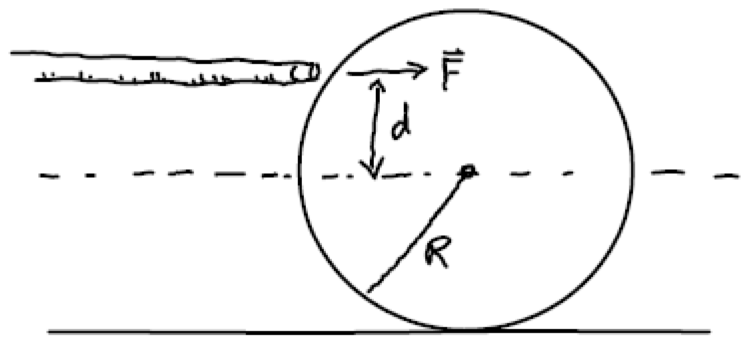
**Problem 7 (16 pts, show work):**

Fast forward a few years. After some years in graduate and medical school, you decide your true calling is playing billiards. One evening you are playing a few games in your favorite local spot called Joe's Billiards and EKG Repair. Joe and some of his friends are watching you and debating the source of your amazing skill at billiards. One guy says it must come from long hours of practice. Another says it has more to do with how much iced tea you drink while you play. Of course, you know it all boils down to your fabulous background in physics. Go you!

You want to strike the cue ball with the stick with a force F in such a way that the cue ball will roll without slipping. If the stick is moving parallel to the surface of the table, as shown in the sketch, what is the distance d above the midpoint of the ball that you should strike the cue ball so that it rolls without slipping? Express your answer in terms of the radius, R, of the cue ball. (*Like all amazing billiards players, you know that the moment of inertia of the spherical ball with uniform density is I = (2/5)MR$^2$, where M is the mass of the ball*).




To role without slipping, Tangential velocity and accel. at outer radius must equal that of center of mass

so $F = Ma \Rightarrow a = F/M$

$$\tau = Fd = I\alpha = I\,a/R$$

$$a = \frac{FdR}{I}$$

$$\frac{FdR}{I} = \frac{F}{M}$$

$$d = I/RM$$

$$d = \tfrac{2}{5}MR^2/RM$$

$$\boxed{d = \tfrac{2}{5}R}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$v = v_o + at$$

$$x = x_o + v_o t + \frac{1}{2}at^2$$

$$x = x_o + \left(\frac{v_o + v}{2}\right)t$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$x - x_o = \int_{t_o}^{t} v\,dt$$

$$v - v_o = \int_{t_o}^{t} a\,dt$$

$$\sum \vec{F} = m\vec{a}$$

$$F_{\text{friction}} = \mu_k N$$

$$F_{\text{friction}} = \mu_s N$$

$$F_{\text{centripetal}} = \frac{mv^2}{r}$$

$$\vec{F}_{grav} = -\frac{Gm_1 m_2}{r^2}\hat{r}$$

$$\vec{F}_{\text{spring}} = -k(\vec{x} - \vec{x}_o)$$

$$\text{work} = \int F \cdot ds$$

$$\text{power} = \frac{dw}{dt}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\theta = \theta_o + \left(\frac{\omega + \omega_o}{2}\right)t$$

$$\omega \;\; = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$KE_{\text{translation}} = \frac{1}{2}MV^2$$

$$KE_{\text{rotation}} = \frac{1}{2}I\omega^2$$

Not for this test no worries ☺

$$I = \sum m_i r_i^2 = \int r^2 dm = \int r^2 \rho dv$$

$$X_{cm} = \frac{\sum x_i m_i}{M} = \frac{\int x\,dm}{M}$$

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\text{circumference of circle} = 2\pi r$$

$$\text{area of circle} = \pi r^2$$

$$\text{quadratic equation} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$