P113 University of Rochester NAME ~~Soln key - SM~~

S. Manly Fall 2013

Final Exam (December 19, 2013)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (4 pts):

A second ball is dropped from a cliff 1 s after a first ball was dropped. As both fall, does the distance between them increase, decrease, or stay the same? Justify your answer with a short sentence.

The distance increases. Both undergo CONSTANT acceleration after starting from rest. The first ball has been accelerating at 9.8 m/s² for 1 s and is already moving at 9.8 m/s when the second ball is dropped. The time evolution of the speed of the 2nd ball will always lag that of the 1st during the fall

Problem 2 (6 pts, 3 pts per part):

Vector $\vec{\mathbf{A}}$ is equal to $-5\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$

Vector $\vec{\mathbf{B}}$ is equal to $+2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

(a) What angle does the vector $\vec{\mathbf{A}}+\vec{\mathbf{B}}$ make with the x-axis?

$\vec{A} + \vec{B} = -3\hat{i} + 11\hat{j}$ $\theta = 15.2^\circ$

$\tan\theta = \frac{3}{11}$

Angle w/ x axis = 90 − 15.2 ≅ 75° w/ −x axis

(b) Determine $\mathbf{A}\cdot\mathbf{B}$

$\vec{A}\cdot\vec{B} = (-5)(2) + (7)(4) = -10 + 28$
$= +18$

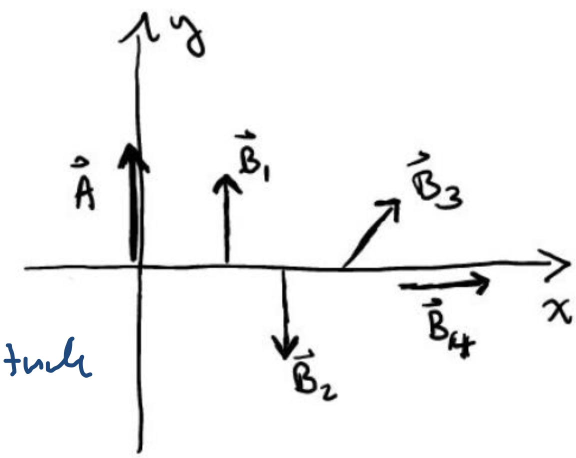
Problem 3 (6 pts, 3 pts per part):

Consider the five vectors shown in the sketch. The vectors $\mathbf{B}_1$ through $\mathbf{B}_4$ are each of the same magnitude and differ only in direction. Consider the vectors $\mathbf{A}\times\mathbf{B}_1$, $\mathbf{A}\times\mathbf{B}_2$, $\mathbf{A}\times\mathbf{B}_3$, and $\mathbf{A}\times\mathbf{B}_4$.

(a) Which of these four vectors has the largest magnitude? $\vec{A}\times\vec{B}_4$ has largest magnitude

(b) Which of these four vectors has the smallest nonzero magnitude?

$\vec{A}\times\vec{B}_3$ has the smallest nonzero magnitude



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Problem 4 (5 pts):

True story

A world's land speed record was set by Col. John P. Stapp when, on March 19, 1954, he rode a rocket-propelled sled that moved down a track at 1020 km/hour. He and the sled were brought to a stop in 1.4 s. What acceleration did he experience during that 1.4 seconds? *You can assume this acceleration is constant.*

$1020 \frac{km}{hr} \times 1000 \frac{m}{km} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ s} = 283\ m/s$

$v = v_0 + at$

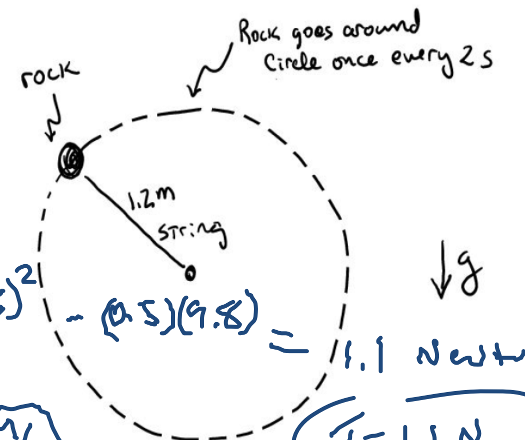
$0 = 283 + a(1.4)$

$a = -202\ m/s^2$

~ 20.6 g's wow!

Problem 5 (5 pts):

A rock of mass 0.5 kg is tied to a string and twirled in a circle in the vertical plane (see sketch) by a person standing on the surface of the earth. The radius of the circle is 1.2 m. If the rock goes around the circle with a period of 2 seconds, what is the tension in the string when the rock is at the top of its motion?



$\frac{mv^2}{R} = Mg + T$

$T = \frac{mv^2}{R} - Mg$

Circumference $= 2\pi(1.2) = 7.5\ m$

$v = \frac{7.5\ m}{2\ s} = 3.8\ m/s$

$T = \frac{0.5(3.8)^2}{1.2} - (0.5)(9.8) = 1.1$ Newton

$T = 1.1\ N$

Problem 6 (4 pts):

Two balls of the same size are completely submerged in water. Ball 1 is made of wood. Ball 2 is made of aluminum. The buoyant force is largest on

(a) Ball 1
(b) Ball 2
(c) Neither ball. The buoyant force is the same on the balls.
(d) Neither ball. There is no buoyant force on objects that are completely submerged.

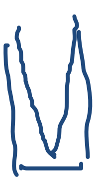
F_b is mass of displaced water – same for each

1)	/4
2)	/6
3)	/6
4)	/5
5)	/5
6)	/4
7)	/7
8)	/8
9)	/8
10)	/10
11)	/10
12)	/11
13)	/12
tot	/100

Problem 7 (7 pts):

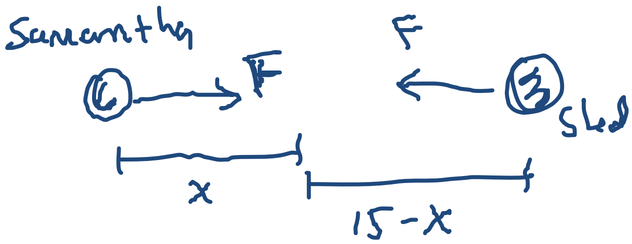
If you blow air across the mouth of an empty soda bottle, you hear a tone. Briefly explain why it is that if you put some water in the bottle, the pitch of the tone increases, i.e.,the frequency of the sound you hear increases.

The sound you hear is the fundamental frequency of the standing sound waves resonating in the bottle. For an open end tube the length of the tube corresponds to $\frac{1}{4}\lambda$ for the fundamental. If you put water in the bottle the wavelength of the fundamental is shorter. Since the speed of sound is unchanged and $v = \lambda f$, f must be larger.



Problem 8 (8 pts):

Samantha Sweetie has a mass of 40 kg and is probably unhappy with me for telling you that. Oh well. Samantha sits on the surface of a smooth, frozen lake 15 meters from an 8.4 kg sled. She pulls on the sled by means of a massless rope with a constant force of 5.2 N. Relative to Samantha's initial position, where does she meet the sled?



$a_{sam} = \frac{5.2}{40} = 0.13\ m/s^2$

$a_{sled} = \frac{5.2}{8.4} = 0.62\ m/s^2$

use $x - x_0 = v_0 t + \frac{1}{2} a t^2$

$x = \frac{1}{2}(.13)t^2$

$15 - x = \frac{1}{2}(.62)t^2$

$x = \frac{\frac{1}{2}(.13)(30 - 2x)}{.62}$

$x = 3.1 - 0.21 = 2.9\ m$

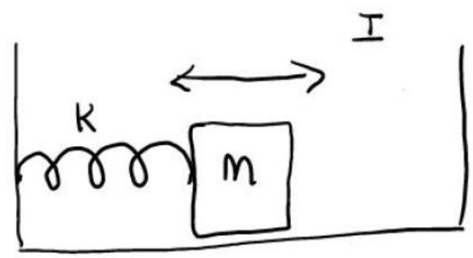
Problem 9 (8 pts):

Consider a mass M = 0.2 kg attached to a spring with spring constant K = 20 N/m. The mass is free to move on a frictionless surface otherwise. The situation is sketched below in figure I. The mass is moved so that the spring is compressed by 0.1 m (relative to its natural length) and then released.

(a) (3 pts) With what period does this mass oscillate?

$\omega = \sqrt{K/M} = 10\ s^{-1}$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = 0.63$ Seconds

(b) (5 pts) Consider the situation sketched in figure II. In this case the same mass is attached to three identical springs with the same spring constant as in part (a). When the mass is not moving, each spring is at its natural length. When the mass is displaced to one side and released, what is the frequency of its oscillation?



$K \rightarrow 3K$

$\omega = \sqrt{\frac{3K}{M}} = 17.3\ s^{-1}$

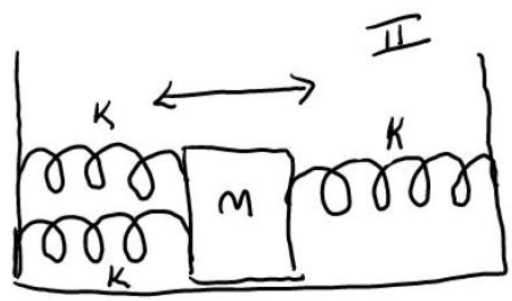
ok if this

or

can switch to

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi} = \frac{17.3}{2\pi} = 2.7\ s^{-1}$



$F = -Kx$

$\frac{d^2x}{dt^2} + \frac{K}{M}x = 0$

$\omega^2 = \frac{K}{M}$

$\omega = \sqrt{K/M}$

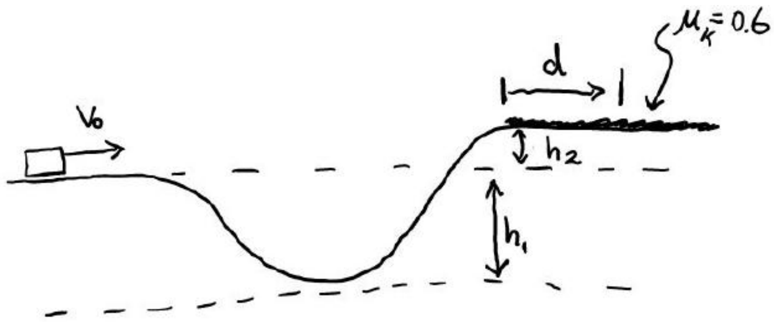
$\omega = 2\pi/T$

$T = \frac{2\pi}{\omega}$

Either ω or f okay as answers

Problem10 (10 pts, 5 pts per part):

Thad Cool is the party chairman of his fraternity. He delivers a case of orange soda to the fraternity house by sliding it along a frictionless track from one level to a higher level. The soda case passes through a depression of depth h_1 (as compared to the initial height) and then rise up to the higher level, which is a height of h_2 above the initial level. At the higher level, the cases encounter a flat surface with a coefficient of kinetic friction of 0.6, which brings them to a stop a distance d from the start of the surface with friction. Suppose the case starts with a velocity of 7.0 m/s and h_1= 1 meter and h_2 = 0.5 meters.



E cons.

(a) Determine d.

$$\frac{1}{2}mv_0^2 = mgh_2 + mg\mu_k d$$

$$\frac{7^2}{2} = (9.8)(0.5) + (9.8)(0.6)d$$

$$\frac{(24.5 - 4.9)}{(9.8)(.6)} = d$$

$$\boxed{d = 3.3\ m}$$

Let $M_{case} = 10\,kg$

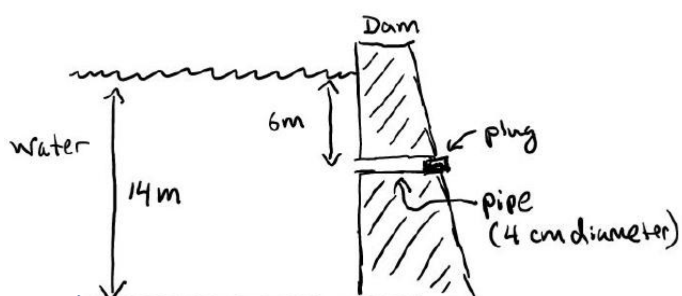
(b) How much work does the force of friction do on the case of orange soda?

$f_{fr} \leftarrow$ $\xrightarrow{d}$ $W = \int F \cdot ds$

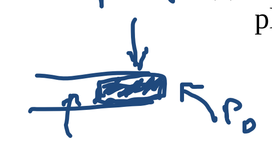
$$\text{Friction} = -mg\mu_k d = -(10)(9.8)(.6)(3.3) = -194 \text{ Joules}$$

Problem11 (10 pts, 5 pts per part):

Fresh water behind a reservoir dam is 14 meters deep. A horizontal pipe 4.0 cm in diameter passes through the dam 6 meters below the water surface. A plug secures the pipe opening.



(a) What is the friction force between the plug and the pipe wall?



$$\Delta P = \rho g h = (1000\ kg/m^3)(9.8\ m/s^2)(6\ m) = 58800\ \text{Newtons}/m^2$$

$$\text{Force of water on plug} = (58800)\,\text{Area} = (58800)\pi(.02)^2 = 74\ N$$

so friction force must be 74 N

(b) The plug is removed. What volume of water flows through the pipe in 1000 seconds?

$$\underbrace{\cancel{P_0} + \cancel{\tfrac{1}{2}\rho v^2}^{\,0} + \rho g h}_{\text{at top}} = \underbrace{\cancel{P_0} + \tfrac{1}{2}\rho v^2 + \cancel{\rho g h}^{\,0}}_{\text{at opening of pipe}}$$

$$\rho g h = \tfrac{1}{2}\rho v^2$$

$$v^2 = \sqrt{2gh} \Rightarrow v = 10.8\ m/s$$

$$\text{Volume} = (\text{Area})(v)(\text{time}) = \pi(.02)^2(10.8)(1000) = \boxed{13.6\ m^3}$$

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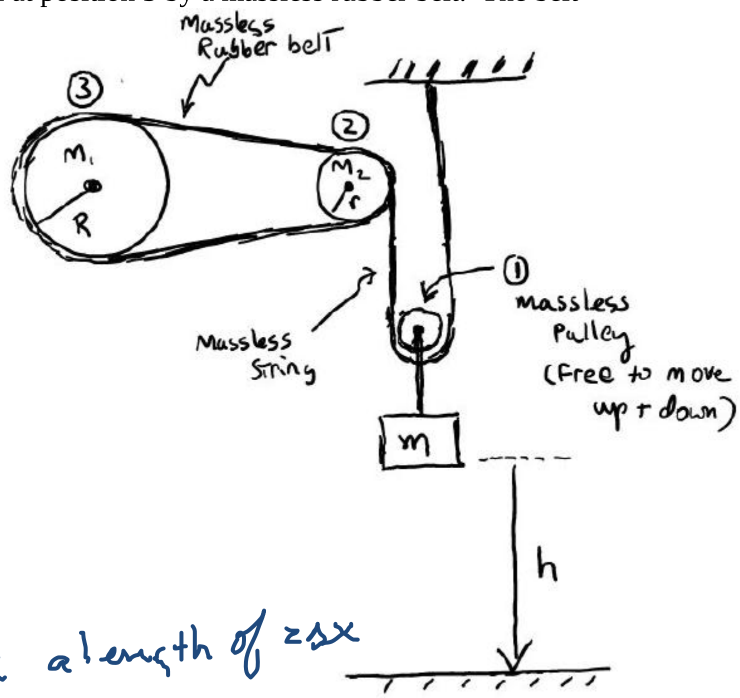
Problem 12 (11 pts):

Mass m hangs from a massless rope connected to a massless pulley that is free to move up and down (position 1 in diagram). Another rope is attached to the ceiling and loops around the pulley at position 1 and then is spooled (wrapped multiple times) about a wheel at position 2 in the diagram. The wheel at position 2 is free to rotate about an axis at its center and it has mass M_2 and radius r. If needed, consider the moment of inertia of the wheel at position 2 to be $(1/2)M_2r_2$. The wheel at position 2 is connected to a wheel at position 3 by a massless rubber belt. The belt goes around the wheel at position 2 in such a way that it does not interfere with the motion of the spooled rope that loops around the pulley at position 1. The wheel at position 3 has mass M_1 and radius R. The wheel at position 3 is free to rotate about an axis through its center and, if needed, assume it to have a moment of inertial of $(1/2)M_1R_2$. The spooled rope and the belt move without slipping on the pulley and wheels. The system described above and in the sketch is released from rest with the mass m a distance h above the floor. Find the speed of mass m just before it hits the floor given M_1=2 kg, M_2=0.5 kg, m=3 kg, and h=0.4 meters.



Try Energy conservation

Note that as m moves down Δx a length of $2\Delta x$ of string is pulled from spool at 2

Initially nothing moving $E \rightarrow mgh$

$s = r\theta$

$v = r\omega$

$v_2 = v_1 = 2v$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{1}{2}M_2r^2\frac{v_2^2}{r^2} + \frac{1}{2}\frac{1}{2}M_1R^2\frac{v_1^2}{R^2}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}M_2 4v^2 + \frac{1}{4}M_1 4v^2 = \left[\frac{1}{2}m + M_2 + M_1\right]v^2$$

$$v^2 = \frac{mgh}{\left[\frac{1}{2}m + M_2 + M_1\right]} = \frac{(3)(9.8)(0.4)}{\left[\left(\frac{1}{2}\right)3 + 0.5 + 2\right]} = 2.9\left(\frac{m}{s}\right)^2$$

$$\boxed{v = 1.7\ \frac{m}{s}}$$

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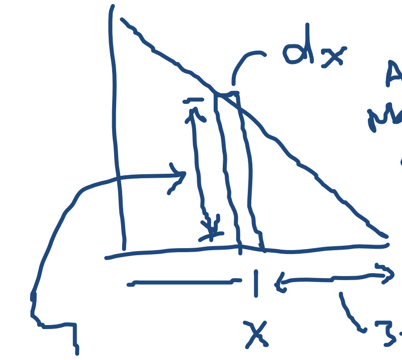
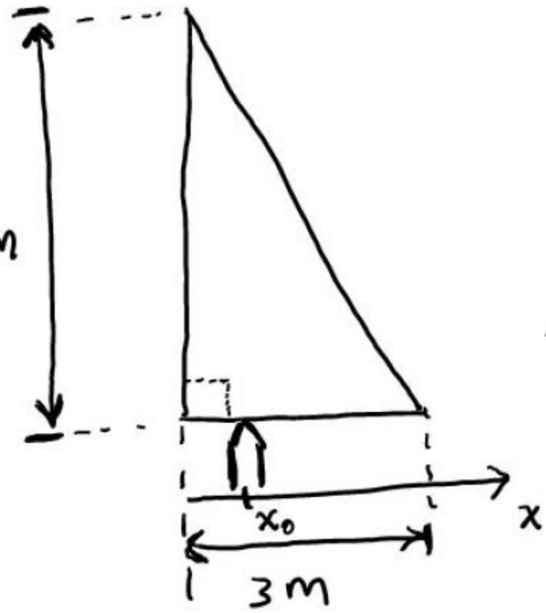
Problem 13 (12 pts):

A right-triangular sheet of metal with sides of length 3 meters and 4 meters, as shown in the sketch, balances on a narrow support placed at a distance x_o from the left side of the triangle as shown in the sketch. (The force of gravity is pointed down on the paper.) If the mass of the triangle is M and it has a uniform density, determine x_o.



Area mass density $= \dfrac{M}{\frac{1}{2}(4)(3)} = \dfrac{M}{6}\ \mathrm{kg/m^2}$



$\frac{4}{3}(3-x)$ Area of sliver at x, width dx

(3-4-5 triangle) $dA = (dx)\frac{4}{3}(3-x)$

dm = mass of sliver dx at position x

= (Area mass density)(dA)

$$dM = \frac{M}{6}\frac{4}{3}(3-x)\,dx$$

Triangle will balance when x_0 is $x_{\text{center-of-mass}}$

$$x_{cm} = \frac{\int_0^3 x\,dM}{\int_0^3 dM} = \frac{\int_0^3 x \frac{M}{6}\frac{4}{3}(3-x)\,dx}{M} = \frac{2}{9}\int_0^3 (3x - x^2)\,dx$$

$$x_{cm} = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{2}{9}\left[\frac{27}{2} - 9\right] = 3 - 2 = 1 \text{ meter}$$

$$\boxed{x_0 = 1 \text{ meter}}$$