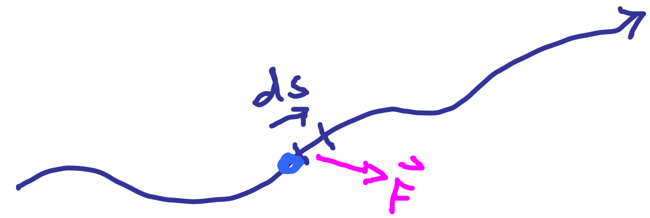
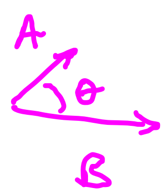
Physics 113 - October 15, 2013

- Exams available for pickup outside my office
- Regrades - happening
  Please make sure you understand the problem/solns
  before you ask for a regrade

Last Time

$$\text{Work} = \int_i^f \underbrace{\vec{F}\cdot d\vec{s}}_{\text{dot product}}$$



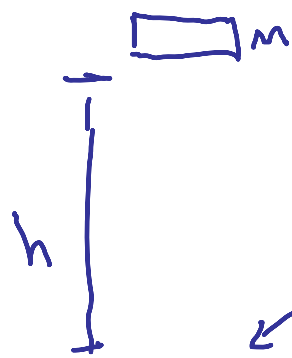


$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Kinetic Energy
of motion

$$KE = \frac{1}{2} m v^2$$



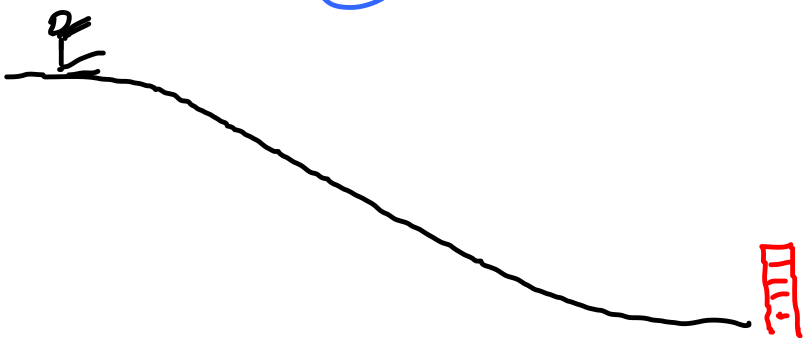
How fast at bottom

$$v^2 = \underbrace{v_0^2}_{0} + 2ah$$

$$v^2 = 2gh$$

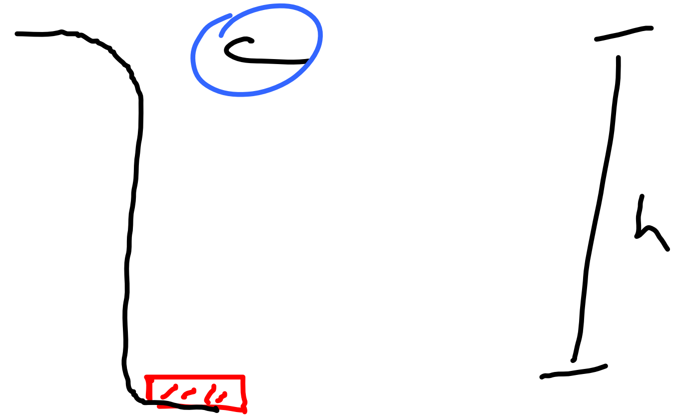
$$\underbrace{\frac{1}{2}mv^2}_{\text{Kinetic energy at Bottom}} = \frac{1}{2}m\,2gh = \overbrace{mgh}^{\text{Potential Energy at Top}}$$

A



B

C



D

Does NOT Matter

$$mgh = \frac{1}{2}mv^2$$

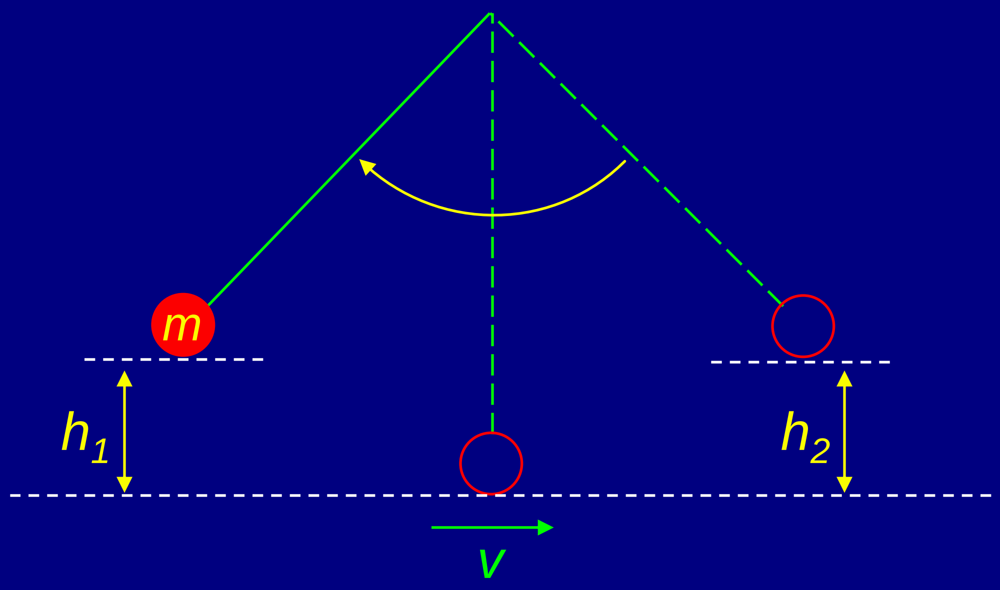
Energy Conservation

$$\sum E_i = \text{CONSTANT}$$

$$= \sum E_f$$

Example: The simple pendulum

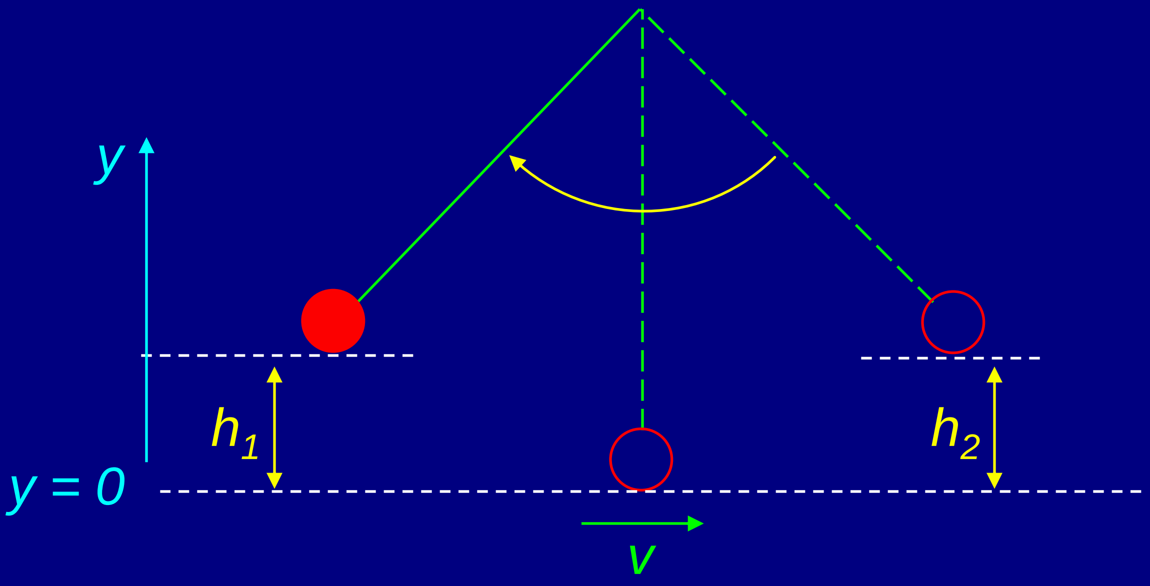
- Suppose we release a mass m from rest a distance h_1 above its lowest possible point.
  - What is the maximum speed of the mass and where does this happen?
  - To what height h_2 does it rise on the other side?



Example: The simple pendulum

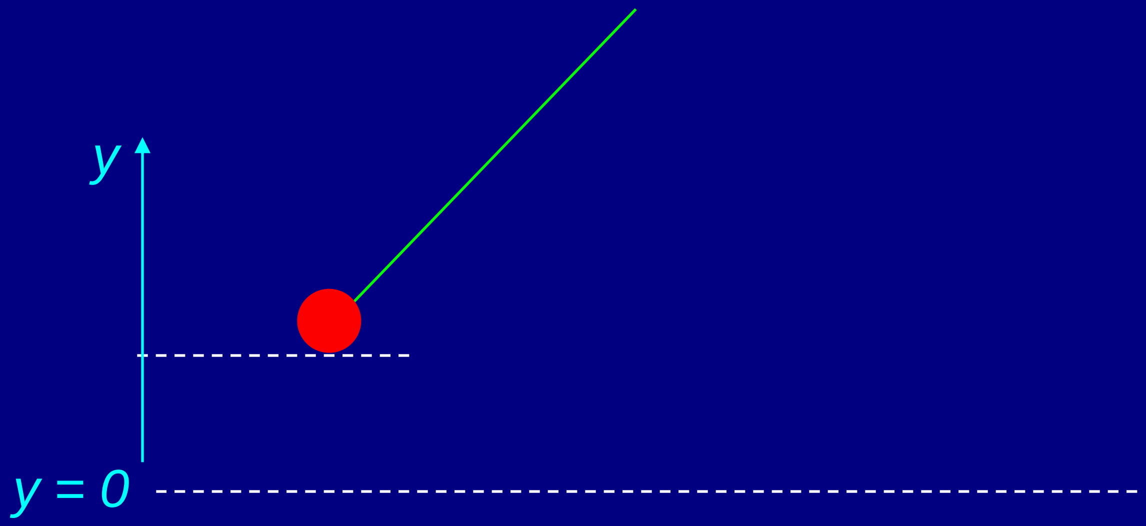
- Kinetic+potential energy is conserved since gravity is a conservative force ($E = K + U$ is constant)
- Choose $y = 0$ at the bottom of the swing, and $U = 0$ at $y = 0$ (arbitrary choice)

$$E = {}^{1}/_{2}mv^2 + mgy$$



Example: The simple pendulum

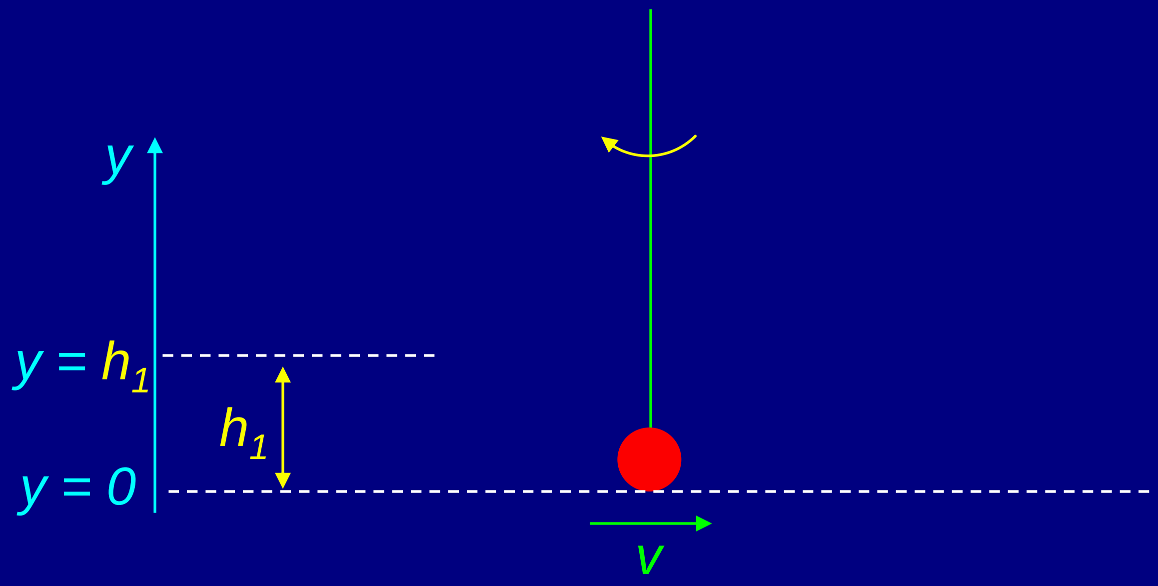
- $E = {}^{1}/_{2}mv^2 + mgy$.
  - Initially, $y = h_1$ and $v = 0$, so $E = mgh_1$.
  - Since $E = mgh_1$ initially, $E = mgh_1$ always since energy is conserved.



Example: The simple pendulum

- $\frac{1}{2}mv^2$ will be maximum at the bottom of the swing.
- So at $y = 0$ $\Rightarrow$ $\frac{1}{2}mv^2 = mgh_1$ $\Rightarrow$ $v^2 = 2gh_1$

$$\boxed{v = \sqrt{2gh_1}}$$



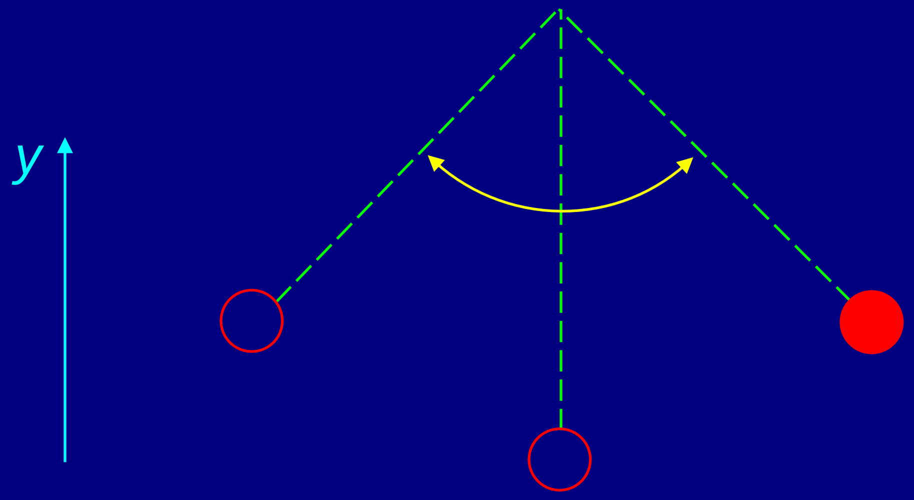
Example: The simple pendulum

- Since $E = mgh_1 = {}^1/_2mv^2 + mgy$ it is clear that the maximum height on the other side will be at $y = h_1 = h_2$ and $v = 0$.
- The ball returns to its original height.

Example: The simple pendulum

- The ball will oscillate back and forth. The limits on its height and speed are a consequence of the sharing of energy between K and U.

  $E = {}^1/_2 mv^2 + mgy = K + U = constant$.



Generalized Work/Energy Theorem:

$$W_{NC} = \Delta K + \Delta U = \Delta E_{mechanical}$$

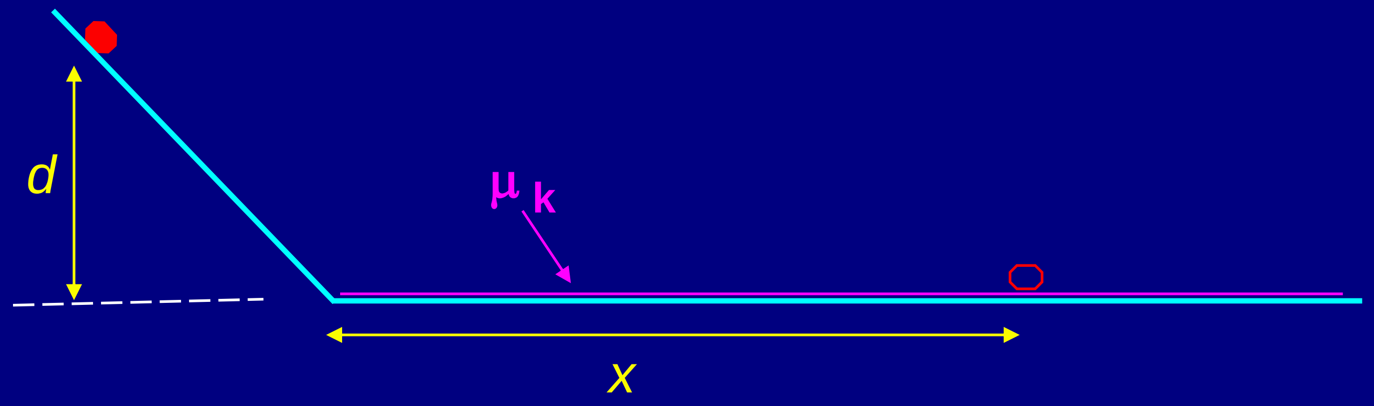
- The change in kinetic+potential energy of a system is equal to the work done on it by non-conservative forces.
  $E_{mechanical}$ =K+U of system not conserved!

⇐If all the forces are conservative, we know that K+U energy is conserved: $\Delta K + \Delta U = \Delta E_{mechanical} = 0$ which says that $W_{NC} = 0$.

⇐If some non-conservative force (like friction) does work, K+U energy will not be conserved and $W_{NC} = \Delta E$.

Problem: Block Sliding with Friction

- A block slides down a frictionless ramp. Suppose the horizontal (bottom) portion of the track is rough, such that the coefficient of kinetic friction between the block and the track is μ_k.
  - How far, x, does the block go along the bottom portion of the track before stopping?

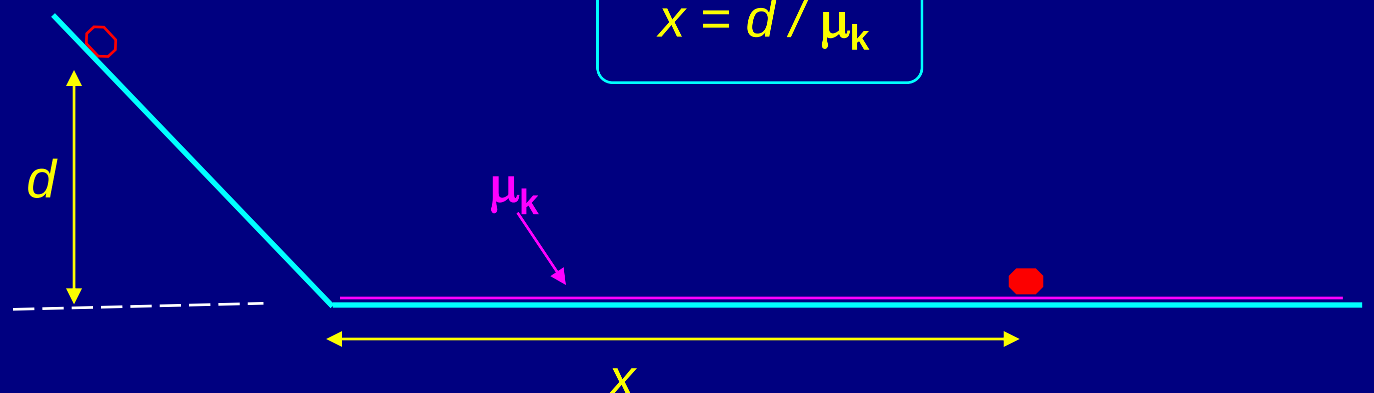


Problem: Block Sliding with Friction...

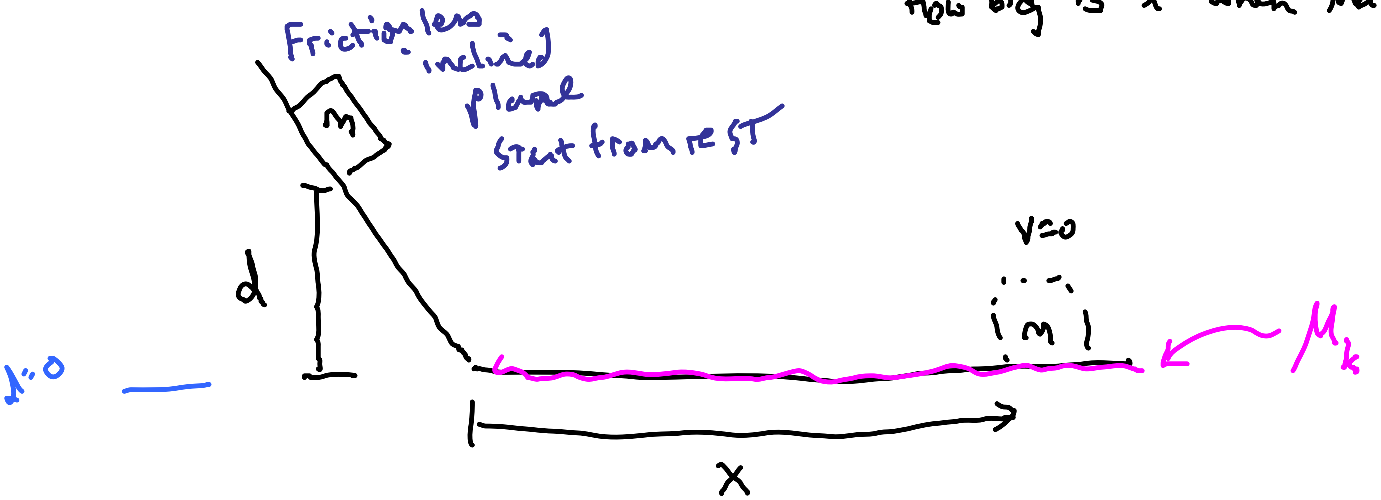
- Using $W_{NC} = \Delta K + \Delta U$
- As before, $\Delta U = -mgd$
- W_{NC} = work done by friction $= -\mu_k mgx$.
- $\Delta K = 0$ since the block starts out and ends up at rest.
- $W_{NC} = \Delta U \quad \Rightarrow \quad -\mu_k mgx = -mgd$

$$\Downarrow$$

$$\boxed{x = d / \mu_k}$$



How big is x when mass comes to rest



$F_{fr} = \mu_k N = \mu_k mg$

Energy lost to friction $\mu_k m g x$

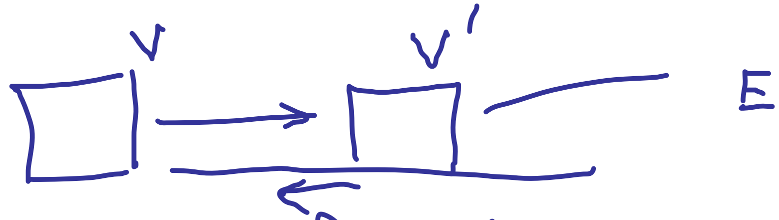
v=0

$E_{start} = \underbrace{PE}_{mgd} + \underbrace{\cancel{KE}}_{0}$

$E_{end} = PE + KE + W_{NC}$

$mgd = 0 + 0 + \mu_k m g x$

$x = d/\mu_k$



E_{bottom}

$F_{fr} = N\mu_k = mg\mu_k$

$$E \rightarrow \Delta PE + \Delta KE + W_{NC}$$

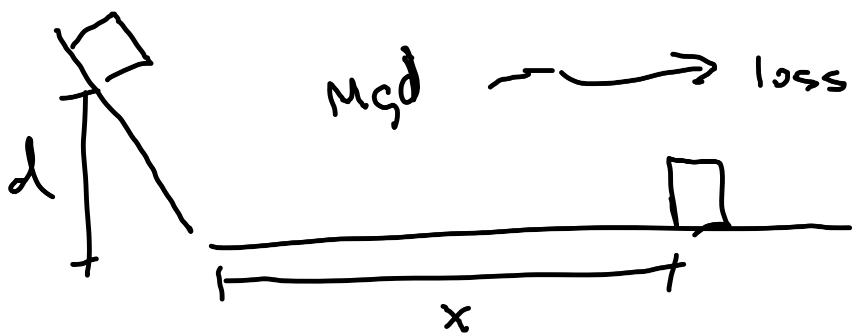
$\Delta PE \rightarrow 0$

ΔKE: $\frac{1}{2}mv^2 \rightarrow \frac{1}{2}mv'^2$

W_{NC}: Energy lost to friction

Work done on object by friction

$mg\mu_k x$



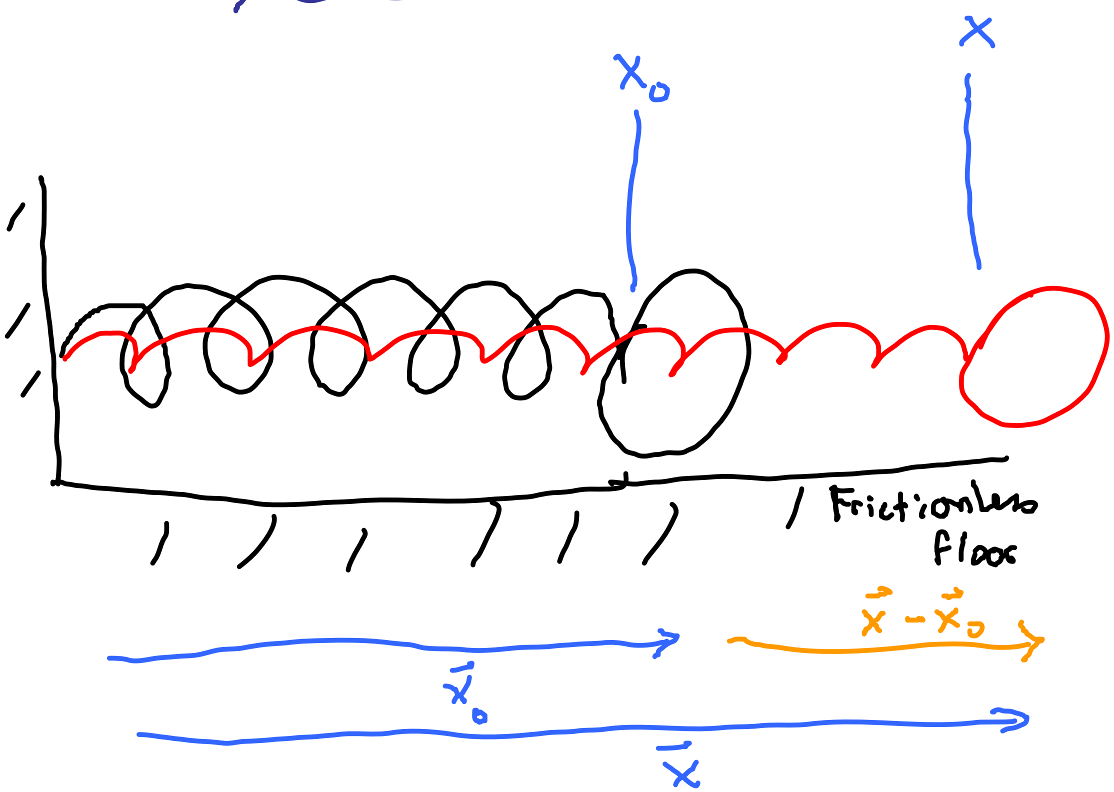
$mgd \longrightarrow$ loss due to fr

$mg\mu_k x$

$$mgd = \mu_k mg x$$

$$x = d/\mu_k$$

Springs



$F_{sp} \propto x - x_0$

restoring force

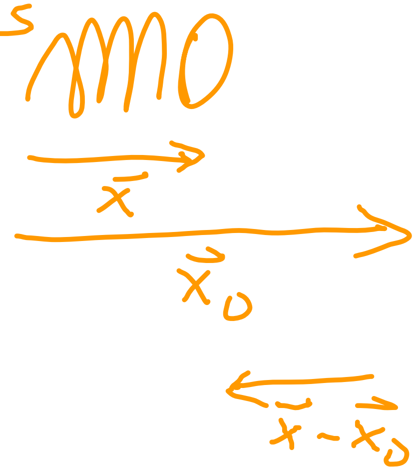
$F_{sp} = k(x - x_0)$

spring constant

$$\vec{F}_{sp} = -k(\vec{x} - \vec{x}_0)$$

Hooke's Law

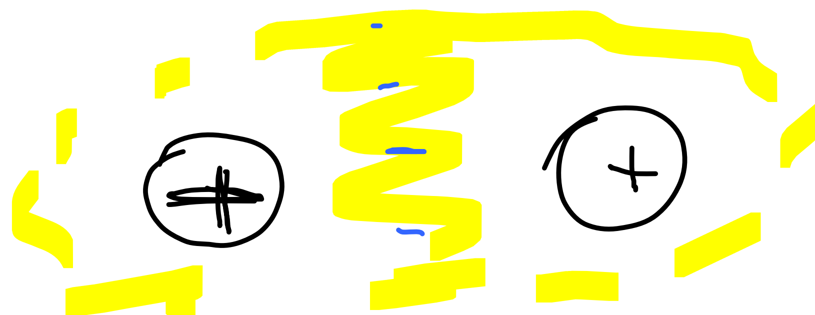
if compress

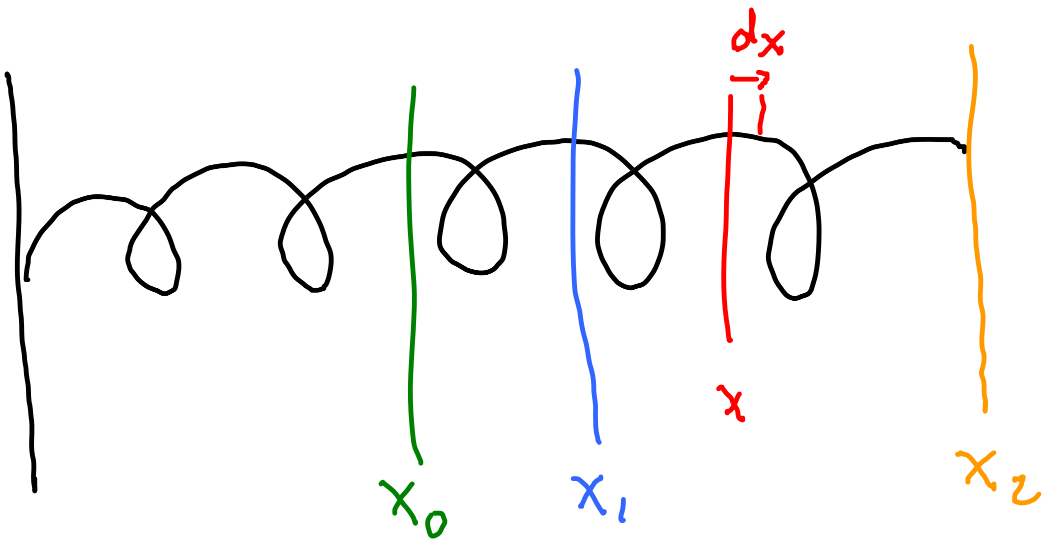


$$\vec{F} = -k\vec{x}$$

$$F = -kx$$

C —— H





$$w = (x_2 - x_1)\underbrace{F_{spring}}_{k(x_2 - x_0)}$$

?

$$dw = k(x - x_0)\,dx$$

$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x}$$