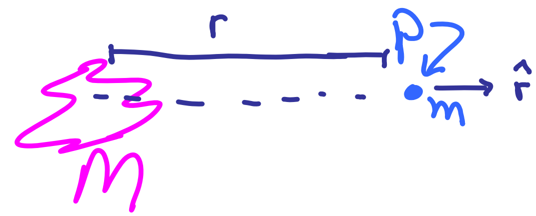
Physics 113 – October 24, 2013

Gravitation

Last Time —

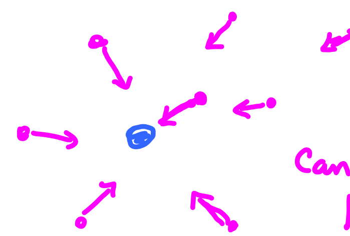


$$\vec{F}_{\text{of M on m}} = -\frac{GMm}{r^2}\,\hat{r}$$

gravitational field

$$\vec{g}(p) = \vec{F}/m = -\frac{GM}{r^2}\,\hat{r}$$

Vector



Can visualize field

$$\Delta PE_{1\to 2} = \left(-\frac{GMm}{r_2}\right) - \left(-\frac{GMm}{r_1}\right) > 0$$

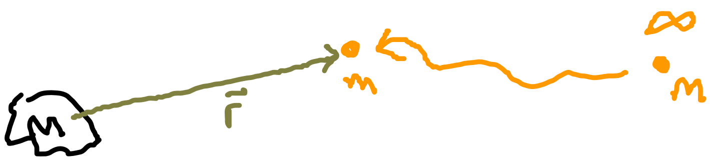
$\Delta PE \sim mgh$ if near surface of Earth

define gravitational potential energy $= -\dfrac{GMm}{r}$

changes in PE are well defined

need "zero" to talk about absolute PE (think height above sea level)

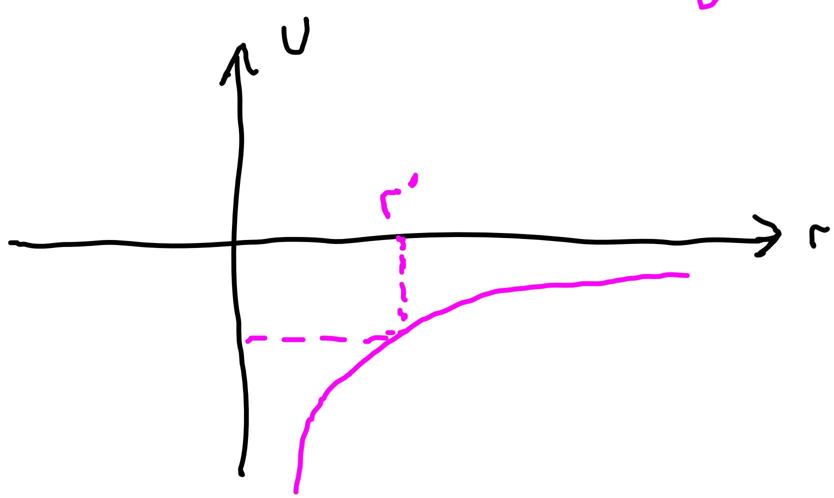
note as $r \to \infty$ $\quad -\dfrac{GMm}{r} \to 0$



How much energy does this take?

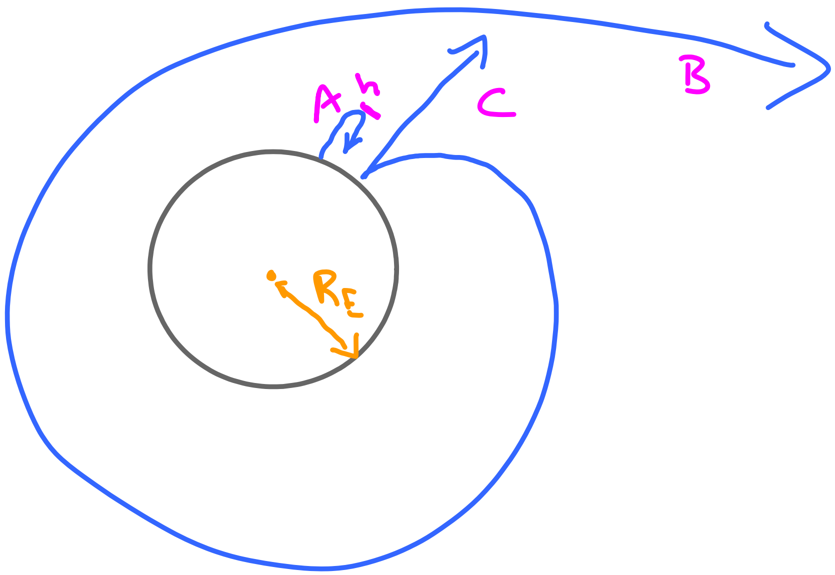
what is PE of m at position r

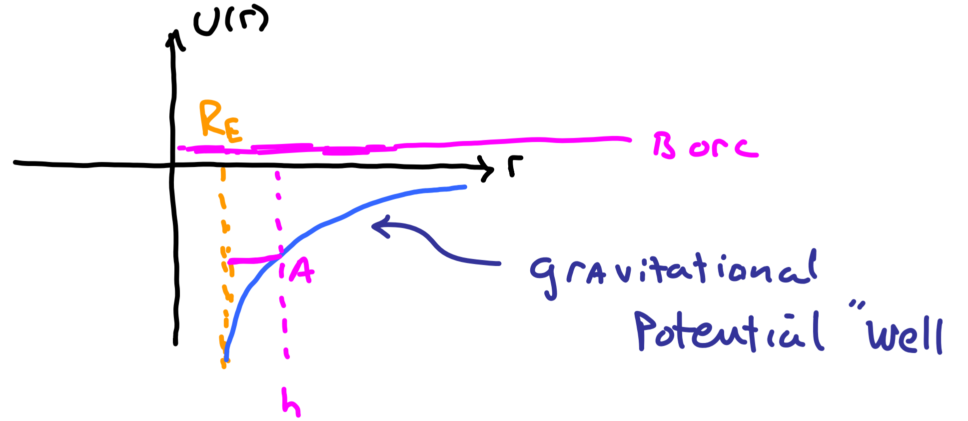
$$U = PE_{at\ \vec{r}} = -\frac{GMm}{r} - \left(-\cancelto{0}{\frac{GMm}{\infty}}\right) = -\frac{GMm}{r}$$



what does it mean that $U(r')$ is negative?

$$U = -\frac{GMm}{r'}$$





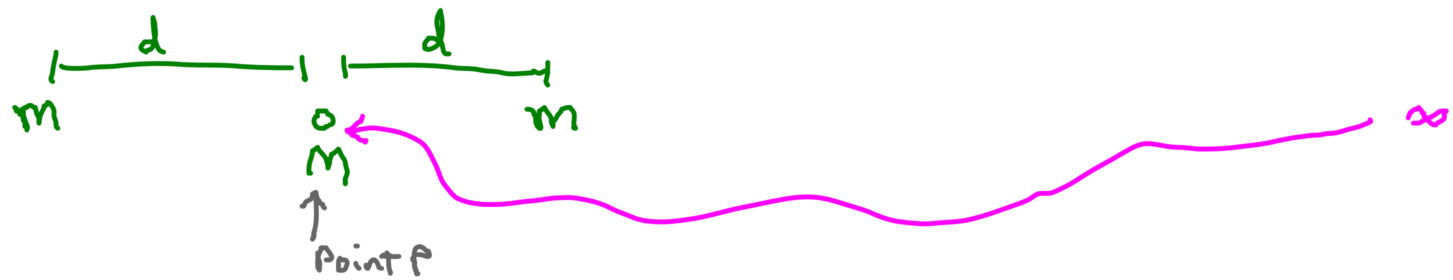
Escape Velocity, V_{es}

KE PE of "bound" state

$$\frac{1}{2}mV_{es}^2 = \frac{GMm}{r}$$

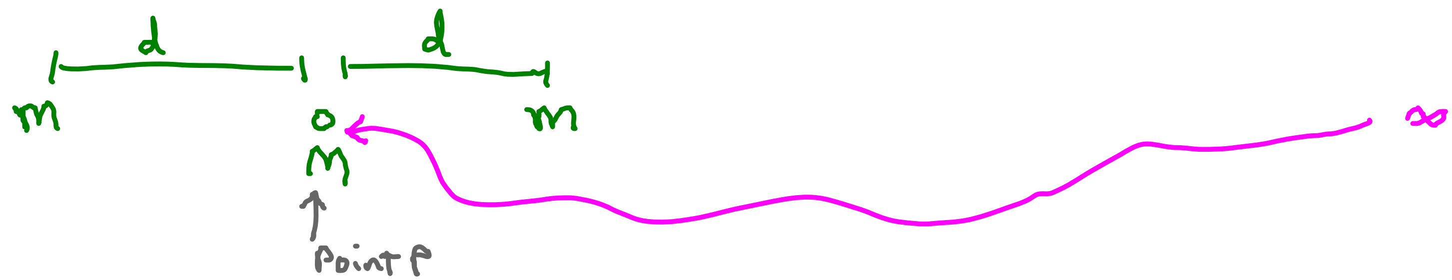
From surface of Earth

$$\frac{1}{2}mV_{es}^2 = \frac{GM_Em}{R_E} \qquad V_{es} = \sqrt{\frac{2GM_E}{R_E}}$$



Mass M moved to point P from ∞ and released
While at point P, what is the gravitational force on M ?

I	II	III	IV	V
$-\frac{2GMm}{d}$	$+\frac{2GMm}{d}$	$-\frac{2GMm}{d^2}$	$+\frac{2GMm}{d^2}$	0

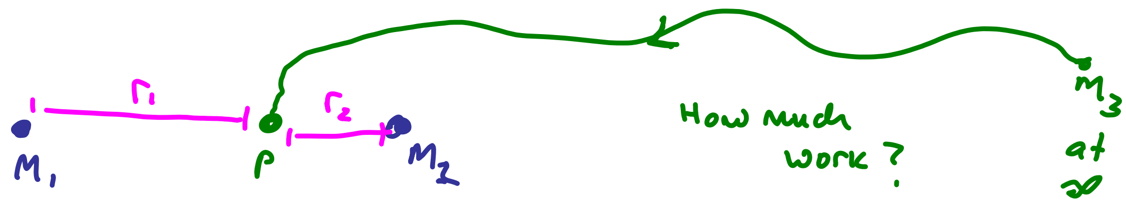


While at point P, what is the Potential Energy Stored in the system?

I	II	III	IV	V
$-\frac{2GMm}{d}$	$+\frac{2GMm}{d}$	$-\frac{2GMm}{d^2}$	$+\frac{2GMm}{d^2}$	0

$$-\frac{2GMm}{d} + -\frac{Gmm}{2d}$$

Why potential?



Energy to move M_3 to point P = $\sum U_i = -\frac{GM_1M_3}{r_1} - \frac{GM_2M_2}{r_2}$

Easier to calculate than $\vec{F}_g$

$$F_s = -\frac{dU_s}{ds}$$

$$\Sigma \vec{F} = m\vec{a}$$

$\vec{p} \equiv$ momentum vector!

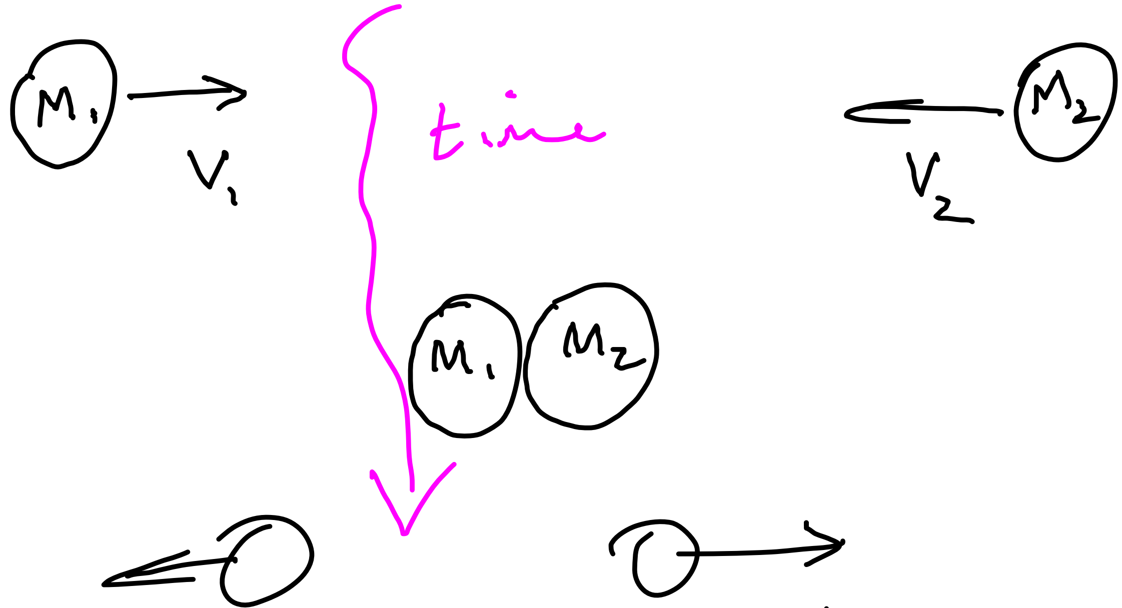
$$\Sigma \vec{F} = \frac{d(m\vec{v})}{dt} = \underbrace{m\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}}_{m\vec{a}} + \underbrace{\frac{dm}{dt}}_{0 \text{ usually}}\vec{v}$$

$$\vec{p} \rightsquigarrow \begin{array}{l} mv_x = p_x \\ mv_y = p_y \\ mv_z = p_z \end{array}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \text{Newton's Second Law}$$

$$d\vec{p} = \vec{F}\,dt$$

$$\vec{p}_{end} - \vec{p}_{start} = \int d\vec{p} = \boxed{\int_{t_1}^{t_2} \vec{F}\,dt \equiv \text{Impulse}}$$



$$\Delta\vec{P}_1 = -\Delta\vec{P}_2$$

$$\Delta\vec{P}_1 = \int_{t_1}^{t_2} \vec{F}_{2\,on\,1}\,dt \qquad \Delta\vec{P}_2 = \int_{t_1}^{t_2} \vec{F}_{1\,on\,2}\,dt \qquad \Delta\vec{P}_1 + \Delta\vec{P}_2 = 0$$

$$\vec{F}_{2\,on\,1} = -\vec{F}_{1\,on\,2}$$

$\vec{\Delta p}_1 + \vec{\Delta p}_2 = 0$

$$\boxed{\sum \vec{\Delta p}_i = 0}$$

Momentum conservation

Total change in momentum (vector) summed over all bodies is zero

If no external forces on system,
the total momentum of system
is conserved.

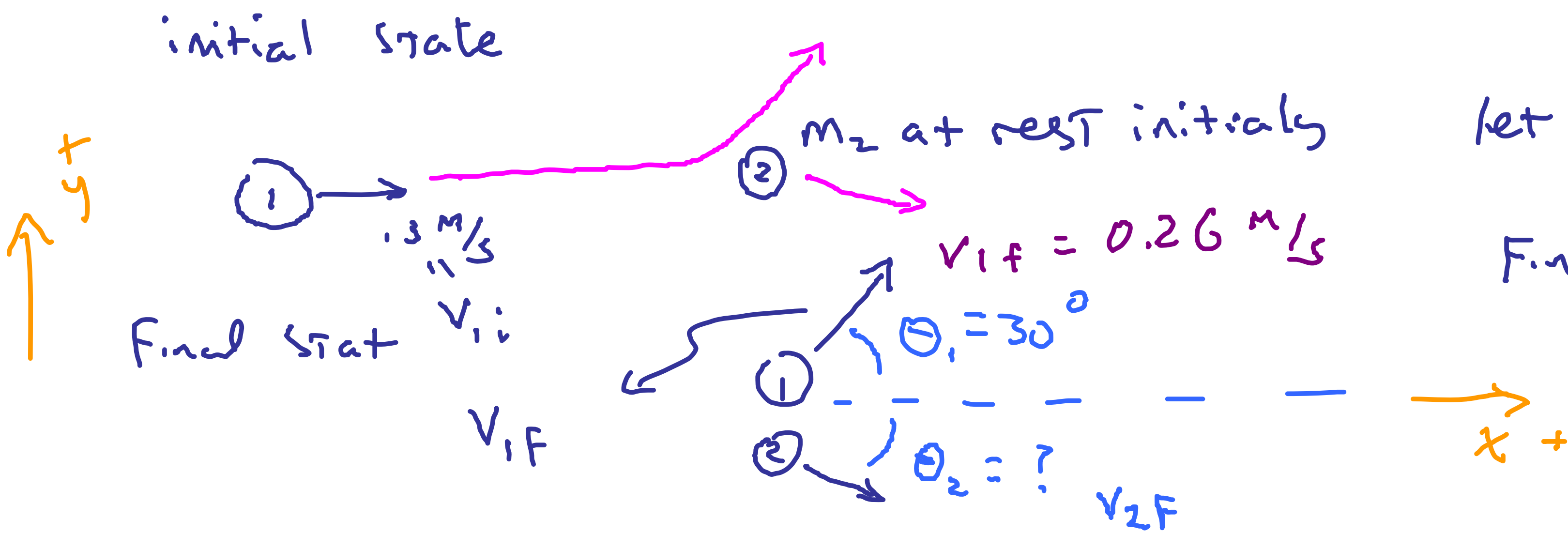
$$\sum \vec{P}_i = \sum \vec{P}_f$$

Elastic Collision $\equiv$ $\longrightarrow$ $\vec{p}$ conserved

KE conserved

Inelastic collision $\equiv$ $\longrightarrow$ $\vec{p}$ conserved

KE NOT necessarily conserved



let $m_1 = m_2 = 0.1$ kg

Find v_{2f} and θ_2

use P cons.

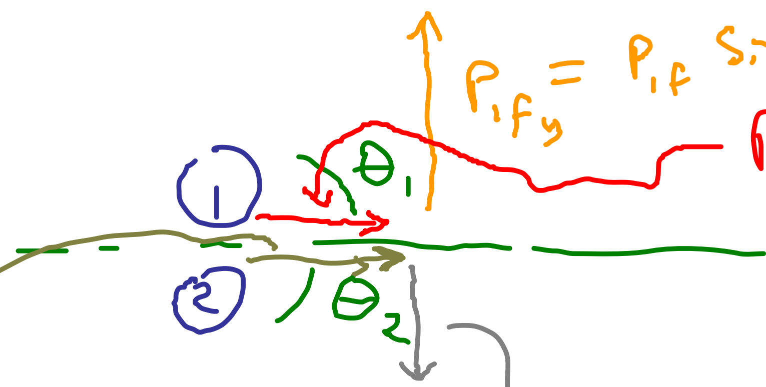
$$\sum \vec{P}_i = \sum \vec{P}_f \Rightarrow \begin{aligned} \sum P_{x_i} &= \sum P_{x_f} \\ \sum P_{y_i} &= \sum P_{y_f} \end{aligned}$$

$P_{1f_y} = P_{1f}\sin\theta_1$

$P_{1f_x} = P_{1f}\cos\theta_1$

$P_{2f_x} = P_{2f}\cos\theta_2$

$P_{2f_y} = P_{2f}\sin\theta_2$



$P_{1f_y} = P_{1f} \sin\theta_1$

$P_{1f_x} = P_{1f} \cos\theta_1$

$P_{2f_x} = P_{2f} \cos\theta_2$

$P_{2f_y} = P_{2f} \sin\theta_2$

x eqn

$$m_1 V_{1i} = \overbrace{m_1 V_{1f}}^{P_{1f}} \cos\theta_1 + \underbrace{m_2 V_{2f}}_{P_{2f}} \cos\theta_2$$

y eqn

$$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2$$

2 eqs

2 unknowns

$$|v_{2f}| = 0.15 \, m/s$$

$\theta_2 = 6^\circ$ down from $+x$ axis

$$\sin^2\theta + \cos^2\theta = 1$$