Physics 113 - October 29, 2013

- Final exam - Thursday, Dec. 19 at 1915 in Hubbell Aud.
- Exam 2 - Nov. 12 - 0800 - Hubbell Aud.

  Will email soonish w/ material coverage details

Last Time

Newton's second law upgrade ...

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d(\vec{p})}{dt} = \underbrace{m\frac{d\vec{v}}{dt}}_{m\vec{a}} + \underbrace{\frac{dm}{dt}}_{\text{not for P113}}\vec{v}$$

$$\vec{p} \equiv m\vec{v} \equiv \text{momentum}$$

For an isolated system, momentum is conserved

$$\sum \vec{P}_{initial} = \sum \vec{P}_{final}$$

$\vec{P}$ is a Vector

$$\sum P_{ix} = \sum P_{fx}$$

$$\sum P_{iy} = \sum P_{fy}$$

$$\sum P_{iz} = \sum P_{fz}$$

$$\frac{d\vec{P}}{dt} = \vec{F}$$

$$d\vec{P} = \vec{F}dt$$

$$\vec{P}_f - \vec{P}_i = \Delta\vec{P} = \int_i^f \vec{F}dt \equiv \text{impulse}$$

In a Collision / interaction / explosion

↳ Momentum Conservation is good if
if system is isolated

If KE conserved ⟷ "Elastic" collision

(Think billiards balls)

If KE NOT conserved ⟷ "inelastic" Collision

(Friction, explosions, etc.)

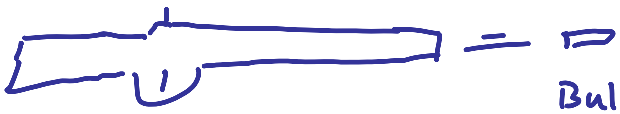
If KE not conserved

Then $\sum KE_i \neq \sum KE_f$

Ballistic Pendulum Example Problem

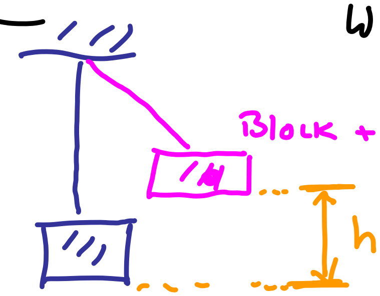
given V_b, m_b, M_w

What is h?



Bullet

m_b



Block + embedded bullet

Block of Wood

M_w

To get V_{w+b}

cannot use E_{cons} → complicated inelastic

if you know V_{w+b} after impact

use E_{cons} to get h

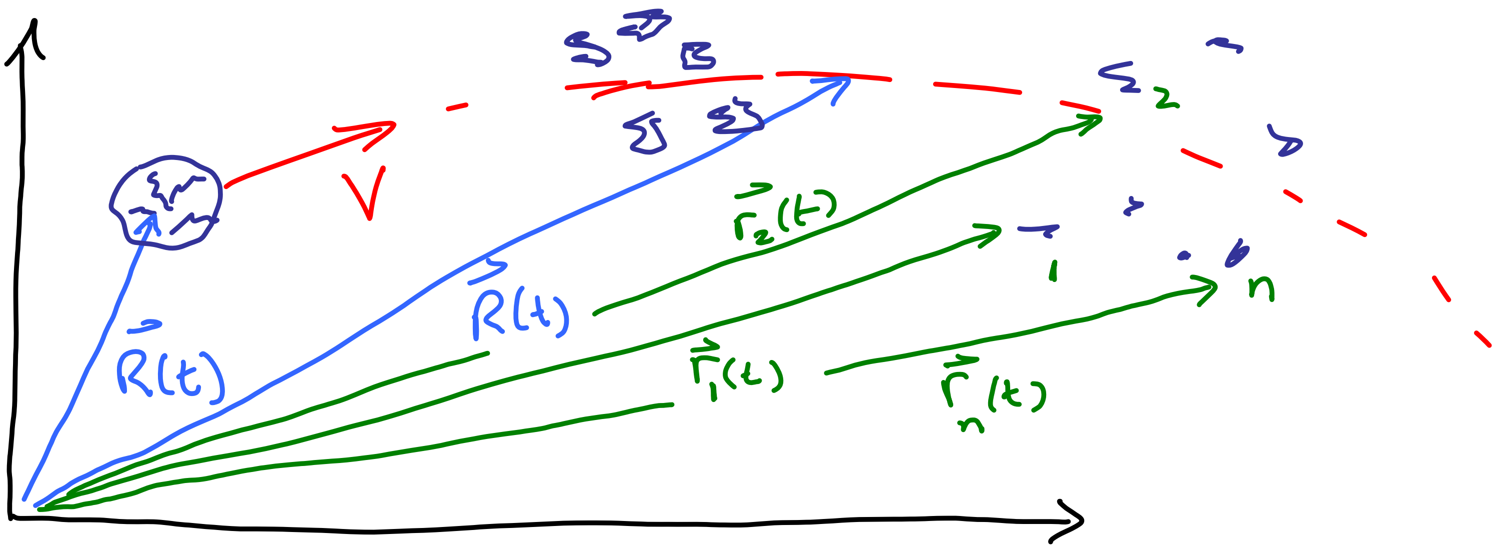
$$\frac{1}{2} M_{w+b} V_{w+b}^2 = mgh$$

use P_{cons}

$$m_b V_b = (m_b + M_w) V_{b+w}$$

$$\frac{1}{2} m_b V_b^2 \neq \frac{1}{2}(M_b + m_w) V_{b+w}^2$$

Center of mass coordinates



momentum is conserved

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n$$

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \cdots + m_n\frac{d\vec{r}_n}{dt}$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n$$

$\vec{R} \equiv (X, Y, Z)$

$\vec{r}_1 \equiv (x_1, y_1, z_1)$

$\vdots$

$$MX = m_1x_1 + m_2x_2 + \cdots + m_nx_n$$

$$MY = m_1y_1 + m_2y_2 + \cdots + m_ny_n$$

$$MZ = m_1z_1 + \cdots + m_nz_n$$

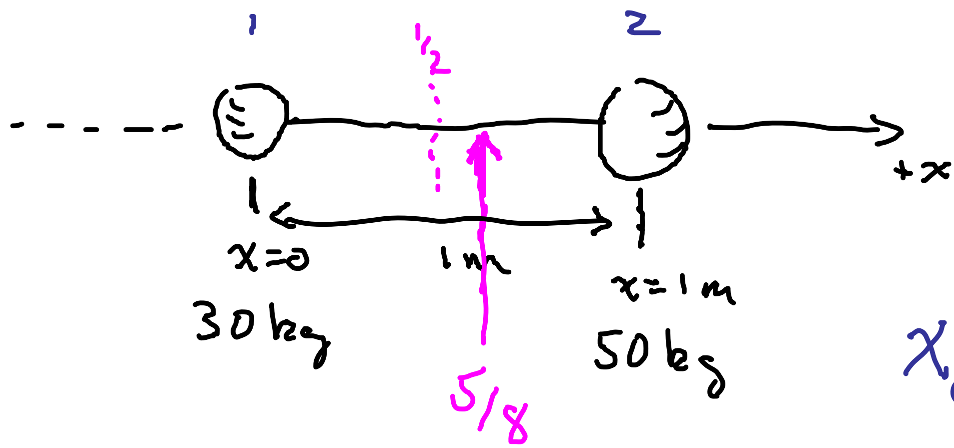
center of mass coordinates

$$X = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{M} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$Y = \frac{m_1 y_1 + \cdots + m_n y_n}{M} = \frac{\sum_i m_i y_i}{\sum m_i}$$

$$Z = \frac{\sum m_i z_i}{\sum m_i}$$

Mass weighted Average x position

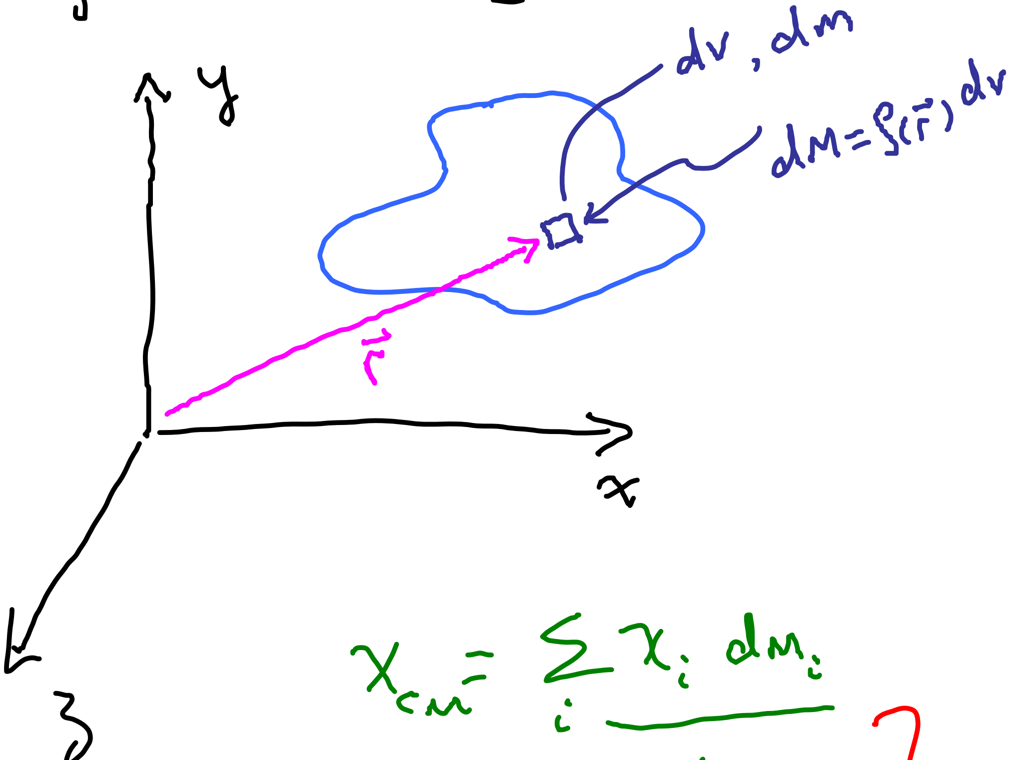


where is x_{cm}

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum m_i} = \frac{(30)(0) + (50)(1)}{30+50}$$

$$x_{cm} = 5/8 \text{ m}$$

go to continuous limit —



where is the center of mass

object has a mass density that is function of $\vec r$

3 dimensional mass density $\equiv \rho(\vec r)$

σ 2d

λ 1d

$$x_{cm} = \frac{\sum_i x_i\, dm_i}{\sum_i dm_i}$$

continuous limit $dv \to 0$

$$\frac{\int_{vol} x\, dm}{\int_{vol} dm}$$

$$x_{cm} = \frac{\int_{vol} x\,\rho(\vec{r})\,dv}{\int_{vol} \rho\,dv}$$

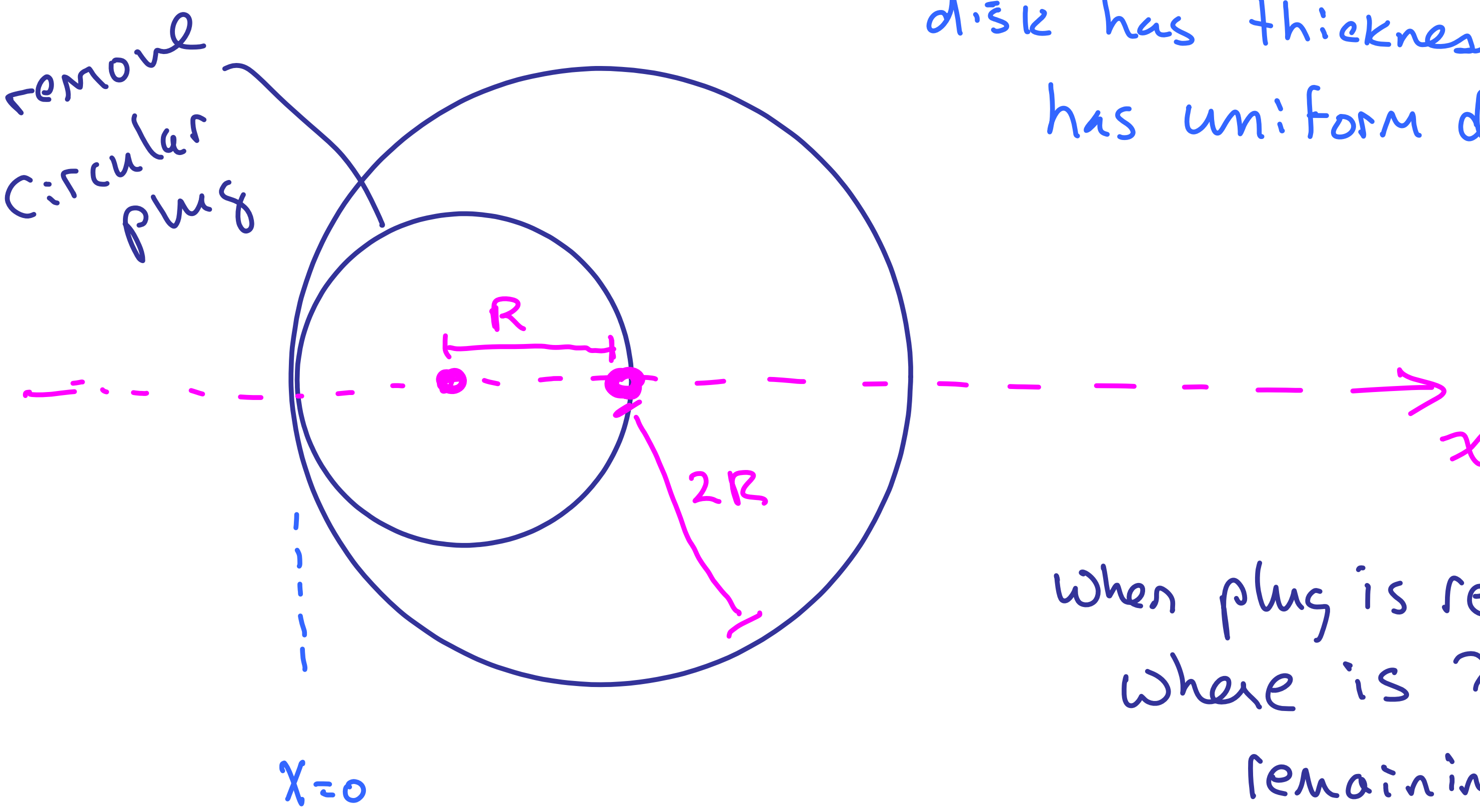
$$y_{cm} = \frac{\int_{vol} y\,\rho(\vec{r})\,dv}{\int_{vol} \rho\,dv}$$

same for z_{cm}

$$\boxed{\vec{R}_{cm} = \frac{\int \vec{r}\,dm}{\int dm} = \frac{\int_{vol} \vec{r}\,\rho(\vec{r})\,dv}{\int_{vol} dm}}$$

disk has thickness t

has uniform density ρ



when plug is removed, where is x_{cm} of remaining part of Disk?

$$X_{cm} = \frac{M_{plug} X_{plug} + M_{disk-plug} \boxed{X_{disk-plug}}}{M_{plug} + M_{disk-plug}}$$

Total disk

Total Mass

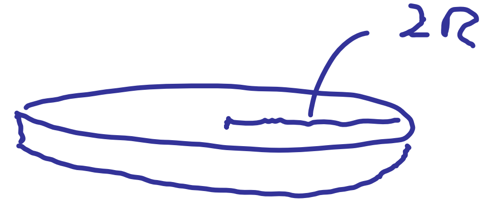
Asked for this

$M - m_{plug}$

$$2R = \frac{M_{plug} R + M_{d-p} X_{d-p}}{\boxed{M}}$$

$M_{plug} = \pi R^2 t \rho$

$M = \pi (2R)^2 t \rho$



$\pi (2R)^2 t$

$$2R = \frac{R^2 \rho \pi t R + 3R^2 \rho \pi t \, x_{d-p}}{4R^2 \rho \pi t}$$

$$2R = \frac{R + 3x_{d-p}}{4} \rightsquigarrow \frac{7}{3} R$$