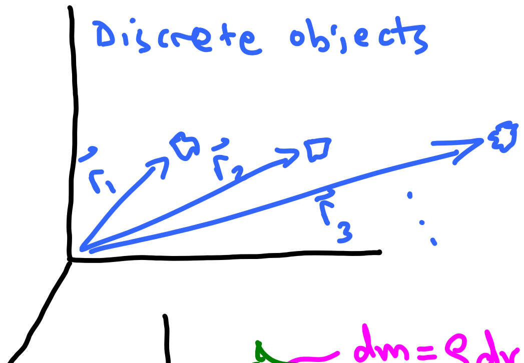
Physics 113 - October 31, 2013

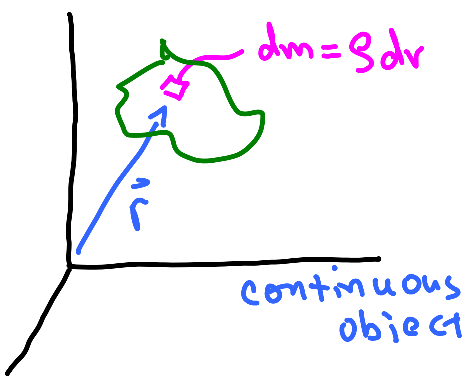
- Exam 2 - Nov 12 0800 Hubble Aud.
- I'm a bit behind ... will post P.S. solns + send out info on exam over the weekend

Happy Halloween!

Last Time

Center-of-mass Coordinates $\rightsquigarrow$ mass Weighted average Position





$$x_{cm} = \frac{\sum_i x_i m_i}{\sum m_i} \quad \text{-- or --} \quad \frac{\int x\, dm}{\int dm}$$

usually $dm = \rho\, dv$

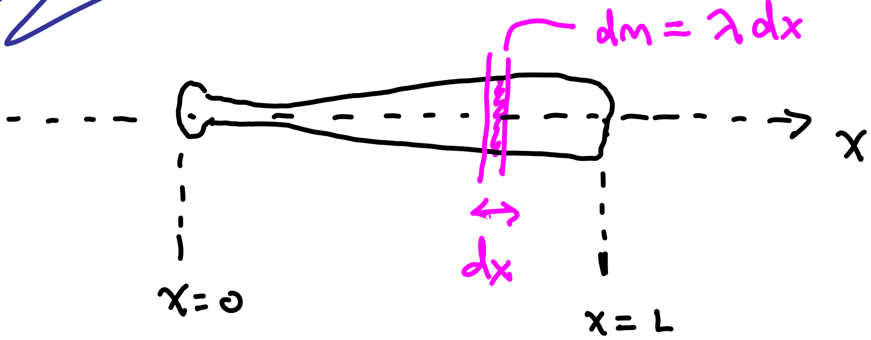
$$y_{cm} = \frac{\sum_i y_i m_i}{\sum m_i} \quad \text{-- or --} \quad \frac{\int y\, dm}{\int dm}$$

$$z_{cm} = \frac{\sum_i z_i m_i}{\sum m_i} \quad \text{-- or --} \quad \frac{\int z\, dm}{\int dm}$$

For discrete case | For continuous case

Example

Find Center of Mass along X



$\lambda \equiv$ linear
$\sigma \equiv$ area
$\rho \equiv$ volume
densities

You are given that bat has "Linear mass density" $\equiv$ mass/length $= \lambda(x) = \lambda_0 \left(1 + \frac{x^2}{L^2}\right)$ where $0 \leq x \leq L$

must have units of kg/m

$$X_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x\,\lambda(x)\,dx}{\int_0^L \lambda(x)\,dx}$$

Denominator

$$M = \int dm = \int_0^L \lambda(x)\, dx = \int_0^L \lambda_0 \left(1 + \frac{x^2}{L^2}\right) dx = \int_0^L \lambda_0\, dx + \int_0^L \lambda_0 \frac{x^2}{L^2}\, dx$$

$$= \lambda_0 x \Big|_0^L + \lambda_0 \frac{x^3}{3L^2} \Big|_0^L = \lambda_0 L + \frac{\lambda_0 L}{3} = \frac{4}{3}\lambda_0 L$$

units are correct

$kg/m \cdot m = kg$

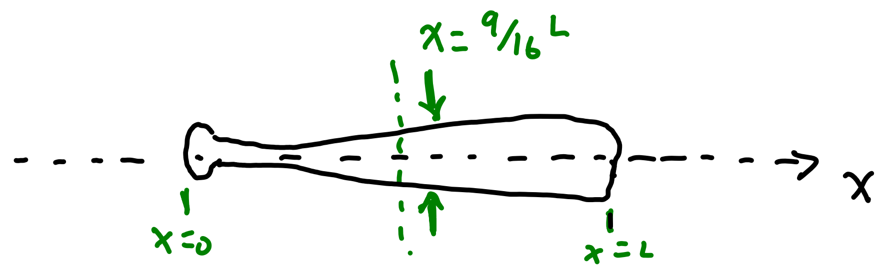
Numerator

$$\int x\, dm = \int_0^L \lambda_0 \left(1 + \frac{x^2}{L^2}\right) x\, dx = \int_0^L \lambda_0 x\, dx + \int_0^L \lambda_0 \frac{x^3}{L^2}\, dx$$

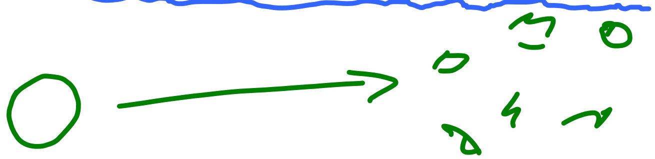
$$= \lambda_0 \frac{x^2}{2} \Big|_0^L + \frac{\lambda_0}{L^2} \frac{x^4}{4} \Big|_0^L = \frac{\lambda_0}{2} L^2 + \frac{\lambda_0 L^2}{4} = \frac{3}{4}\lambda_0 L^2$$

SO

$$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\frac{3}{4}\lambda_0 L^2}{\frac{4}{3}\lambda_0 L} = \boxed{\frac{9}{16} L}$$



a final note on CM coords



$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_n\vec{r}_n$$

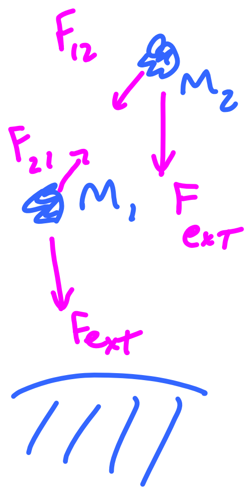
$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + \dots + m_n\frac{d\vec{r}_n}{dt}$$

$$M\vec{V}_{cm} = m_1\vec{v}_1 + \dots + m_n\vec{v}_n$$

Momentum Conservation

$$M\vec{a}_{cm} = m_1\vec{a}_1 + \dots + m_n\vec{a}_n$$

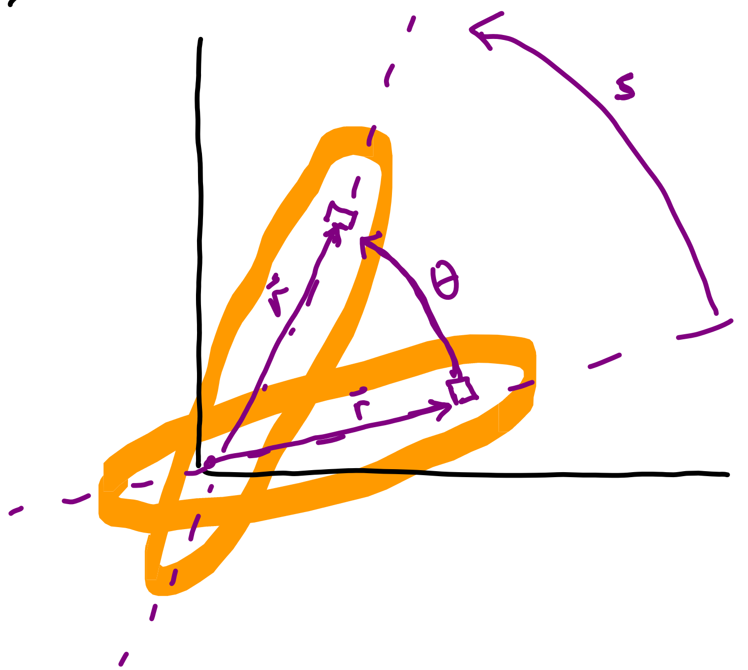
$F_{21} = -F_{12}$

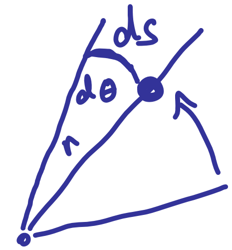


$$\sum\vec{F} = \underbrace{\sum\vec{F}_1}_{F_{1ext} + F_{21}} + \underbrace{\sum\vec{F}_2}_{F_{2ext} + F_{12}} + \dots + \sum\vec{F}_n = \sum\vec{F}_{external} + \underbrace{\cancel{\sum\vec{F}_{internal}}}_{0}$$

$F_{12} = -F_{21}$

Rotational Kinematics





$$S = r\theta$$

Arclength = (radius)(Angle in radians)

$$ds = r\,d\theta$$

Tangential Linear velocity in m/s

$$\boxed{\frac{ds}{dt}} = r\boxed{\frac{d\theta}{dt}}$$

Angular velocity $\equiv \omega$ in rad/s

Tangential Linear Acceleration in m/s^2

$$\boxed{\frac{d^2s}{dt^2}} = r\boxed{\frac{d^2\theta}{dt^2}} = r\boxed{\frac{d\omega}{dt}}$$

Angular Acceleration in rad/s^2 $\equiv \alpha$

If someone were to hit you w/ a bat ... would you rather be hit by the end of the bat or by part of the bat closer to handle?

$$\boxed{\begin{aligned} s &= r\theta \\ v &= r\omega \\ a &= r\alpha \end{aligned}}$$

Recall

$$\frac{dx}{dt} = v$$

$$dx = v\,dt$$

$$\int dx = \int v\,dt$$

$$x - x_0 = \int v\,dt$$

For rotational motion

$$\frac{d\theta}{dt} = \omega$$

$$d\theta = \omega\,dt$$

$$\int d\theta = \int \omega\,dt$$

$$\boxed{\theta - \theta_0 = \int \omega\,dt}$$

True in general

$$\frac{d\omega}{dt} = \alpha$$

$$d\omega = \alpha\, dt$$

True in general

$$\boxed{\omega - \omega_0 = \int \alpha\, dt}$$

Analogue to $v - v_0 = \int a\, dt$ for linear motion

Let us Assume $\alpha = \text{constant}$

Constant angular acceleration

$$\omega - \omega_0 = \alpha \int dt$$

$$\omega - \omega_0 = \alpha (t - t_0)$$

$$\omega = \omega_0 + \alpha t \qquad (t_0 = 0)$$

Seem familiar? $v = v_0 + a t$

$$v = v_0 + at$$

$$r\omega = r\omega_0 + r\alpha t \rightarrow \omega = \omega_0 + \alpha t$$

Constant a eqns

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

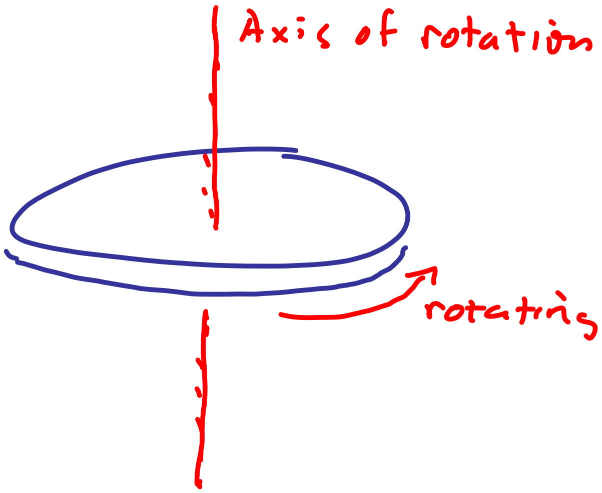
Const α eqns

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \left(\frac{\omega + \omega_0}{2}\right)t$$



Disk initially rotating at 120 rad/s slows down with a constant angular accel. of 4 rad/s^2.

How much time elapses before disk stops rotating?

$$\omega = \omega_0 + \alpha t$$

$$0 = 120 - (4)t$$

$$t = 30 \text{ seconds}$$

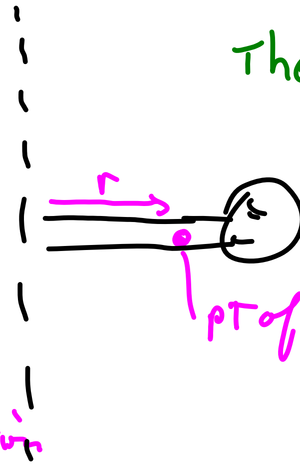
$$F = ma$$

$$\Rightarrow \quad F = m r \alpha$$

$$\Rightarrow \quad \underbrace{rF}_{\text{Angular Force}} = \underbrace{(m r^2)}_{\text{Angular Mass}} \underbrace{\alpha}_{\text{Angular Acceleration}}$$

Angular Mass $\longrightarrow$ moment of inertia

Angular Force ↳ gives Angular Acceleration



pt of Application of force

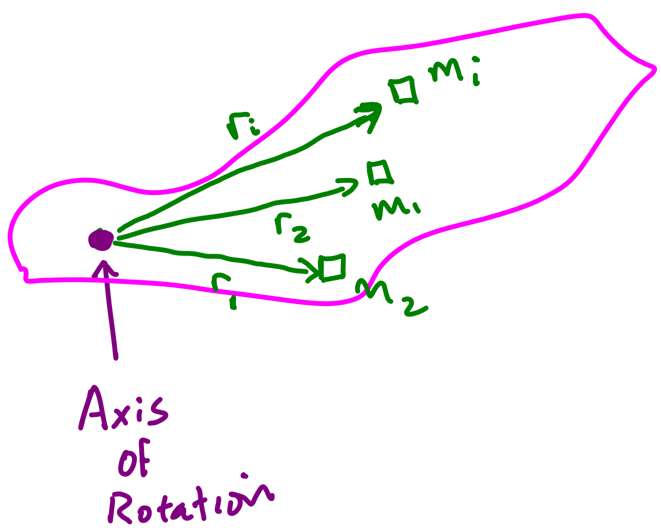
The point of application of the force relative to the Axis of Rotation MATTERS

$$\boxed{\tau = I \alpha}$$

τ: Torque

I: Moment of Inertia

α: Angular Acceleration



$$\tau = I\alpha$$

$$rF = (mr^2)\alpha$$

For i mass elements

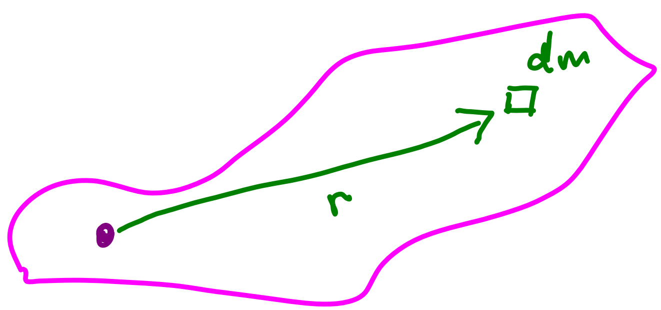
$$\sum (rF)_i = \sum (mr^2)_i \, \alpha$$

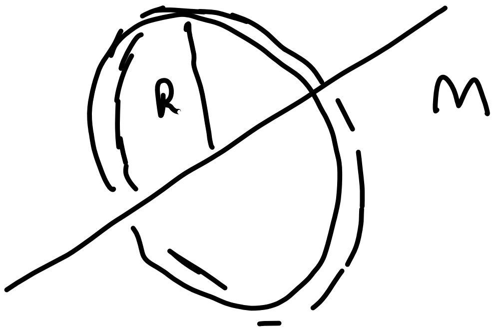
$$dI = r^2 \, dm$$

$$\int_{vol} dI = \int_{vol} r^2 \, dm$$

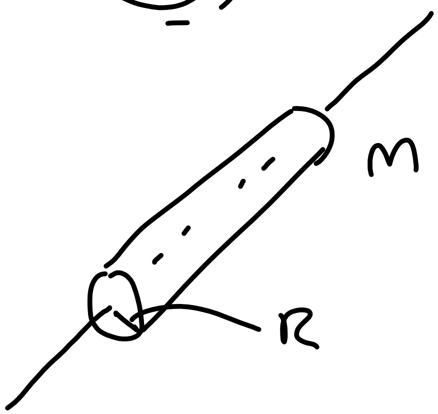
$$I = \int_{vol} r^2 \, dm$$

$\rho \, dv$





$$I = MR^2$$

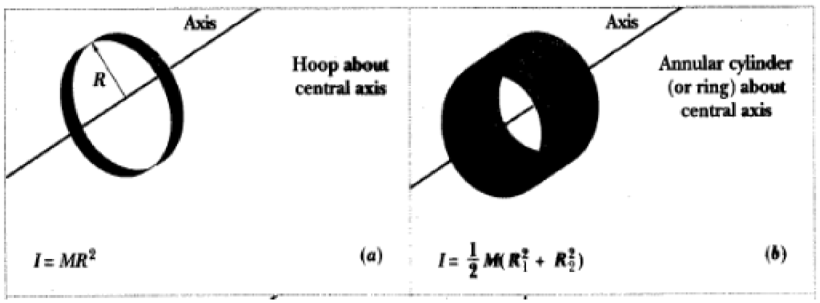


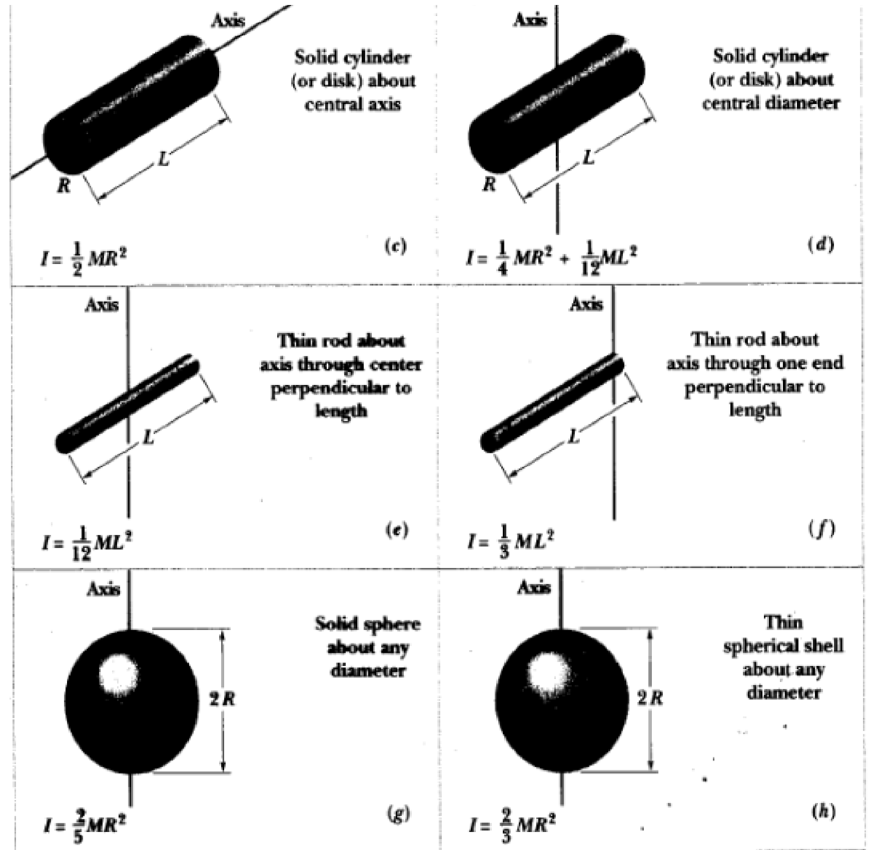
$$I = \frac{1}{2}MR^2$$

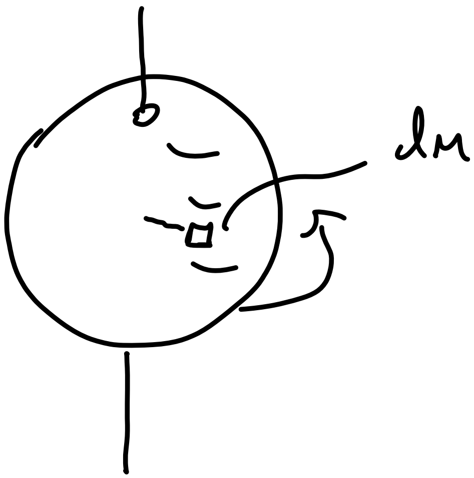
different shapes
different distributions of mass about axis of rotation

Have Different Moments of Inertia

Tables of Moments of Inertia



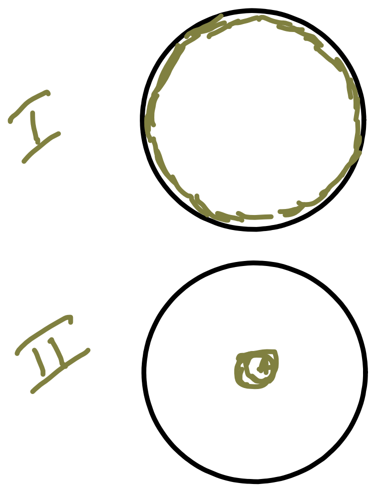




$$KE = \sum_i \frac{1}{2} m_i v_i^2$$

$$\frac{1}{2} \underbrace{m_i r_i^2}_{I} \omega_i^2$$

$$KE = \frac{1}{2} I \omega^2$$



Two Solid wheels
Same mass & radius
Different mass Distribution

Which wins the race to the bottom of the ramp

a) I

b) II

c) Both reach bottom at ~ Same time