

Physics 113 - November 5, 2013

- Exam 2 - Nov 12 at 0800 in Hubbell Aud.
- 1 Side of 8.5 x 11 inch sheet
- Material coverage sent out in email
- Q+A session ... need to schedule
- No prob set due this week
- Probably will have workshops next week ... but have not decided yet

LAST TIME

Rotational
Motion

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

Linear

$$x - x_0 = \int v dt$$

$$v - v_0 = \int a dt$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

angular

$$\theta - \theta_0 = \int \omega dt$$

$$\omega - \omega_0 = \int \alpha dt$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{(\omega + \omega_0)t}{2}$$

general

CONSTANT
 a α

general

$$F = ma$$

$$KE = \frac{1}{2} mv^2$$

$$\tau = I\alpha$$

$$KE = \frac{1}{2} I \omega^2$$

$$F = m a$$

$$\vec{L} = I \alpha$$

Angular Acceleration
 $a = r \alpha$

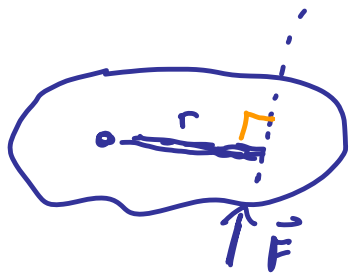
Torque

Moment of Inertia

$$I = \sum_i (m_i r_i^2)$$

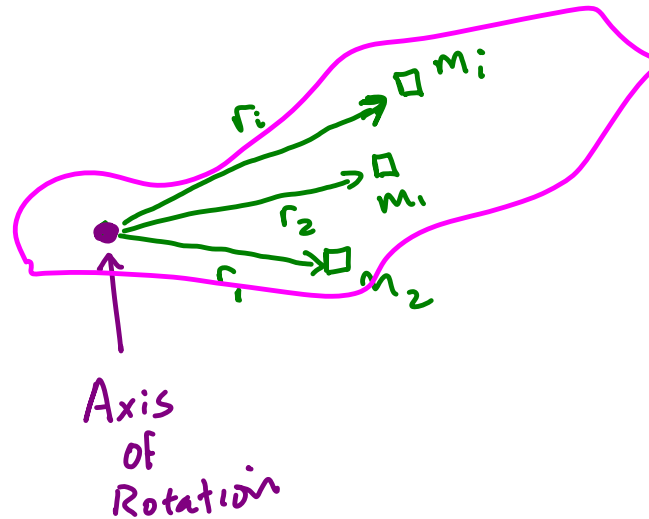
$$I = \int r^2 dm$$

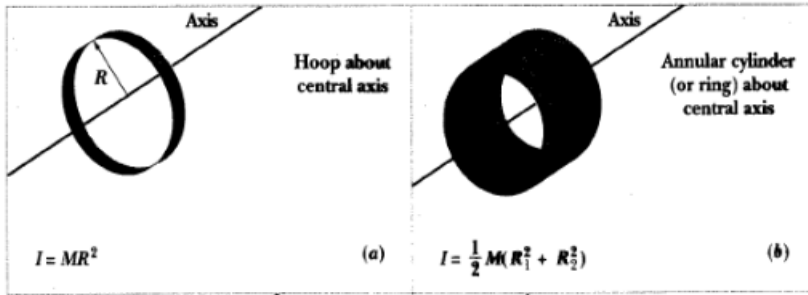
$$I = \int r^2 \rho dv$$



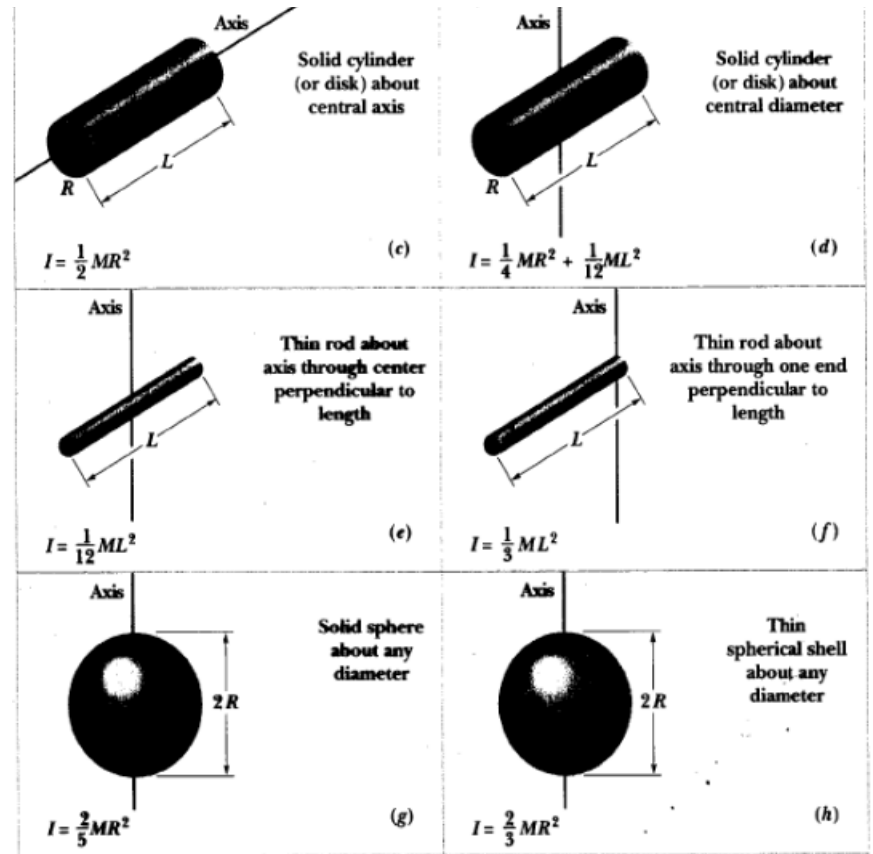
Vector equations

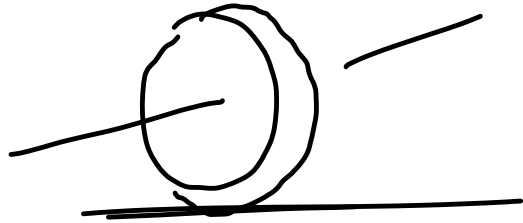
... Vector part coming soon



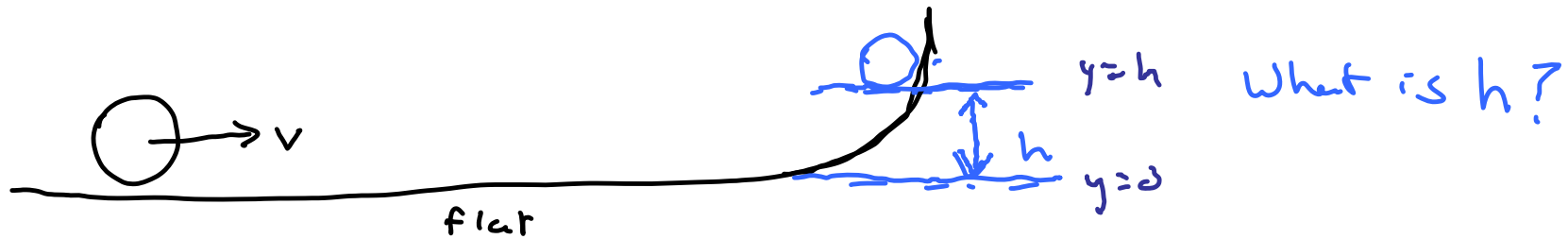


Moments of Inertia





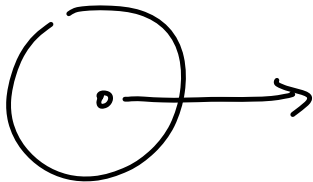
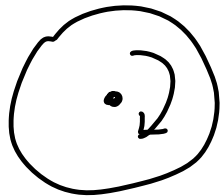
How far up ramp
will wheel go?





cylinder (wheel) rotates w/out slipping

$$\frac{1}{2}mv^2 = mgh$$

$$h = \frac{v^2}{2g}$$



TOTAL KE wheel moving w/out slipping (Method I)

Total KE =  + 

Linear
TRANS
KE

Rotation
About
c.M.

$$\frac{1}{2} M V^2$$

$$S = R\theta$$
$$v = R\omega$$

$$\frac{1}{2} I \omega^2$$

$$\frac{1}{2} I \left(\frac{v}{R}\right)^2$$

$$\text{Total KE} = \frac{3}{4} M V^2$$

$$\frac{1}{4} M V^2$$

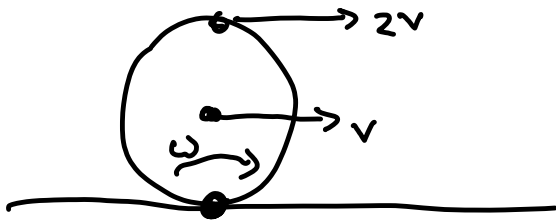
$$= \frac{1}{2} \left(\frac{1}{2} M R^2\right) \frac{v^2}{R^2}$$



$$\frac{3}{4}mv^2 = mgh$$

$$h = \frac{3v^2}{4g}$$

TOTAL KE wheel moving w/out slipping (Method II)



$$v = R\omega$$

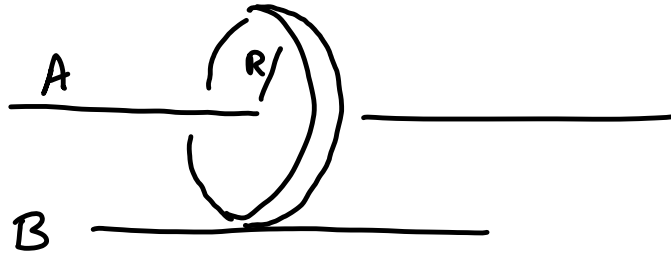
$$\omega = \frac{v}{R}$$

Think of "rotation only"
about bottom point

$$KE = \frac{1}{2} I \omega^2 = \frac{3}{4} Mv^2$$

↓ ↓
? $\left(\frac{v}{R}\right)^2$
 $\frac{3}{2} MR^2$

Parallel
Axis
Theorem



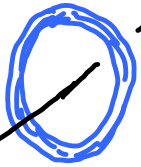
$$I_A = \frac{1}{2} M R^2$$

$r \equiv$ Dist bet Axes

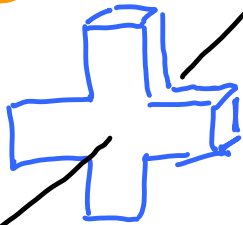
Parallel axis Theorem

$$I_B = I_A + M r^2 = \frac{1}{2} M R^2 + M R^2 = \frac{3}{2} M R^2$$

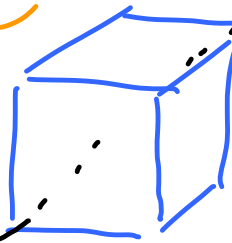
A



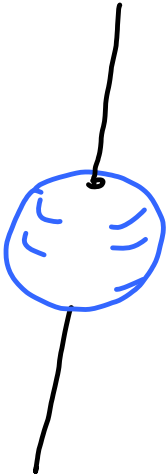
B



C



D



E



All objects have
the same Mass.
Each is solid and
has uniform
Density.

Which object has the largest
Moment of inertia about
the Axes shown?



$$M_1 = 25.0 \text{ kg}$$

$$M_2 = 38.0 \text{ kg}$$

Pulley: uniform cylinder
w/ radius $R_0 = 0.3 \text{ m}$
and Mass $M = 4.8 \text{ kg}$

Init M_1 on ground M_2 at rest 2.5 m above ground
Assume rope massless and does not slip

System released \rightarrow what's speed of M_2 just before it hits the ground?

Two ways to solve this problem \rightarrow Energy conservation
 \rightarrow Newton's Laws

Energy Conservation

$$E_{\text{START}} = E_{\text{end}}$$

$\uparrow v$ $\downarrow v$

$$m_2 g h = m_1 g h + \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2 + \frac{1}{2} I \omega^2$$

$$v_1 = v_2 = v = R_0 \omega$$

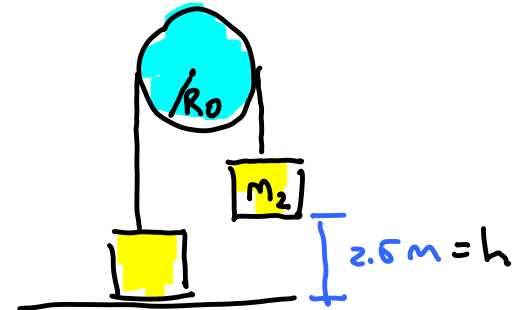
$$I = \frac{1}{2} M R_0^2$$

$$M_2 g h = M_1 g h + \frac{1}{2} (M_1 + M_2) v^2 + \frac{1}{2} \left(\frac{1}{2} M R_0^2 \right) \frac{v^2}{R_0^2}$$

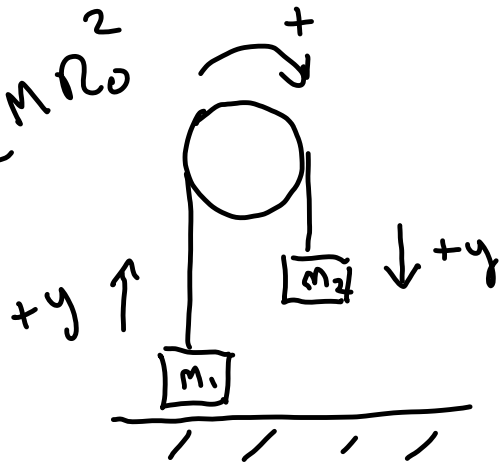
Solve for v

$$v = \pm \sqrt{\frac{(M_1 - M_2) g h}{\frac{M}{4} + \frac{M_2}{2} + \frac{M_1}{2}}} = \pm 1.4 \text{ m/s}$$

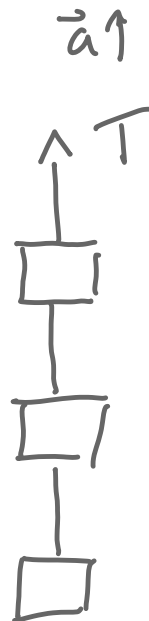
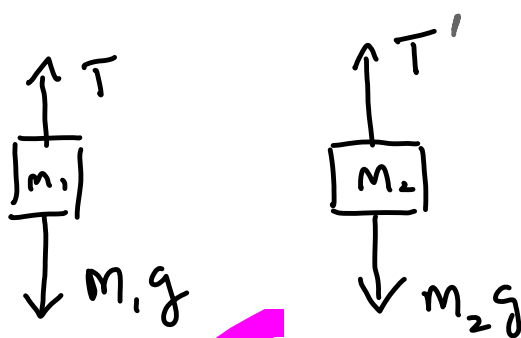
(1.4 m/s)



$$I = \frac{1}{2} M R_0^2$$

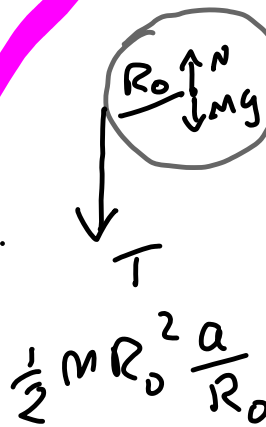


FBD's



$$\sum F_y = m_1 a = T - m_1 g$$

$$\sum F_y = m_2 a = m_2 g - T'$$



$$\sum \tau = I \alpha$$

$$= I \alpha = R_0 T' - R_0 T$$

$$\frac{1}{2} M R_0 a = R_0 T' - R_0 T$$

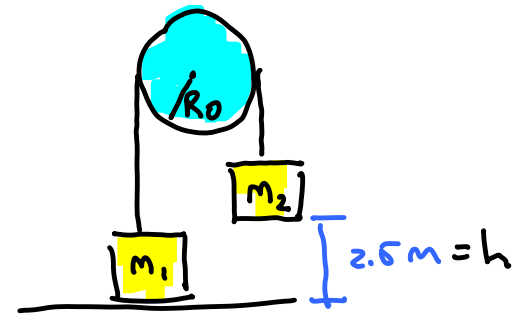
$$a = \frac{(m_2 - m_1)g}{\frac{1}{2}M + m_1 + m_2}$$

$$a = 0.4 \text{ m/s}^2$$

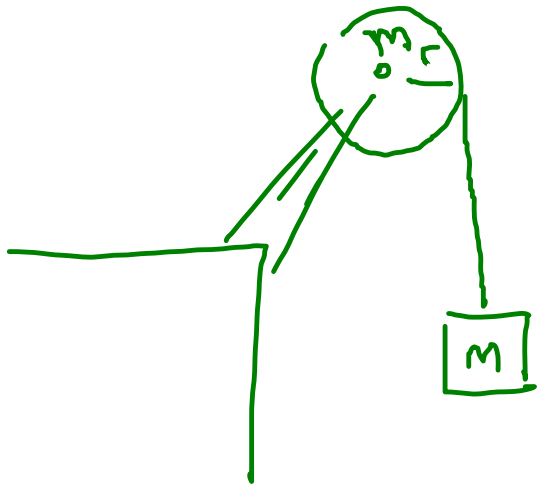
$$V_2^2 = \cancel{V_0^2} + 2ah$$

$$V_2^2 = 2(0.4)(2.5) = 2$$

const a



$$V_2 = 1.4 \text{ m/s}$$



$$I = \frac{1}{2} m r^2$$

Correct expr.
for a

- 1) g
- 2) $\left(\frac{M-m}{M+m} \right) g$
- 3) $\left(\frac{1}{2} m r^2 + M \right) g$
- 4) $\left(\frac{2M}{2M+m} \right) g$