

Physics 113 - November 12, 2013

LAST
Time
Rotational
Dynamics

Linear world

Rotational world

$$s \xrightarrow{s = r\theta}$$

$$v \xrightarrow{v = r\omega}$$

$$a \xrightarrow{a = r\alpha}$$

$$M \xrightarrow{\quad}$$

$$\vec{\tau} \xrightarrow{\vec{L} = \vec{r} \times \vec{F}}$$

$$\frac{1}{2}mv^2 \xrightarrow{\quad}$$

$$\vec{p} = m\vec{v}$$

$$\theta$$

$$\vec{\omega}$$

$$\vec{\alpha}$$

$$I = \int r^2 dm$$

$$\vec{L}$$

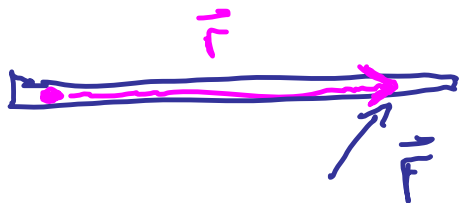
$$\frac{1}{2}I\omega^2$$



$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \quad \text{out of board}$$

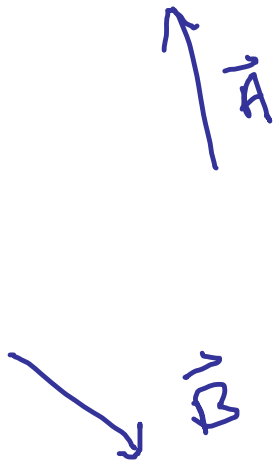


use
RH Rule
to get
direction



$$\vec{L} = \vec{r} \times \vec{F} \quad \leftarrow \text{out of board}$$
$$\vec{L} = I \vec{\alpha} \quad \text{gives } \vec{L} \text{ also } \uparrow$$

makes $\vec{\omega}$ be also out of board
+ rotation counter-clockwise



Direction of $\vec{B} \times \vec{A}$ is what?

- (I) into board
- (II) out of board ✓
- (III) left
- (IV) Right
- (V) up
- (VI) down

A



Direction of $\vec{B} \times \vec{A}$ is what?

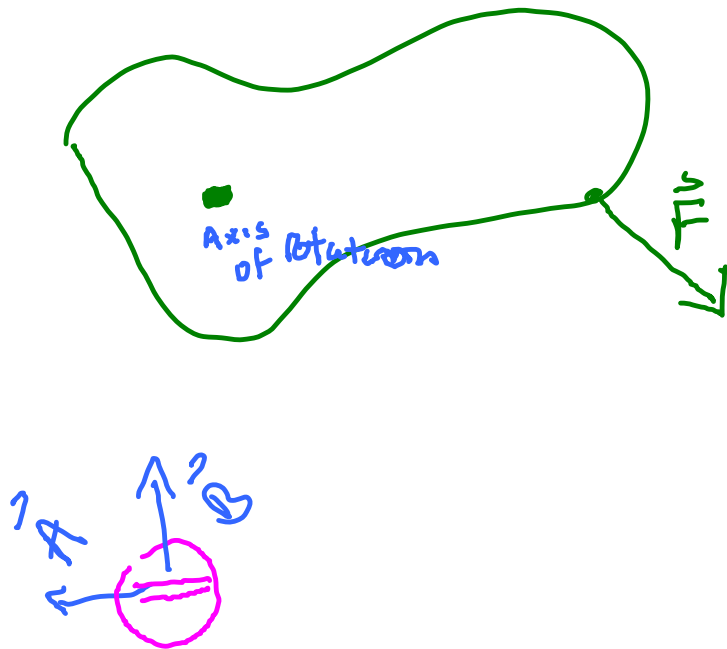
- (I) into board
- (II) out of board
- (III) left ✓
- (IV) Right
- (V) up
- (VI) down

$\times \vec{B}$

$\uparrow \vec{A}$

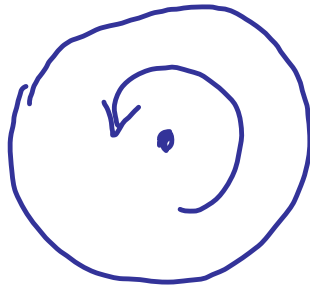
Direction of $\vec{A} \times \vec{B}$ is what?

- (I) into board
- (II) out of board
- (III) left ✓
- (IV) Right
- (V) up
- (VI) down



Direction of Torque is

- Ⓘ into board ✓
- Ⓜ out of board
- ⓓ left
- ⓓ Right
- ⓓ up
- ⓓ down



Slowing down

what is direction of $\vec{\omega}$?

what is direction of \vec{L} ?

- (I) into board ✓ \vec{L}
- (II) out of board ✓ ω
- (III) left
- (IV) Right
- (V) up
- (VI) down

Angular Momentum $\equiv \vec{L} = I \vec{\omega}$ (recall linear momentum is $m\vec{v}$.)

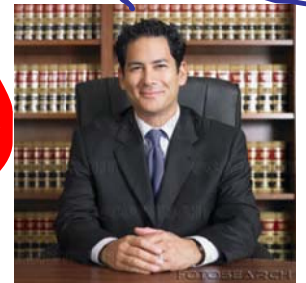
$$\vec{F} = m\vec{a} = \frac{d\vec{P}}{dt} = m \frac{d\vec{v}}{dt}$$

$$\vec{L} = \frac{d\vec{L}}{dt} = \frac{d(I\vec{\omega})}{dt} = I \frac{d\vec{\omega}}{dt}$$

Angular momentum is conserved

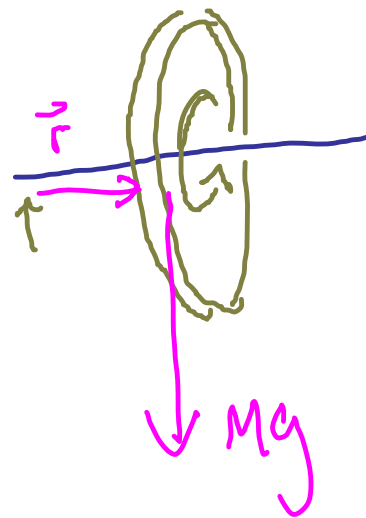
$$\vec{L}_{\text{init}} = \vec{L}_{\text{final}}$$

in an
isolated
system

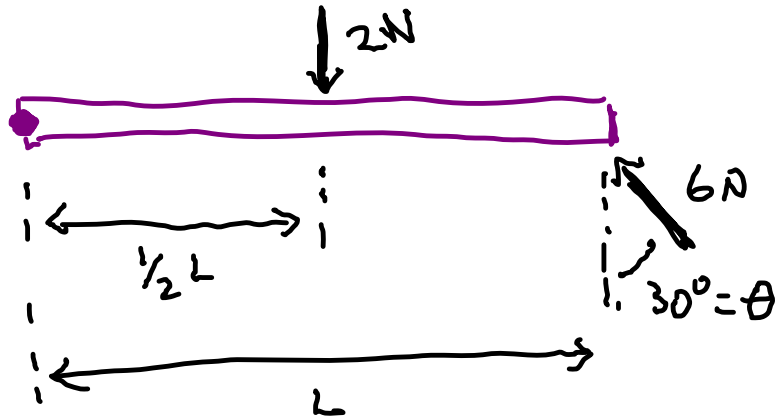


Precession

$$\vec{L} = \frac{d\vec{L}}{dt} = \frac{d(I\vec{\omega})}{dt} = I \frac{d\vec{\omega}}{dt}$$



Precesses
into
Board



Thin rod length L

Free to rotate abt
axis thru end

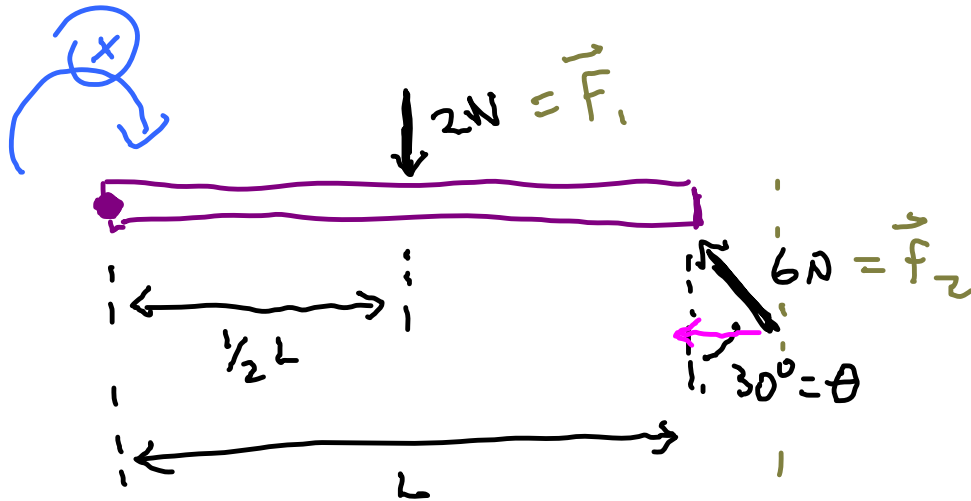
Mass rod is 3 kg

$$L = 2\text{ m}$$

Find $\vec{\alpha}$ for rod
at moment shown

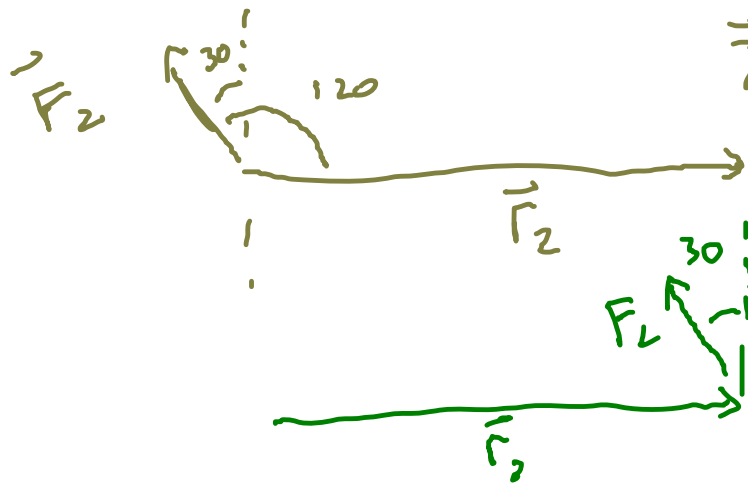
$$\text{give } \frac{I}{\text{rod}} = \frac{1}{3} ML^2$$

ignore all other forces



$$\sum \tau = I \alpha$$

$$I = \frac{1}{3} mL^2 \alpha$$



$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = |\vec{r}_2| |\vec{F}_2| \sin 120^\circ$$

$$\sum \tau = (r_1 |F_1| - (r_2 |F_2| \cos 30$$

$$\frac{1}{3} mL^2 \alpha = \frac{1}{2} L(2) - L(6) \cos 30$$

$$\alpha = -2.5 \text{ rad/s}^2$$