Physics 113
Fall 2013
Solutions to PS 10

11-111 AGAIN, CONSERVE ANGULAR MOMENTUM.

A) $I_i = \frac{1}{2}MR^2$ $\qquad \omega_i = 0.95\ \text{rad/s}$

$$I_f = \underbrace{\tfrac{1}{2}MR^2}_{\text{MERRY-GO-ROUND}} + \underbrace{mR^2}_{\text{PERSON}}$$

$$\omega_f = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + mR^2}\left(0.95\ \text{rad/s}\right)$$

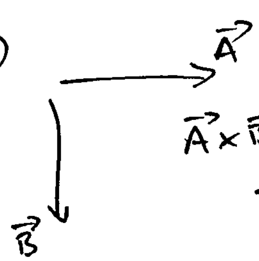
$$\boxed{\omega_f = 0.55\ \text{rad/s}}$$

B) $KE_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{4}MR^2\omega_i^2 = \boxed{420\ J}$

$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega_f^2 = \boxed{240\ J}$

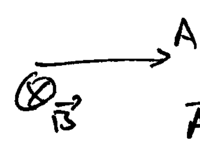
11-22

A)



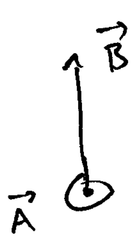
$\vec{A} \times \vec{B} = \otimes$ (~~DOWN~~ INTO PAGE)

(B)



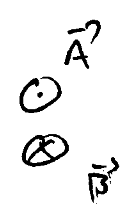
$\vec{A} \times \vec{B} = \uparrow$ (NORTH)

C)



$\vec{A} \times \vec{B} = \leftarrow$ (WEST)

D)



$\vec{A} \times \vec{B} = 0$

11-41

A) $L = L_{PULLEY} + L_A + L_B$

$= I_{PULLEY}\omega_{PULLEY} + M_A V R_o + M_B V R_o$

$(L = mvr)$

~~PER~~ PERPENDICULAR DISTANCE TO VELOCITY ~~PERPENDICULAR~~

$\omega_{PULLEY} = \frac{V}{R_o}$

$L = \frac{I_P V}{R_o} + M_A R_o V + M_B R_o V$

$$\boxed{L = \cancel{R_o} V\left(\frac{I_P}{R_o} + M_A R_o + M_B R_o\right)}$$

B) YOU COULD USE FREE BODY DIAGRAMS AND NEWTONS SECOND LAW ($\sum \vec{F} = m\vec{a}$, $\sum \tau = I\alpha$) TO GET THIS. BUT WE ALSO KNOW THAT $\sum \vec{\tau} = \frac{d\vec{L}}{dt}$ $\cancel{= I\alpha}$ AND THAT GRAVITY IS THE ONLY EXTERNAL FORCE (AND THEREFORE ~~GR~~ THE ONLY EXTERNAL TORQUE).

SO $\sum \tau = M_B g R_o = \frac{d}{dt}\left\{V\left(\frac{I}{R_o} + R_o(M_A + M_B)\right)\right\}$

$= a\left(\frac{I}{R_o} + R_o(M_A + M_B)\right)$

$$\Longrightarrow \boxed{a = \frac{M_B g}{\frac{I}{R_o^2} + M_A + M_B}}$$

11-48 | ANGULAR MOMENTUM WILL BE CONSERVED

$$\left.\begin{aligned} L_i &= m_{BULLET} V_0 \left(\tfrac{1}{4}\ell\right) \\ L_f &= I_{STICK}\,\omega + m_{BULLET} V_f \left(\tfrac{1}{4}\ell\right) \end{aligned}\right\} \quad L_i = L_f$$

$$m_{BULLET} V_0 \left(\tfrac{1}{4}\ell\right) = I_{STICK}\,\omega + m_{BULLET} V_f \left(\tfrac{1}{4}\ell\right)$$

$$\omega = \frac{m_{BULLET}\,\ell}{4I}(V_0 - V_f) = \frac{m_{BULLET}\,\ell}{4\left(\frac{1}{12} M_{STICK}\,\ell^2\right)}(V_0 - V_f$$

$$\boxed{\omega = 3.7\ \text{rad/s}}$$

11-53 |

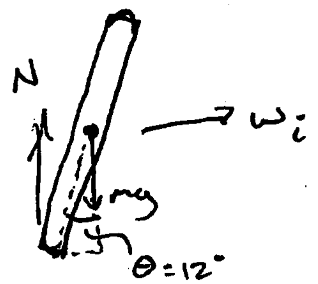
$$\Omega = \frac{Mgr}{I\omega} \qquad \text{(FROM PAGE 299)}$$

$$\hookrightarrow I = \frac{Mgr}{\Omega\omega} = \frac{(0.22\,\text{kg})(9.8\,\text{m/s}^2)(0.035\,\text{m})}{\left(\frac{1\,\text{rev}}{6.5\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right)\left(\frac{.15\,\text{rev}}{1\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)}$$

$$\boxed{I = 8.3\times10^{-7}\ \text{kg}\cdot\text{m}^2}$$

11-71

(a) FRONT VIEW



PIVOT POINT IS CONTACT POINT ON GROUND THUS TORQUE RESULTS FROM mg

TORQUE DIRECTION IS $\otimes$, AND SO IS $\Delta\vec{\omega}$'s.

THE WHEEL WILL TURN TO THE RIGHT

(b) $\vec{\tau} = \vec{r} \times \vec{F} = (0.32\,m)(8\,kg)(10\,m/s^2)\sin(\theta) = 5.3\ N{\cdot}m$

SO $\Delta\vec{L} = \tau\Delta t = (5.3\ N{\cdot}m)(0.2\,s) = \boxed{1.1\ \frac{kg\,m^2}{s} = \Delta L}$

THE ORIGINAL $L = I\omega$ $\quad \omega = \frac{v}{r} = \frac{2\ m/s}{0.32\,m} = 6.25\ rad/s$

$L = (0.83\ kg\,m^2)(6.25\ rad/s) = \boxed{5.2\ kg\,m^2/s = L}$

THEN $\frac{\Delta L}{L} = \frac{1.1}{5.2} = \boxed{0.2 \text{ or } 20\%}$

11-74

$L_{SPIN} = I\omega = \frac{2}{5} m r^2 \omega_{SPIN} = \frac{2}{5} m r^2 \omega$ (r = radius of moon

$L_{ORBIT} = (m R_{ORBIT}^2)\,\omega_{orbit} = m R_{ORBIT}^2\,\omega$

$\frac{L_{SPIN}}{L_{orbit}} = \frac{\frac{2}{5} m r^2 \omega}{m R_{orbit}^2 \omega} = \frac{2}{5}\frac{r^2}{R_{ORBIT}^2} = 8.2 \times 10^{-6}$

$\boxed{L_{ORBIT} >>> L_{SPIN}}$

11-73 THE ANGULAR MOMENTUM GAINED BY THE WATERWHEEL IS LOST BY THE WATER

(A) $\Delta L_{WH} = -\Delta L_{WAT} = L_{WAT_i} - L_{WAT_f} = m v_1 R - m v_2 R$

$$\frac{\Delta L_{WH}}{\Delta t} = \frac{m v_1 R - m v_2 R}{\Delta t} = \frac{mR}{\Delta t}(v_1 - v_2) = \boxed{816\ kg \cdot m^2/s^2}$$

(B) $\tau = \frac{\Delta L}{\Delta t} \rightarrow \boxed{\tau = 816\ m \cdot N}$

(C) $P = \tau\omega = (816\,N\cdot m)\left(\frac{2\pi\,rev}{5.5\,s}\right) = \boxed{930\,W}$

11-81

(A) IF $\Delta L = 0$ THEN $L_i = L_f \rightarrow I_e\omega_i = I_f\omega_f$

$$\left(\tfrac{2}{5}\, 8 M_{SUN} R_{SUN}^2\right)\omega_i = \left(\tfrac{2}{5}\, 2 M_{SUN} R_f^2\right)\omega_f$$

$$\omega_f = \frac{\frac{2}{5}\cdot 8 M_{SUN} R_{SUN}^2}{\frac{2}{5}\cdot 2 M_{SUN} R_f^2}\,\omega_i \Rightarrow \boxed{\omega_f = 17{,}000\ \frac{rev}{s}}$$

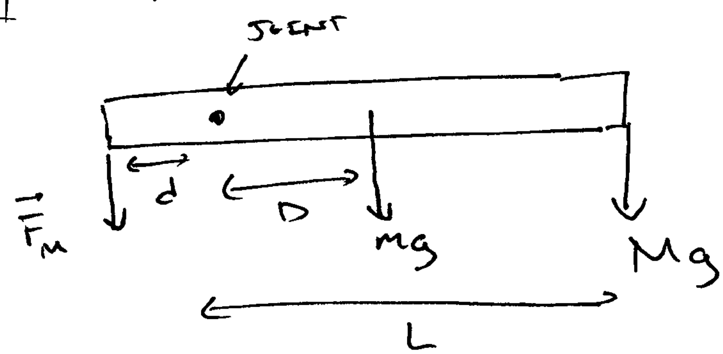
(b) THE $\frac{3}{4}$ OF THE MASS THAT LEAVES ALSO CARRIES AWAY $\frac{3}{4}$ OF THE ANG. MOMENTUM

SINCE $L \propto \omega$ WE KNOW THAT

$$\omega_f = \tfrac{1}{4}(17{,}000) = \boxed{4300\ \frac{rev}{s}}$$

12-2

DRAW A FREE BODY DIAGRAM

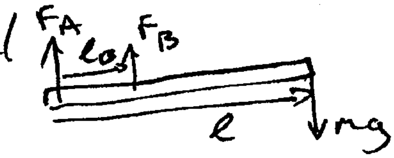


NOW SUM THE TORQUES ABOUT THE JOINT

$$\Sigma\tau = 0 = F_M d - mgD - MgL$$

$$F_M = \left(\frac{mD + ML}{d}\right)g = \boxed{970\ \text{N}}$$

12-5 (a)



$\circlearrowleft +$ (IGNORING WEIGHT OF BOARD)

SUM FORCES $\sum F = F_A + F_B - mg = 0$

SUM TORQUES ABOUT A (B WOULD ALSO WORK!)

$$\sum \tau = F_B \ell_B - mg\ell = 0$$

(ASSUME UNKNOWN TORQUES ARE POSITIVE!)

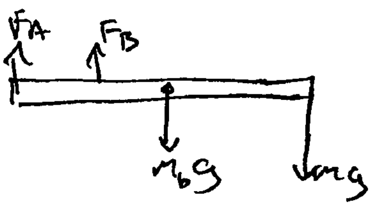
$$F_B = \frac{mg\ell}{\ell_B} = 2080\,N \quad \text{UP}$$

NOW USE THIS IN

$$F_A + F_B - mg = 0$$

$$F_A = mg - F_b = \boxed{-1560\,N = F_A}$$

(b)



$$\sum F = F_A + F_B - m_b g - mg = 0$$

SUM TORQUES ABOUT A

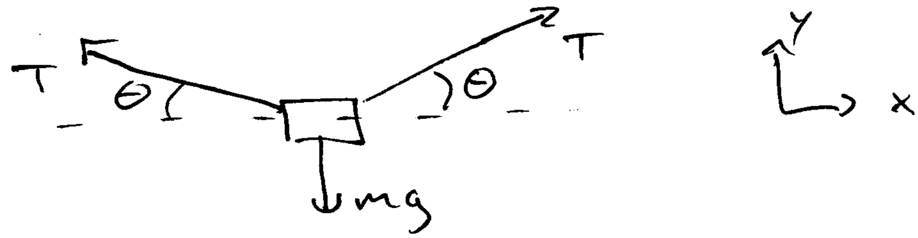
$$\sum \tau = F_B \ell_B - m_b g\left(\frac{\ell}{2}\right) - mg\ell = 0$$

$$F_b = \frac{m_b g(\frac{\ell}{2}) + mg\ell}{\ell_B} = \boxed{2640\,N = F_B}$$

AND

$$F_A = m_b g + mg - F_B = \boxed{-1840\,N = F_A}$$

12-7



A) $\theta = \arctan\left(\frac{1.5\ m}{3.3\ m}\right) = 24.4^\circ$

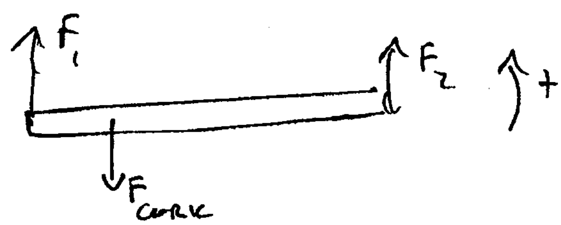
$\Sigma F_x = 0 \qquad \Sigma F_y = 2T\sin\theta - mg = 0$

$T = \frac{mg}{2\sin\theta}$ $\boxed{T = 230\ N}$

B) $\theta = \arctan\left(\frac{0.15\ m}{3.3\ m}\right) = 2.6^\circ$

$T = \frac{mg}{2\sin\theta} = \boxed{2050\ N}$

12-14



SUM TORQUES ABOUT ~~F~~ LEFT END (F_1)

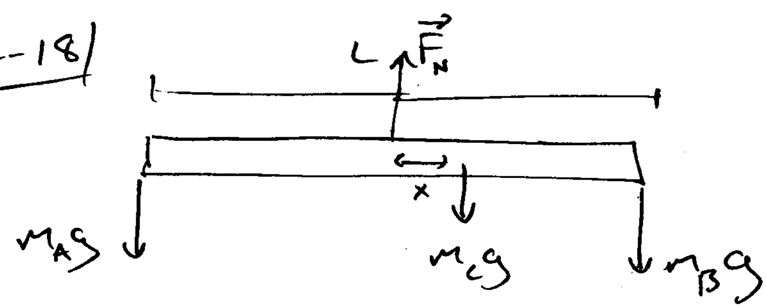
$$\sum \tau = -F_{cork}\, l_{cork} + F_2\, l_{end} = 0$$

$$F_2 = \frac{F_{cork}\, l_{cork}}{l_{end}} = \frac{9}{79}(200\,N)$$

$$\text{or} \quad \frac{9}{79}(400\,N)$$

$$\boxed{F_2 = 23\,N \text{ TO } 46\,N}$$

12-18



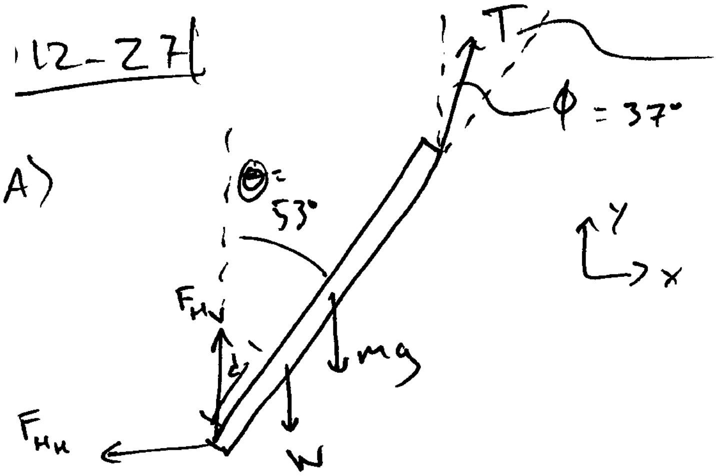
$$\sum \tau = 0 = m_B g \frac{L}{2} + m_c g x - m_A g \frac{L}{2} = 0$$

$$x = \frac{\frac{L}{2}(m_A g - m_B g)}{m_c g} \qquad x = \frac{L(m_A - m_B)}{2 m_c}$$

$$\boxed{x = 0.64\,m}$$

12-27

A)



$53° - \phi = 16°$

B) EQUILIBRIUM $\longrightarrow$ $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma \tau = 0$

SO

$$\Sigma F_x = T \sin\phi - F_{HH} = 0$$

$$F_{HH} = (85\,N)\sin(37°) = \boxed{51N}$$

$$\Sigma F_y = T\cos\phi + F_{HV} - mg - W = 0$$

$$F_{HV} = mg + W - T\cos\phi$$

$$\boxed{F_{HV} = -9N}$$

NEGATIVE SIGN MEANS THE VERTICAL FORCE FROM THE HINGE ACTUALLY POINTS DOWNWARD!

~~(C) $\Sigma\tau = 0$ $T\ell\sin(16°) + mg\left(\frac{\ell}{2}\right)\sin 53°$~~

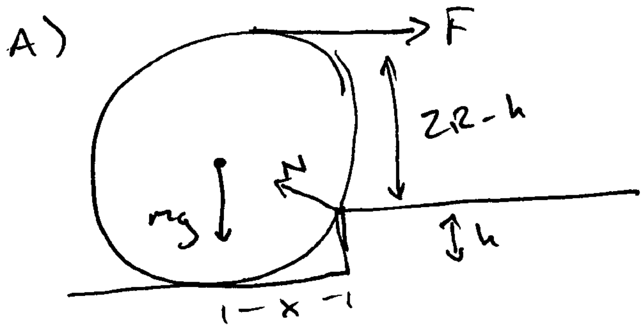
(C) $\Sigma \tau = 0 = W d \sin\theta + mg\frac{\ell}{2}\sin\theta - T\ell\sin(\theta - \phi)$

$$d = \frac{T\ell\sin(16°) - mg\frac{\ell}{2}\sin 53°}{W\sin 53°}$$

$$\boxed{d = 2.4\,m}$$

112-59/ HAVE TO ASSUME THERE IS NO FRICTION! OTHERWISE YOU DON'T HAVE ENOUGH INFO.

A)

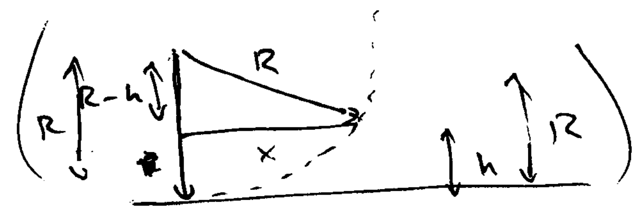


IF WE TAKE THE MOMENT THAT THE WHEEL LOSES CONTACT WITH THE FLAT PART OF THE FLOOR, WE JUST NEED TO BALANCE THE TORQUE DUE TO F AND mg.

N CONTRIBUTES NO TORQUE AS IT IS AT THE PIVOT POINT

$$\sum_i \tau = F(2R-h) - mgx = 0$$

$$x = \sqrt{R^2-(R-h)^2} = \sqrt{h(2R-h)}$$



$$F = \frac{mg\left(\sqrt{h(2R-h)}\right)}{2R-h}$$

$$\boxed{F = \frac{mg\sqrt{h}}{\sqrt{2R-h}} = Mg\sqrt{\frac{h}{2R-h}}}$$

B) SAME DEAL, BUT NOW $\vec{F}$ ACTS AT A PERPENDICULAR DISTANCE OF R-h

$$\sum_i \tau = 0 = F(R-h) - mg\sqrt{h(2R-h)} = 0$$

$$\boxed{F = \frac{mg\sqrt{h(2R-h)}}{R-h}}$$