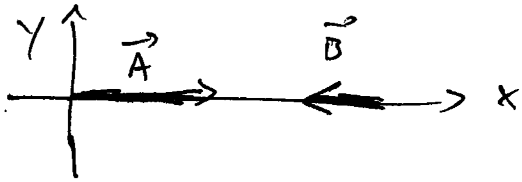
P113 Fall 2013
PS 3 solutions

3-61



$\vec{A} = 6.8\,\hat{i}$

$\vec{B} = -5.5\,\hat{i}$

(a) $\vec{C} = \vec{A} + \vec{B} = 6.8\,\hat{i} + (-5.5\,\hat{i}) = 1.3\,\hat{i}$

$|\vec{C}| = 1.3$ DIRECTION IS $+x$ $(+\hat{i})$

(b) $\vec{C} = \vec{A} - \vec{B} = 6.8\,\hat{i} - (-5.5\,\hat{i}) = 12.3\,\hat{i}$

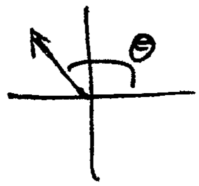
$|\vec{C}| = 12.3$ DIRECTION IS $+x$ $(+\hat{i})$

(c) $\vec{C} = \vec{B} - \vec{A} = (-5.5\,\hat{i}) - (6.8\,\hat{i}) = -12.3\,\hat{i}$

$|\vec{C}| = 12.3$ DIRECTION IS $-x$ $(-\hat{i})$

3-81 $\vec{V}_1 = -6\hat{i} + 8\hat{j}$ $\vec{V}_2 = 4.5\hat{i} - 5\hat{j}$

(a) $|\vec{V}_1| = \sqrt{(-6)^2 + (8)^2} = \sqrt{100} = \boxed{10}$

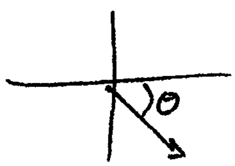


$\theta = \arctan\left(\frac{y}{x}\right) = -53°$

BUT THIS IS OFF BY 180°

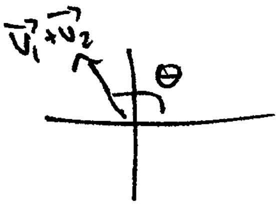
$\rightarrow \theta = 180° + (-53°) = \boxed{127° \text{ FROM x-AXIS}}$

(b) $|\vec{V}_2| = \sqrt{(4.5)^2 + (-5)^2} = \boxed{6.7}$



$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{-5}{4.5}\right) = \boxed{-48° \text{ (or } 312°) \text{ FROM x-axis}}$

(c) $|\vec{V}_1 + \vec{V}_2| = \sqrt{(-6+4.5)^2 + (8+(-5))^2} = \boxed{3.4}$

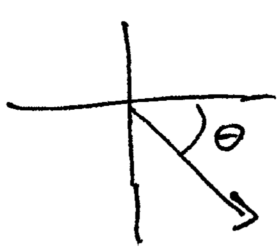


$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{3}{-1.5}\right) = -63$

BUT NEED TO ADD 180°

$\boxed{\theta = 117°}$

(d) $|\vec{V}_2 - \vec{V}_1| = \sqrt{(4.5-(-6))^2 + (-5-8)^2} = \boxed{16.7}$



$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{-13}{10.5}\right)$

$\boxed{\theta = -51° \text{ OR } 309° \text{ FROM x-axis}}$

3-9 | $\vec{v}_1 = 4\hat{i} - 8\hat{j}$ $\qquad \vec{v}_3 = -2\hat{i} + 4\hat{j}$

(2) $\vec{v}_2 = \hat{i} + \hat{j}$

(a) $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = (4\hat{i} - 8\hat{j}) + (1\hat{i} + 1\hat{j}) + (-2\hat{i} + 4\hat{j})$

$= (4\hat{i} + 1\hat{i} - 2\hat{i}) + (-8\hat{j} + 1\hat{j} + 4\hat{j})$

$\boxed{= 3\hat{i} - 3\hat{j}}$

$|3\hat{i} - 3\hat{j}| = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18} \boxed{= 4.2}$

$\theta = \arctan\left(\frac{-3}{3}\right) \boxed{= 315°}$

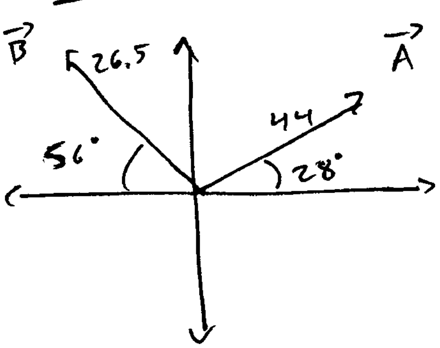
(b) $\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = (4\hat{i} - 8\hat{j}) - (1\hat{i} + 1\hat{j}) + (-2\hat{i} + 4\hat{j})$

$= 1\hat{i} - 5\hat{j}$

$|1\hat{i} - 5\hat{j}| = \sqrt{1^2 + 5^2} = \sqrt{26} \boxed{= 5.1}$

$\theta = \arctan\left(\frac{-5}{1}\right) \boxed{= 280°}$

12) 3-11 |



$\vec{A} = |\vec{A}|\cos(28°)\hat{i} + |\vec{A}|\sin(28°)\hat{j}$

$= 44(\cos(28°)\hat{i} + \sin(28°)\hat{j})$

$\vec{A} = 39\hat{i} + 21\hat{j}$

$\vec{B} = -26.5\cos(56°) + 26.5\sin(56°)\hat{j}$

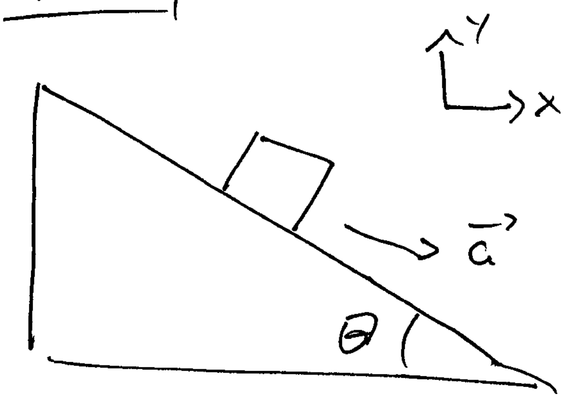
$= -15\hat{i} + 22\hat{j}$

(c) $\vec{B} - \vec{A} = (-15 - 39)\hat{i} + (22 - 21)\hat{j} = \boxed{-54\hat{i} + 1\hat{j} = \vec{B} - \vec{A}}$

(b) $\vec{A} - \vec{B} = (39 - (-15))\hat{i} + (21 - 22)\hat{j} = \boxed{54\hat{i} - 1\hat{j} = \vec{A} - \vec{B}}$

THEY ARE OPPOSITES.

3-22



WE ARE ASKED FOR THE VERTICAL COMPONENT OF $\vec{a}$, SO WE NEEDN'T TILT THE COORDINATE SYSTEM

(a) $\vec{a} = |\vec{a}| \cos\theta \,\hat{i} - |\vec{a}| \sin\theta \,\hat{j}$

WE WANT a_y SO

$$a_y = |\vec{a}| \sin\theta = -(1.8\,m/s^2)\sin(30°)$$

$$\boxed{a_y = -0.9\,m/s^2}$$

(b) IF WE LET $y_0 = 325\,m$, $y = 0\,m$, $a_y = -0.9\,m/s^2$

WE CAN USE $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$$\rightarrow 0 = 325\,m - \tfrac{1}{2}(0.9)t^2$$

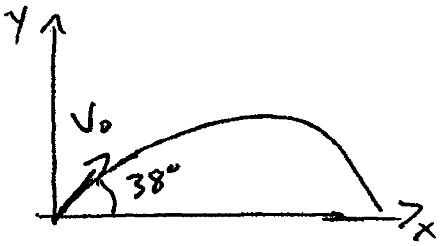
$$t = \left(\frac{(325\,m)(2)}{(0.9\,m/s^2)}\right)^{1/2}$$

$$\boxed{t = 27\,s}$$

3-33

WE KNOW $\vec{V}_0 \Rightarrow$ WE KNOW V_{oy}, V_{ox}

$\vec{a} = -g\hat{j}$, $Y_0 = 0$, $Y_f = 0$



WE WANT t_{LAND}.

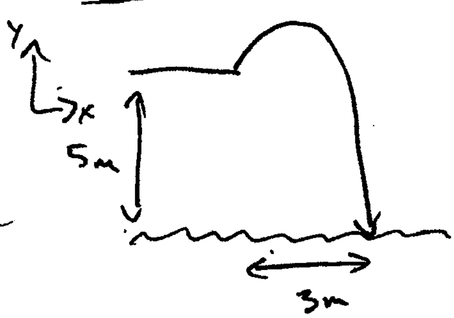
WE CAN USE $Y = Y_0 + V_{oy}t + \frac{1}{2}a_y t^2$

$$0m = 0m + (18 m/s)\sin(38°)t - \frac{1}{2}gt^2$$

$$\hookrightarrow V_0 \sin\theta\, t = \frac{1}{2}gt^2 \rightarrow t = \frac{2V_0 \sin\theta}{g}$$

$$t = \frac{2(18 m/s)\sin(38°)}{10 m/s^2} \quad \boxed{t = 2.2\,s}$$

3-45



TAKE $Y_0 = 5m$ $x_0 = 0m$ $t = 1.3\,s$

$Y_f = 0m$ $x_f = 3m$

$a_y = -g$ $a_x = 0$

(a)

WE MUST USE Y - DIRECTION TO GET V_{oy}, X - DIRECTION TO GET V_{ox}

$$Y = Y_0 + V_{oy}t + \frac{1}{2}a_y t^2 \Rightarrow Y - Y_0 - \frac{1}{2}a_y t^2 = V_{oy}t$$

$$V_{oy} = \frac{Y - Y_0}{t} - \frac{1}{2}gt^2 = \frac{0m - 5m}{1.3s} + \frac{1}{2}(10 m/s^2)(1.3s)^2$$

$$V_{oy} = 2.6 m/s$$

~~4x~~ $x = x_0 + V_{ox}t + \frac{1}{2}\cancel{a_x}t^2$ $\quad V_{ox} = \frac{x - x_0}{t} = \frac{3m}{1.3s}$

$V_{ox} = 2.3 m/s$

$$\boxed{\vec{V}_0 = (2.3 m/s)\hat{i} + (2.6 m/s)\hat{j}}$$

3.45 / CONTINUED

(b) MAX HEIGHT IS WHEN $V_y = 0$

USE $(V_y^2 - V_{0y}^2) = -2g(y - y_0)$ SOLVE FOR y

$V_y \to 0$

$$\frac{-V_{0y}^2}{-2g} = y - y_0 \Rightarrow y = \frac{V_{0y}^2}{2g} + y_0 = \frac{(2.6\,m/s)^2}{20\,m/s^2} + 5m$$

$$\boxed{y = 5.3\,m}$$ (ABOVE WATER)

(c) SINCE $a_x = 0$ WE KNOW $V_{fx} = V_{0x} = 2.3\,m/s$

FOR V_y WE CAN USE

$$V_y = V_{0y} \mp gt$$

$t = 1.3s$

$a_y = -g$

$V_{0y} = 2.6\,m/s$

$$V_{fy} = 2.6\,m/s - (10\,m/s^2)(1.3s)$$

$$V_{fy} = -10.4\,m/s$$

$$\boxed{\vec{V}_f = (2.6\,m/s)\hat{i} - (10.4\,m/s)\hat{j}}$$

3-61

THE LIFEGAURD AND CHILD WILL BE CARRIED DOWNSTREAM AT THE SAME RATE.

SHE WILL REACH THE CHILD AFTER

$$t = \frac{d}{v} = \left(\frac{45\,m}{2\,m/s}\right) \boxed{= 23\,s}$$

IN THAT TIME THE CURRENT WILL TAKE THEM $d = (23\,s)(1\,m/s) \boxed{= 23\,m}$

DOWNSTREAM.

3-62

THE VELOCITIES OF THE PERSON AND THE BOAT ADD COMPONENTWISE

SO $\vec{V} = \left\{(0.6\,m/s)\cos 45^\circ\right\}\hat{i} + (1.7\,m/s)\,\hat{i} + \left\{(0.6\,m/s)\sin 45^\circ\right\}\hat{j}$

$$\boxed{\vec{V} = (2.12\,\hat{i} + 0.42\,\hat{j})\,m/s}$$

3-77

FIRST, WE KNOW THAT $a_y = -9.8\,m/s^2$

AND THAT ~~the~~ V_y AT 8m HEIGHT

WILL BE ZERO, SO WE CAN FIND V_0 VIA

$$V_{f_y}^2 - V_{0_y}^2 = 2a_y(y - y_0) \quad \text{with} \begin{cases} V_f = 0\,m/s \\ a_y = -9.8\,m/s^2 \\ y = 8m \\ y_0 = 0m \end{cases}$$

$$V_{0_y} = \left\{2(9.8\,m/s^2)(8m)\right\}^{1/2}$$

$$V_{0y} = 12.5\,m/s$$

WITH THIS WE CAN FIND THE TIME OF

FLIGHT:

$$V_y = V_{0y} + a_y t \qquad \begin{cases} V_y = 0\,m/s \\ V_{0y} = 12.5\,m/s \\ a_y = -9.8\,m/s^2 \end{cases}$$

$$t = \frac{V_y - V_{0y}}{a_y} = \frac{-12.5\,m/s}{-9.8\,m/s^2} \qquad t = 1.28\,s$$

SO NOW WE KNOW THAT WE MUST TRAVEL

9m HORIZONTALLY IN 1.28s, SO USING

$$x = x_0 + V_{0x}t + \tfrac{1}{2}a_x t^2 \quad \text{with} \begin{cases} x = 9m \\ x_0 = 0m \\ a_x = 0 \\ t = 1.28s \end{cases}$$

SO $9m = V_{0x}(1.28s)$

$$V_{0x} = \frac{9m}{1.28s} \approx 7\,m/s$$

SINCE IT IS ONLY MOVING HORIZONTALLY

AT THE TIME OF IMPACT THIS MUST BE

THE TOTAL VELOCITY

$$\boxed{\vec{V}_f = (7\,m/s)\,\hat{\imath}}$$