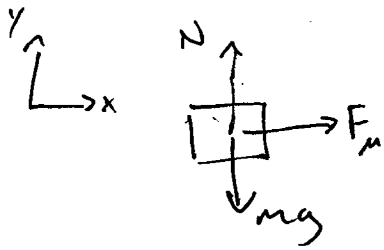
Physics 113 - Fall 2013 - solutions to problem set 5

1) 5-31

START BY DRAWING A FREE BODY DIAGRAM



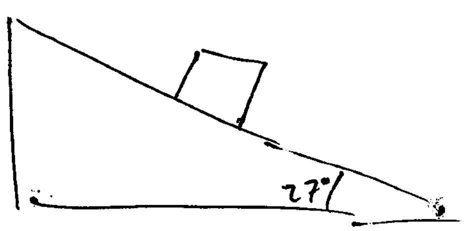
$\sum_i F_y = N - mg = 0$

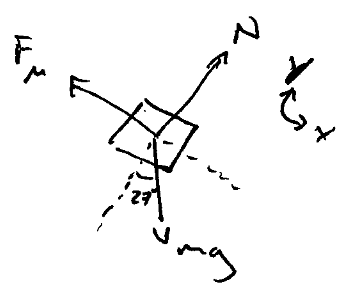
$\hookrightarrow N = mg$

$\sum_i F_x = F_\mu = \mu N = \mu m g = m a$

so $\boxed{\mu = \frac{a}{g} = \frac{0.2g}{g} = 0.2}$

2) 5-7





$\sum_i F_x = mg\sin\theta - F_\mu = ma = mg\sin\theta - \mu N$

$\sum_i F_y = N - mg\cos\theta = 0$

$\hookrightarrow N = mg\cos\theta$

so THE FORCE OF FRICTION IS

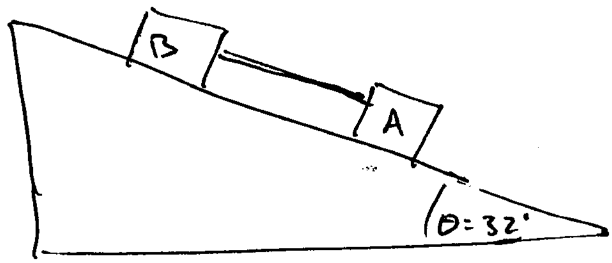
$F_\mu = mg\sin\theta - ma = (25\,kg)(9.8\,m/s^2 \sin 27° - 0.3\,m/s^2)$

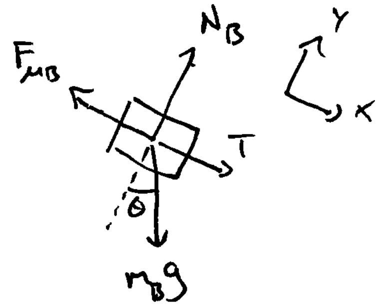
$\boxed{F_\mu \approx 100\,N}$

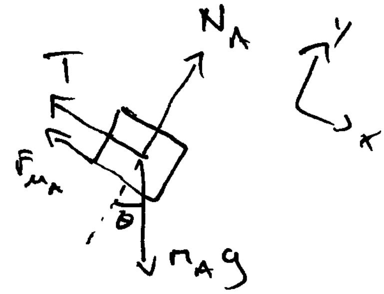
AND $F_\mu = \mu N = \mu m g \cos\theta \rightarrow \boxed{\mu = 0.48}$

3) 5-20

I'M GOING TO DRAW THE RAMP GOING DOWN TO THE RIGHT (NOTHING WRONG WITH THIS!)







WE MUST SUM THE FORCES ON EACH BLOCK:

A) $\sum F_{x_A} = M_A g \sin\theta - T - F_{\mu A} = M_A a$

$\sum F_{y_A} = N_A - M_A g \cos\theta = 0 \longrightarrow N_A = M_A g \cos\theta$

$\hookrightarrow F_{\mu A} = \mu_A M_A g \cos\theta$

B) $\sum F_{x_B} = M_B g \sin\theta + T - F_{\mu B} = M_B a$

$\sum F_{y_B} = N_B - M_B g \cos\theta = 0 \longrightarrow N_B = M_B g \cos\theta$

$\hookrightarrow F_{\mu B} = \mu_B M_B g \cos\theta$

SUBSTITUTING $F_{\mu A}, F_{\mu B}$ IN TO THE X EQUATIONS GIVES

$M_A a = M_A g \sin\theta - T - \mu_A M_A g \cos\theta$

$M_B a = M_B g \sin\theta + T - \mu_B M_B g \cos\theta$

WE CAN ADD THESE TWO EQUATIONS TO GET

$M_A a + M_B a = M_A g \sin\theta - T - \mu_A M_A g \cos\theta + M_B g \sin\theta + T - \mu_B M_B g \cos\theta.$

THIS SIMPLIFIES TO

$$a(m_A + m_B) = m_A(g\sin\theta - \mu_A g\cos\theta) + m_B(g\sin\theta - \mu_B g\cos\theta)$$

SO ACCELERATION IS JUST:

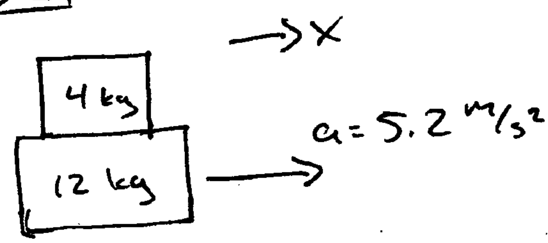
$$a = \frac{m_A(g\sin\theta - \mu_A g\cos\theta) + m_B(g\sin\theta - \mu_B g\cos\theta)}{(m_A + m_B)}$$

PLUGGING IN THE NUMBERS GIVES

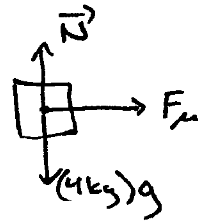
$$\boxed{a \approx 3.1\ m/s^2}$$

$\mu = \mu_s = \mu_k$

4) <u>5-32</u>



(a)



THE TOP BLOCK MUST HAVE $a = 5.2\ m/s^2$ IF IT IS TO NOT SLIDE

so $\sum F_x = F_\mu = (4kg)\,g\,\mu = (4kg)\,5.2\ m/s^2$

$$\mu = \frac{5.2\ m/s^2}{9.8\ m/s^2} \approx 0.53$$

(b) IF $\mu = 0.26$

$F_\mu = \mu N = 0.26\,(4kg)\,g = 10.2\ N$

$F_\mu = ma \Rightarrow a = \frac{10.2N}{m} = \boxed{2.5\ m/s^2}$

(c) acceleration ADDS LIKE A VECTOR, BUT THE ACCELERATIONS ARE ~~...~~ PARALLEL

so

ACCELERATION OF top RELATIVE TO bottom

$a_{tb} = a_t - a_b$

RELATIVE TO TABLE

$a_{tb} = 2.5\ m/s^2 - 5.2\ m/s^2$

$\boxed{a_{tb} = -2.7\ m/s^2}$

(d) IN (a) TREAT SYSTEM AS ONE BOX WITH $m = 16\ kg$

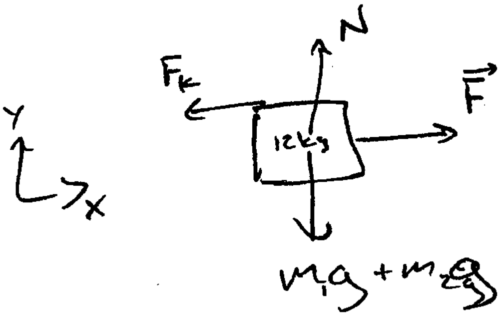
$F = ma = (16kg)(5.2\ m/s^2)$

$\vec{F} = (83.2\ N)\hat{\imath}$

2

5-3C (CONT

IN (b) THERE IS A FORCE OPPOSING THE ACCELERATION DUE TO THE FRICTION BETWEEN THE BOXES



IN (b) WE FOUND THAT THE TOP BOX ACCELERATED AT $2.5\ m/s^2$ SO

$$|F_k| = (4\,kg)(2.5\ m/s^2) = 10\ N$$

THIS IS THEN THE MAGNITUDE OF F_k (BY THIRD LAW) IN THE FBD ABOVE

$$\sum F_x = \vec{F} - \vec{F}_k = m a_x$$

$$\cancel{F = (4\,kg)(5.2\ m/s^2) + 10\ N = \boxed{30.8\ N = F}}$$

$$\vec{F} = (12\,kg)(5.2\ m/s^2)\hat{i} + 10\ N\hat{i} = \boxed{72.4\ N\hat{i}}$$

5) 5-34 FRICTION MUST PROVIDE ALL OF THE CENTRIPETAL FORCE

SO $\mu N = m\frac{v^2}{r} \longrightarrow v = \sqrt{\frac{\mu N r}{m}} = \sqrt{\mu g r}$

$\mu = 0.65$

$N = mg$ ($m = 1200$ kg, $g = -9.8$ m/s)

$N = 11{,}760$ N

$r = 80$ m

$v = \sqrt{(0.65)(9.8\,m/s^2)(80\,m)}$

$v = \sqrt{509.6} = \boxed{22.6\ m/s = v}$

6) 5-40 AT THE TOP OF THE CIRCLE THEY MUST BE ACCELERATING AT LEAST $9.8\,m/s^2$ DOWNWARD. SO $F_c \geq mg$

$m\frac{v^2}{r} \geq mg \qquad v^2 \geq gr$

$v \geq \sqrt{gr} = \sqrt{(9.8\,m/s^2)(7.6\,m)}$

$\boxed{v \geq 8.6\ m/s}$

7) 5-42 | SIMILAR TO PROBLEM 5-34

$$\mu N = \mu m g = \frac{m v^2}{r}$$

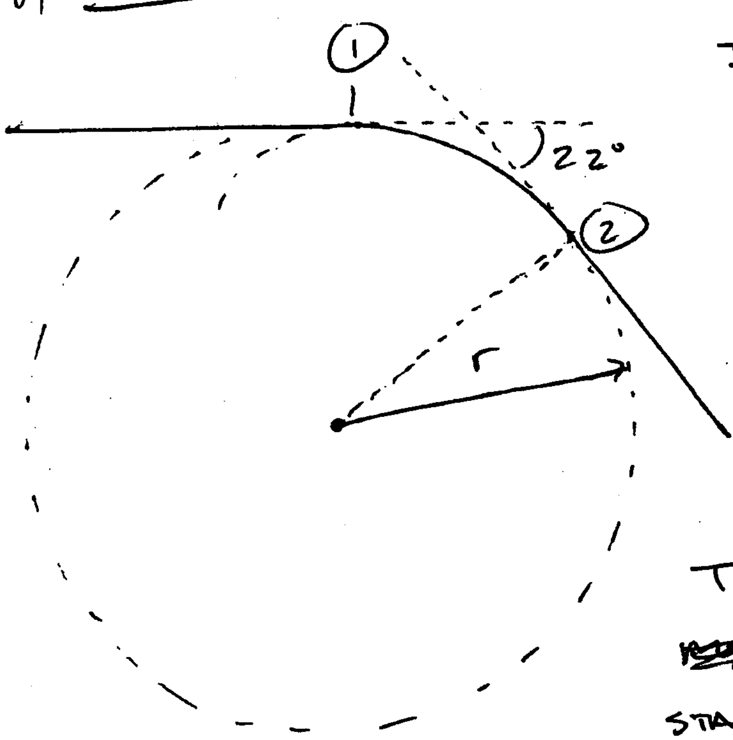
$$\mu = \frac{v^2}{gr}$$

$v = 95\ \text{km/hr}\ (= 26.4\ \text{m/s})$

$r = 85\ \text{m}$

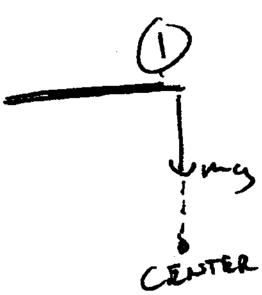
$$\mu = \frac{(26.4\ \text{m/s})^2}{(85\ \text{m})(9.8\ \text{m/s}^2)} \approx \boxed{0.8 \approx \mu}$$

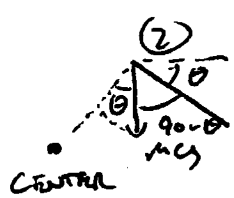
8) 5-52 |



THE ONLY FORCE PRESENT IS GRAVITY (AND SOME NORMAL FORCE) SO GRAVITY MUST PROVIDE THE CENTRIPETAL ACCELERATION THAT HOLDS THE CAR DOWN IN THE CIRCULAR SEGMENT.

AS THE CAR TRAVERSES THIS SEGMENT THOUGH THE FORCE OF GRAVITY STARTS AS POINTING TOWARD THE CENTER OF THE CIRCLE AND ENDS POINTING AT THE CENTER





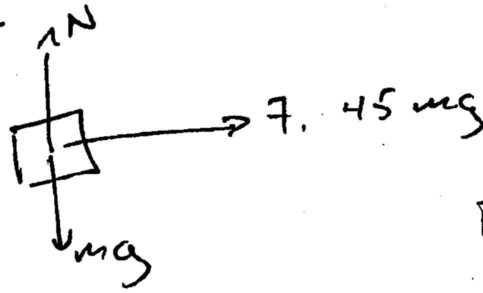
AT 2 THEREFORE,

$$m\frac{v^2}{r} = mg\cos(22^\circ)$$

SO $r = \dfrac{v^2}{g\cos(22^\circ)} = \dfrac{(26.4\ \text{m/s})^2}{(9.8\ \text{m/s}^2)\cos(22^\circ)}$

$$\boxed{r = 76.7\ \text{m}}$$

9) 5-831



$F_c = m\frac{v^2}{r} = 7.48\ mg$

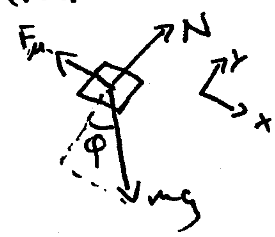
~~$r = v^2$~~ $v = \sqrt{7.45(9.8\ m/s^2)(11\ m)}$

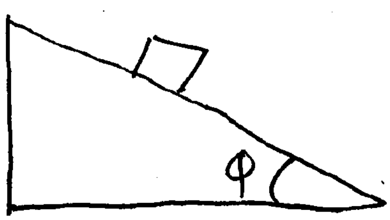
$v = \sqrt{803\ m^2/s^2} \approx \boxed{28.4\ m/s}$

~~$f = 2\pi r v$~~ $f = \frac{v}{2\pi r} = \frac{28.4\ m/s}{2\pi(11\ m)}$

$\boxed{f \approx 0.4\ \frac{rev}{s}}$

10) 5-87 THINK OF THIS AS AN INCLINED PLANE PROBLEM. THE TRICK IS UNDERSTANDING WHY THE PLANE ANGLE IS φ





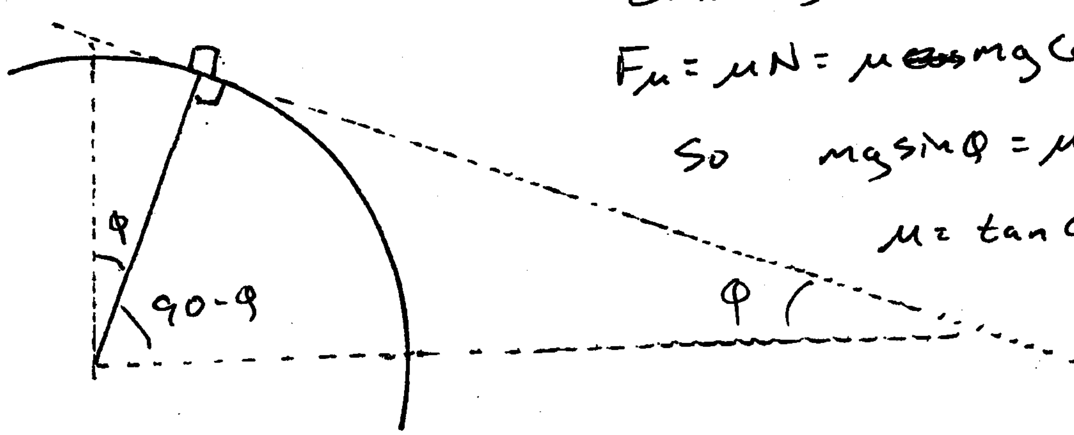
THEN, TO PREVENT SLIP

$\sum F_x = mg\sin\varphi - F_\mu = 0$

$F_\mu = \mu N = \mu\ \cancel{\cos}\ mg\cos\varphi$

So $mg\sin\varphi = \mu mg\cos\varphi$

$\mu = \tan\varphi$



~~tan~~ $\varphi = \arctan\mu$

$\varphi = \arctan(0.7)$

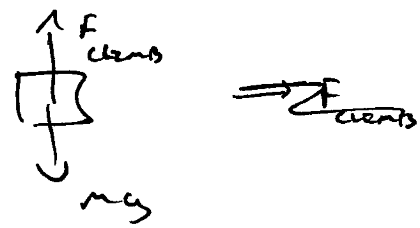
$\boxed{\varphi = 35^\circ}$

11) 7-31

SINCE $W = \vec{F}\cdot\vec{d} = |\vec{F}||\vec{d}|\cos\theta$

AND $\theta = 0^\circ$ (FORCE AND DISTANCE IN SAME DIRECTION)

~~THEN~~ A FBD FOR THE FIREFIGHTER IS



ASSUMING HE CLIMBS AT A CONSTANT RATE

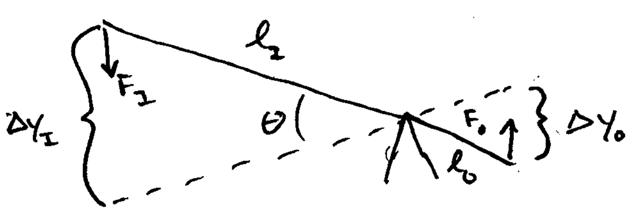
$$\Sigma F = F_{climb} - mg = 0 \longrightarrow F_{climb} = mg$$

SO

$$W = mgd = (75\ kg)(9.8\ m/s^2)(20\ m)$$

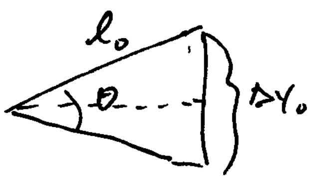
$$\boxed{W = 1.47\times10^4\ J}$$

12) 7-61



AS THE INPUT ARM MOVES ΔY_I, THE OUTPUT ARM MOVES ΔY_o

BUT WE CAN ZOOM IN ON ~~THIS~~ ONE TRIANGLE TO SEE



$$\frac{\Delta Y_o}{2} = l_o \sin\left(\frac{\theta}{2}\right) \quad \text{SO} \quad \begin{cases} \Delta Y_I = 2 l_I \sin\left(\frac{\theta}{2}\right) \\ \Delta Y_o = 2 l_o \sin\left(\frac{\theta}{2}\right) \end{cases}$$

THEN THE WORK DONE ON ~~THE~~ EACH SIDE IS

$$W_I = F_I \left(2 \ell_I \sin \frac{\theta}{2}\right)$$

$$W_o = F_o \left(2 \ell_o \sin \frac{\theta}{2}\right)$$

BUT THE TWO MUST BE EQUAL,

SO

$$2 F_I \ell_I \sin \frac{\theta}{2} = 2 F_o \ell_o \sin \frac{\theta}{2}$$

SO $\longrightarrow$ $\boxed{\dfrac{F_o}{F_I} = \dfrac{\ell_I}{\ell_o}}$