Physics 113 - Fall 2013 - Solutions to problem set 6

7-12

(A) $W_{MOTOR} = F_{MOTOR}\, d \cos 0° = mg\,d = (2250\,kg)(9.8\,m/s^2)(3345\,m - 2150\,m)$

$$\boxed{W_{MOTOR} = 2.63 \times 10^7\ J}$$

(B) THE FORCE OF GRAVITY IS IN THE OPPOSITE DIRECTION AS THE DISPLACEMENT SO

$$W_{GRAVITY} = F_{GRAVITY}\, d \cos 180° = -mg\,d = -2.63 \times 10^7\ J$$

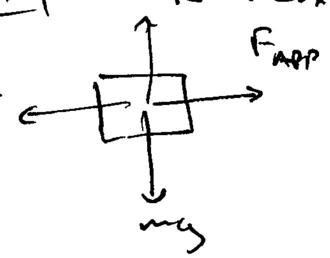
(C) IF THE FORCE OF THE MOTOR WAS 10% MORE THEN THERE WOULD BE A NET FORCE,

$$F_{NET} = F_{MOTOR} - F_{GRAV.} = 1.1\,mg - mg = 0.1\,mg = ma$$

SO $a = 0.1\,g = \boxed{0.98\ m/s^2 = a}$

7-14

(A)



$$\Sigma F_x = F_{APP} - F_\mu = 0$$

$$\Sigma F_y = N - mg = 0$$

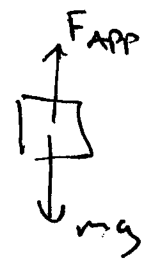
$$\hookrightarrow N = mg$$

$$F_{App} = F_\mu = \cancel{\mu N = \mu mg}$$

$$F_{App} = 230\,N$$

$$W = (230N)(4m) = \boxed{920\ J = W}$$

(B)



$$\Sigma F = F_{APP} - mg = 0$$

$$F_{App} = mg = 2200\,N$$

$$W = (2200N)(4m) = \boxed{8800\ J = W}$$

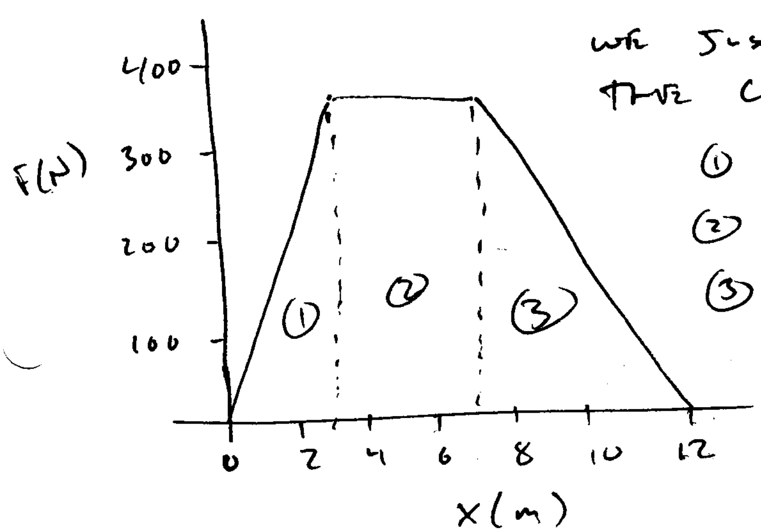
7-

$$W = \int \vec{F} \cdot d\vec{s} = \int F dx$$

(1-DIMENSION.)

YOU COULD INTEGRATE THREE FUNCTIONS OVER THE THREE INTERVALS, BUT IT IS EASY TO THINK ABOUT GRAPHICALLY

WE JUST NEED THE AREA UNDER THE CURVE,



① $A_1 = \frac{1}{2}(3m)(380N) = 470\ J$

② $A_2 = (380N)(4m) = 1520\ J$

③ $A_3 = \frac{1}{2}(380N)(5m) = 950\ J$

$$A = A_1 + A_2 + A_3 = \boxed{3040\ J = W}$$

7-40 $W = \int_{x_i}^{x_f} F(x)dx$ (IN 1-D)

(a) AREA $= \frac{1}{2}(3m)(400N) + (4m)(400N) + \frac{1}{2}(3m)(400N)$

AREA $= 2800$ N·m $= \boxed{2800 \text{ J} = W}$

(b) $W(0 \to 10m) = 2800$ J

$W(10m \to 15m) = -\frac{1}{2}(1.5m)(200N) - (2m)(200N) - \frac{1}{2}(1.5m)(200N)$

$= -700$ J

$\boxed{W(0m \to 15m) = 2100 \text{ J}}$

7-58 IF BRAKING FORCE (I.E. FRICTION) IS CONSTANT, THEN THE NET FORCE WHILE BRAKING IS $W_{NET} = F \cdot d$

TO STOP $W_{NET} = \Delta KE$.

IF $V_1 \rightarrow 1.5\, V_0$ THEN

~~$KE_1 \rightarrow \frac{1}{2} m$~~ $KE_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m (1.5 v_0)^2$

$= \frac{1}{2} m (2.25) v_0^2$

~~KE~~ $KE_1 = 2.25\, KE_0$

SO $\boxed{W_1 = 2.25\, W_0}$

BUT $F_1 = F_0$ SO

$F_1 \Delta x_1 = (F_0 \Delta x_0)\, 2.25$

$\rightarrow \Delta x_1 = 2.25\, \Delta x_0$

THE BRAKING DISTANCE IS 2.25 TIMES LONGER IF THE VELOCITY GOES UP BY 50%

8-16 ENERGY MUST BE CONSERVED

(a) $E_i = mgh_0 + \frac{1}{2}mv_0^2$ $h_0 = 2m$ $V_0 = 4.5 \ m/s$

$E_f = mgh_f + \frac{1}{2}mv_f^2$ $h_f = 0 \ m$

So

$$mgh_0 + \frac{1}{2}mv_0^2 = \cancel{mgh_f} + \frac{1}{2}mv_f^2$$

$$gh_0 + \frac{1}{2}v_0^2 = \frac{1}{2}v_f^2$$

$$v_f = (2gh_0 + v_0^2)^{1/2} = \boxed{7.8 \ m/s = v_f}$$

(b) AT THE BOTTOM HE HAS NO VELOCITY.

$$E_i = \frac{1}{2}mv^2 \left(= \frac{1}{2}m(7.8 \ m/s)^2\right)$$

$$E_f = \frac{1}{2}ky^2$$ (IGNORE mgh COMPONENT FOR SIMPLICITY)

$$\frac{1}{2}mv^2 = \frac{1}{2}ky^2 \rightarrow y = \left(\frac{mv^2}{k}\right)^{1/2}$$

$$\boxed{y = -0.3 \ m}$$

8-20

IN ALL CASES

$E_i = E_f = E$, $E = \frac{1}{2}mv^2 + mgy$

SO, $E_1 = \frac{1}{2}mv_1^2 + mgy_1$

$v_1 = 0$ (RELEASED FROM REST)

$y_1 = 32m$

$E_1 = mg(32m)$

IN GENERAL THEN

$E_f = mgy_1 = \frac{1}{2}mv_f^2 + mgy_f$

SO FOR ANY f

$v_f^2 = 2g(y_1 - y_f)$ $v_f = \sqrt{2g(y_1 - y_f)}$

$v_f = \sqrt{2g(32m - y_f)}$

SO

$$\boxed{\begin{aligned} v_2 &= \sqrt{2g(32m - 0m)} = 25\ m/s \\ v_3 &= \sqrt{2g(32m - 26m)} = 11\ m/s \\ v_4 &= \sqrt{2g(32m - 14m)} = 19\ m/s \end{aligned}}$$

8-21 WE KNOW $E_i = E_f$

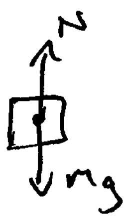
LET'S FIND D

$$E_i = mgD$$

$$E_f = \frac{1}{2}kx^2 \quad \text{BUT } x = D \text{ SO}$$

$$mgD = \frac{1}{2}kD^2 \qquad \boxed{D = \frac{2mg}{k}}$$

NOW, FOR d



BUT $N = kd$ SO $\sum F = kd - mg = 0$

THEN $\boxed{d = \frac{mg}{k}}$

$$D \neq d \rightarrow \boxed{D = 2d}$$

THE POSITION d IS WHEN THE RESTORING FORCE OF THE SPRING CANCELS OUT GRAVITY, ALLOWING THE SYSTEM TO REMAIN AT REST.

THE POSITION D REFLECTS ADDITIONAL COMPRESSION DUE TO THE FACT THE SYSTEM STARTED UNBALANCED $(mg > \frac{1}{2}kx_0)$. THIS SYSTEM WILL BOUNCE BETWEEN $x = D$ AND $x = x_0$.

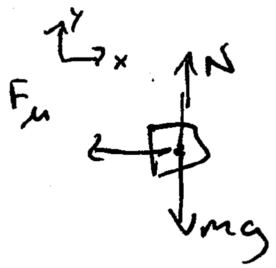
8-361

(a) $A \to B$ $\quad E_A = E_B$

$$mgh_A = \frac{1}{2}mv_B^2 \qquad h_A = r = 2m$$

$$\boxed{v_B = \sqrt{2gr} = 6.3\ m/s}$$

(b) $B \to C$ ENERGY IS DISSIPATED BY THE WORK FRICTION DOES



$$\sum_i F_x = -F_\mu = -\mu_k N = F_{NET}$$

$$\sum_i F_y = N - mg = 0 \qquad N = mg$$

so $\vec{F}_{NET} = -mg\mu_k \hat{i}$

$$W_{NET} = \vec{F}_{NET} \cdot \Delta\vec{x} = -mg\mu_k d \qquad (d = 3m)$$

$$W = -7.5\ J \rightarrow$$ 7.5 J OF THERMAL ENERGY ARE PRODUCED

(c) $W_{NET} = \Delta K.E. = \frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2 = -\mu_k g m d$

so $v_C^2 = -2\mu_k g d + v_B^2$

$$v_C = \sqrt{v_B^2 - 2\mu_k g d} = \boxed{5.0\ m/s = v_C}$$

(d) $C \to D$ $\quad E_C = \frac{1}{2}mv_C^2 \quad E_D = \frac{1}{2}kx^2 \quad x = 0.2m$

$$\frac{1}{2}mv_C^2 = \frac{1}{2}kx^2 \rightarrow k = \frac{mv_C^2}{x^2} \qquad \boxed{k = 625\ N/m}$$

12) 8-58

$$P = \frac{W}{\Delta t} = \frac{\Delta K.E.}{\Delta t}$$

$$\Delta K.E. = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_f^2 - v_i^2)$$

$v_i = 0 \, m/s$ $\quad$ $v_f = 95 \, km/hr = 26.4 \, m/s$

$$P = \frac{\frac{1}{2}(1400 \, kg)(26.4 \, m/s)^2}{7.4 \, s} = \boxed{66{,}000 \, W}$$

$$\boxed{P = 66 \, kW}$$