

1) 6-2/

$$M_{\text{moon}} = 7.35 \times 10^{22} \text{ kg}$$

$$R_{\text{moon}} = 1.74 \times 10^6 \text{ m}$$

$$g_{\text{moon}} = \frac{G M_{\text{moon}}}{R_{\text{moon}}^2} = \frac{(6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}) (7.35 \times 10^{22} \text{ kg})}{(1.74 \times 10^6 \text{ m})^2}$$

$$g_{\text{moon}} = 1.6 \text{ m/s}^2$$

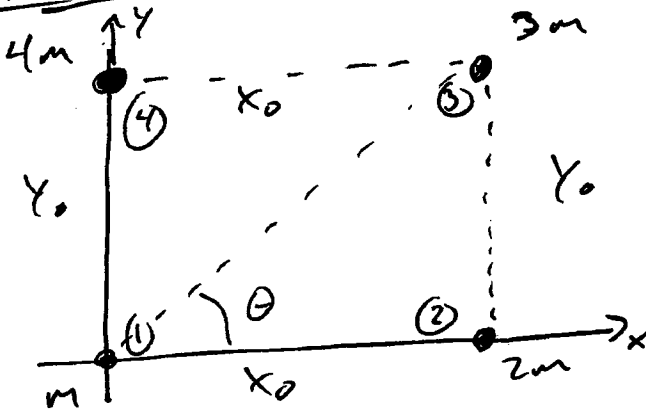
2) 6-7/

$$g_{\text{SATELLITE}} = \frac{G M_{\oplus}}{(R_{\oplus} + 3 \times 10^5 \text{ m})^2} = \frac{(6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}) (5.98 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m} + 3 \times 10^5 \text{ m})^2}$$

$$g_{\text{SATELLITE}} = 8.9 \text{ m/s}^2$$

STILL PRETTY BIG

3) 6-11/



$$\vec{F}_{\text{NET}} = \sum_{i=2}^4 \vec{F}_{i1}$$

$$\vec{F}_{i2} = +G \frac{(m)(2m)}{x_0^2} \hat{y}$$

$$\vec{F}_{i4} = +G \frac{(m)(4m)}{y_0^2} \hat{x}$$

$$|\vec{F}_{i3}| = G \frac{(m)(3m)}{(x_0^2 + y_0^2)}$$

WE NEED

TO GET THE DIRECTION OF  $\vec{F}_{i3}$

~~WE NEED TO GET THE DIRECTION OF  $\vec{F}_{i3}$~~



3) 6-11 | (CONTINUED)

$$\text{proj}_x \vec{F}_{13} = F_{13} \cos \theta \hat{i} = F_{13} \frac{x_0}{\sqrt{x_0^2 + y_0^2}} \hat{i}$$

$$\text{proj}_y \vec{F}_{13} = F_{13} \sin \theta \hat{j} = F_{13} \frac{y_0}{\sqrt{x_0^2 + y_0^2}} \hat{j}$$

$$\text{SO, } \vec{F}_{13} = G \frac{3m^2}{(x_0^2 + y_0^2)^{3/2}} (x_0 \hat{i} + y_0 \hat{j})$$

THEN,

$$\vec{F}_{\text{NET}} = \sum_{i=2}^4 \vec{F}_{1i} = Gm^2 \left[ \left( \frac{2}{x_0^2} + \frac{3x_0}{(x_0^2 + y_0^2)^{3/2}} \right) \hat{i} + \left( \frac{4}{y_0^2} + \frac{3y_0}{(x_0^2 + y_0^2)^{3/2}} \right) \hat{j} \right]$$

H) IF THE EARTH'S ORBIT IS CIRCULAR THEN THE FORCE OF GRAVITY ON THE EARTH FROM THE SUN IS A CENTRIFUGAL FORCE

THEN

$$G \frac{M_{\odot} M_{\oplus}}{R^2} = M_{\oplus} \frac{v^2}{R}$$

IF WE SOLVE FOR THE SUN'S MASS ( $M_{\odot}$ ) WE HAVE

$$M_{\odot} = \frac{Rv^2}{G}$$

SINCE THE ORBIT IS CIRCULAR,  $v = \frac{2\pi R}{T}$

$$\rightarrow M_{\odot} = \frac{R \left( \frac{2\pi R}{T} \right)^2}{G} = \frac{4\pi^2 R^3}{T^2 G} \quad \text{PLUGGING}$$

INS THE #'S

$$M_{\odot} = \frac{4\pi^2 (1.5 \times 10^8)^3}{(6.17 \times 10^{-11}) [(365)(24)(3600)]^2} \approx \boxed{2 \times 10^{30} \text{ kg}}$$

5) 6-29/



$$\sum \vec{F} = N - mg = ma$$

SPRING SCALE MEASURES  $N \rightarrow$

$$\rightarrow N = m(a + g)$$

$$(a) N = m(a + g) = (53 \text{ kg})(9.8 \text{ m/s}^2)$$

$$\boxed{N = 520 \text{ N}}$$

$$(b) N = m(a + g) = (53 \text{ kg})(0.33g + g) = (53 \text{ kg})(1.33 \cdot 9.8 \text{ m/s}^2)$$

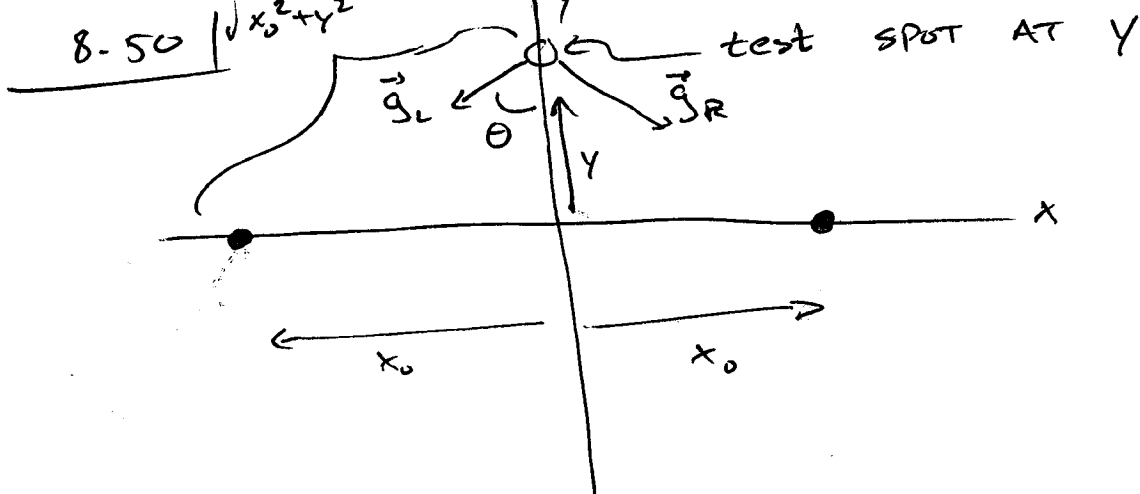
$$\boxed{N = 690 \text{ N}}$$

$$(c) N = m(-0.33g + g) = (53 \text{ kg})(0.67 \cdot 9.8 \text{ m/s}^2)$$

$$\boxed{N = 350 \text{ N}}$$

$$(d) N = m(-g + g) = 0$$

$$\boxed{N = 0}$$



IN GENERAL, FOR A THE FIELD FROM A SINGLE POINT MASS IS

$$\vec{g}(\vec{r}) = -G \frac{M}{r^2} \hat{r}$$

IN THIS CASE, THE X-COMPONENTS ARE EQUAL AND OPPOSITE, AND THUS CANCEL

THE Y-COMPONENTS ARE EQUAL AND IN THE SAME DIRECTION

$$g_y = 2 \left( -G \frac{M}{x_0^2 + y^2} \cos \theta \right) \quad \text{BUT} \quad \cos \theta = \frac{y}{\sqrt{x_0^2 + y^2}}$$

SO

$$g_y = -2G \frac{m y}{(x_0^2 + y^2)^{3/2}}$$

THAN

$$\vec{g} = -2G \frac{m y}{(x_0^2 + y^2)^{3/2}} \hat{j}$$

(b) AT MAXIMA (OR MINIMA)  $\frac{dg}{dy} = 0$

SO,  $0 = \frac{d}{dy} \left( 2Gm \frac{y}{(x_0^2 + y^2)^{3/2}} \right)$  { USE PRODUCT RULE }

$$0 = \frac{d}{dy} (2Gm y) \left( \frac{1}{(x_0^2 + y^2)^{3/2}} \right) + (2Gm y) \frac{d}{dy} \left( \frac{1}{(x_0^2 + y^2)^{3/2}} \right)$$

{ (USE CHAIN RULE!) }

$$0 = 2Gm \left( \frac{1}{(x_0^2 + y^2)^{3/2}} \right) + 2Gm y \left( \frac{3}{2} (x_0^2 + y^2)^{-5/2} \right) 2y$$

$$\rightarrow \boxed{y = \frac{x_0}{\sqrt{2}}}$$



11) 8-50 | (CONTINUED)

PLUG THIS INTO  $g$  TO GET  $g_{\max}$

$$g_{\max} = 2 G m \frac{x_0 / \sqrt{2}}{(x_0^2 + x_0^2 / 2)^{3/2}}$$

$$= \frac{2 G m x_0}{(2)^{3/2} (x_0^2 + x_0^2 / 2)^{3/2}}$$

NOW MULTIPLY  
BY  $\frac{2}{2}$

$$g_{\max} = \frac{4 G m x_0}{(2)^{3/2} (x_0^2 + x_0^2 / 2)^{3/2}} = \frac{4 G m x_0}{(2x_0^2 + x_0^2)^{3/2}}$$

$$g_{\max} = \frac{4 G m x_0}{(3)^{3/2} x_0^3} = \boxed{0.77 \frac{G m}{x_0^2}}$$

9-13 | THERE IS NOT NET EXTERNAL FORCE  
SO MOMENTUM IS CONSERVED

$$P_f = m_b v_b + m_p v_p$$

$$P_i = 0$$

so  $m_b v_b + m_p v_p = 0$

$$\rightarrow m_b v_b = -m_p v_p$$

$$m_b = 59 \text{ kg}$$

$$m_p = 5.7 \text{ kg}$$

$$v_p = 10 \text{ m/s}$$

$$v_b = \frac{-(5.7 \text{ kg})(10 \text{ m/s})}{59 \text{ kg}}$$

$$v_b = -0.97 \text{ m/s}$$

$\uparrow$   $v_b$  IN OPPOSITE DIRECTION  
OF  $v_p$

9-27

RAIN IS FALLING AT A RATE OF

$$5 \text{ cm/hr} \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) = 1.39 \times 10^{-5} \frac{\text{m}}{\text{s}}$$

RATE DEPTH IS  
ADDED, NOT A VELOCITY!

THEN THE VOLUME COLLECTED EVERY SECOND

$$\text{IS } (1.39 \times 10^{-5} \frac{\text{m}}{\text{s}}) (1 \text{ m}^2) = 1.39 \times 10^{-5} \text{ m}^3/\text{s}$$

THEN THE MASS ADDED PER SECOND IS

$$m = (1.39 \times 10^{-5} \text{ m}^3/\text{s}) (1 \times 10^3 \frac{\text{kg}}{\text{m}^3}) = 1.39 \times 10^{-2} \text{ kg/s}$$

NOW, WE CAN FIND THE MOMENTUM  
OF THIS WATER THAT IS LOST EVERY SECOND

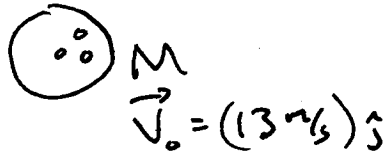
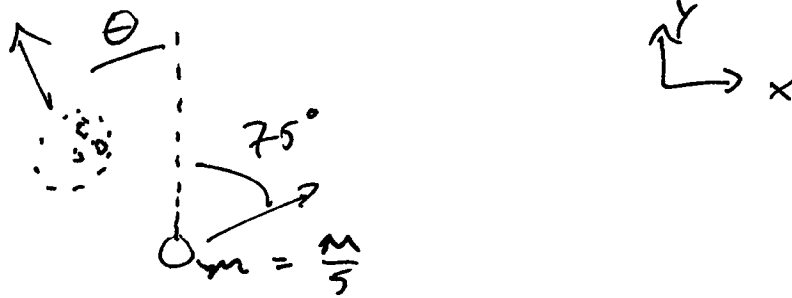
$$\Delta p = (1.39 \times 10^{-2} \text{ kg}) (8.0 \text{ m/s})$$

$$\Delta p = 0.11 \frac{\text{kg m}}{\text{s}}$$

THE AVERAGE FORCE IS THEN

$$\frac{\Delta p}{\Delta t} = \frac{0.11 \text{ kg m/s}}{1 \text{ s}} = 0.11 \frac{\text{kg m}}{\text{s}^2} = \boxed{0.11 \text{ N}}$$

9-88



PROBLEM STATES THAT THIS COLLISION IS ELASTIC

SO WE CAN CONSERVE ENERGY AND MOMENTUM

$$E_i = \frac{1}{2} M v_0^2$$

$$E_f = \frac{1}{2} M |v_{bf}|^2 + \frac{1}{2} \left(\frac{M}{5}\right) |v_{pf}|^2$$

~~$P_i = M v_0$~~

$$\vec{P}_i = M \vec{v}_0$$

$$\vec{P}_f = M \vec{v}_{bf} + \frac{M}{5} \vec{v}_{pf}$$

BREAK THESE INTO X, Y COMPONENTS

$$P_{ix} = 0$$

$$P_{fx} = M v_{bfx} + \frac{M}{5} v_{pfx} \quad \leftarrow \begin{matrix} v_{bf} \sin \theta \\ v_{pf} \sin 75^\circ \end{matrix}$$

~~$P_{iy}$~~   $P_{iy} = M v_0$

$$P_{fy} = M v_{bfy} + \frac{M}{5} v_{pfy} \quad \leftarrow \begin{matrix} v_{bf} \cos \theta \\ v_{pf} \cos 75^\circ \end{matrix}$$





9-88 (CONT)

SO WE HAVE:

$$\left\{ \begin{aligned} \frac{1}{2} M V_0^2 &= \frac{1}{2} M V_{bf}^2 + \frac{1}{2} \left(\frac{M}{5}\right) V_{Pf}^2 & (1) \\ 0 &= -M V_{bf} \sin \theta + \frac{M}{5} V_{Pf} \sin 75^\circ & (2) \\ M V_0 &= M V_{bf} \cos \theta + \frac{M}{5} V_{Pf} \cos 75^\circ & (3) \end{aligned} \right.$$

3 EQUATIONS WITH 3 UNKNOWN (V<sub>Pf</sub>, V<sub>bf</sub>, θ)

A) FIND V<sub>Pf</sub>:

FROM (2)  $M V_{bf} \sin \theta = \frac{M}{5} V_{Pf} \sin 75^\circ$

SO  $V_{bf} \sin \theta = \frac{1}{5} V_{Pf} \sin 75^\circ$   ~~$5 V_{bf} \sin \theta = V_{Pf} \sin 75^\circ$~~

FROM (1)  $V_0^2 = V_{bf}^2 + \frac{1}{5} V_{Pf}^2 \rightarrow 5 V_{bf}^2 = 5 V_0^2 - V_{Pf}^2$  (\*)

FROM (3)  $5 V_0 = 5 V_{bf} \cos \theta + V_{Pf} \cos 75^\circ$

$\hookrightarrow 5 V_{bf} \cos \theta = 5 V_0 - V_{Pf} \cos 75^\circ$

SO WE HAVE

$$5 V_{bf} \sin \theta = V_{Pf} \sin 75^\circ$$

$$5 V_{bf} \cos \theta = 5 V_0 - V_{Pf} \cos 75^\circ$$

SQUARE BOTH TO GET

$$25 V_{bf}^2 \sin^2 \theta = V_{Pf}^2 \sin^2 75^\circ$$

$$25 V_{bf}^2 \cos^2 \theta = (5 V_0 - V_{Pf} \cos 75^\circ)^2 = 25 V_0^2 + V_{Pf}^2 \cos^2 75^\circ - 10 V_0 V_{Pf} \cos 75^\circ$$

AND ADD THEM (REMEMBER  $\sin^2 \theta + \cos^2 \theta = 1$ )  $\rightarrow$

9-88 (CONT)

$$25V_{bf}^2 = 25V_0^2 + V_{Pf}^2 - 10V_0V_{Pf}\cos 75^\circ$$

$\rightarrow$  PLUG IN  $(\star)$  FROM PREVIOUS PAGE ...

$$5(5V_0^2 - V_{Pf}^2) = 25V_0^2 + V_{Pf}^2 - 10V_0V_{Pf}\cos 75^\circ$$

$$25V_0^2 - 5V_{Pf}^2 = 25V_0^2 + V_{Pf}^2 - 10V_0V_{Pf}\cos 75^\circ$$

REARRANGE TO GET

$$-6V_{Pf}^2 = 10V_0V_{Pf}\cos 75^\circ$$

$$V_{Pf} = \frac{10V_0\cos 75^\circ}{6}$$

$$V_{Pf} = \frac{10}{6}V_0\cos 75^\circ = 5.6 \text{ m/s} = V_{Pf}$$

B) NOW THAT WE KNOW  $|V_{Pf}|$ , WE CAN USE  $(\star)$  TO GET  $V_{bf}$

$$V_{bf}^2 = V_0^2 - \frac{1}{5}V_{Pf}^2 = (13)^2 - \frac{1}{5}(5.6)^2$$

$$V_{bf} = 12.8 \text{ m/s}$$

C) SINCE  $V_{Pf}\sin 75^\circ = V_{bf}\sin \theta$

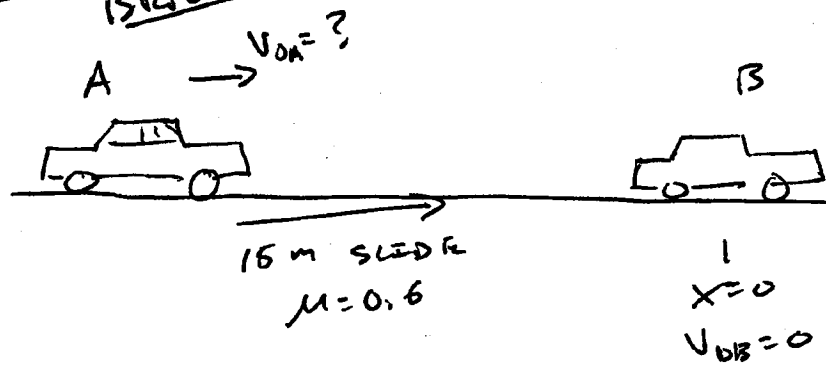
$$\theta = \arcsin\left(\frac{V_{Pf}\sin 75^\circ}{V_{bf}}\right) = 4.9^\circ = \theta$$

9.95/

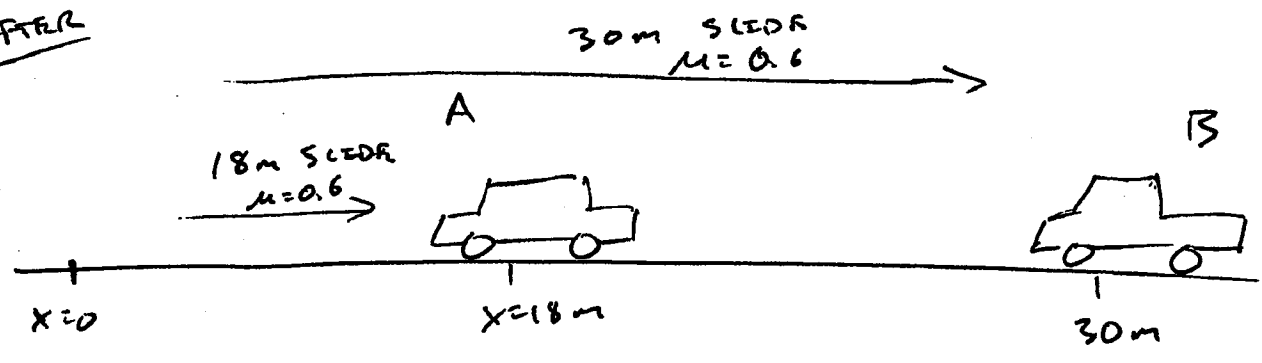
BEFORE

$$m_A = 1500 \text{ kg}$$

$$m_B = 1100 \text{ kg}$$



AFTER



WE NEED TO FIND  $v_{OA}$  = VELOCITY OF A BEFORE HE APPLIES THE BRAKES

FIRST WE CAN FIND IT IN TERMS OF THE VELOCITY OF A JUST BEFORE THE COLLISION.

$$\Delta K.E. = \left[ \frac{1}{2} m_A v_{fA}^2 - \frac{1}{2} m_A v_{oA}^2 = -\mu m_A g d \right] \quad (1)$$

WE ALSO KNOW THAT MOMENTUM IS CONSERVED DURING THE COLLISION. SO

$$m_A v_{fA} = m_A v_A + m_B v_B \quad (2) \quad (v_A, v_B \text{ ARE VELOCITIES RIGHT AFTER THE COLLISION})$$



9.95 | (CONTINUED)

AND AFTER THE COLLISION WE KNOW THAT

$$-m m_A g d_A = -\frac{1}{2} m_A v_A^2 \quad (3)$$

$$-m m_B g d_B = -\frac{1}{2} m_B v_B^2 \quad (4)$$

WE NOW NEED TO USE (1), (2), (3), (4) TO FIGURE OUT  $v_{A0}$ .

FROM (3) AND (4) WE KNOW

$$v_A = \sqrt{2m_A g d_A} = 14.7 \text{ m/s}$$

$$v_B = \sqrt{2m_B g d_B} = 18.9 \text{ m/s}$$

NOW, USING THESE IN (2)

$$m_A v_{A\text{f}} = m_B (18.9 \text{ m/s}) + m_A (14.7 \text{ m/s})$$

$$v_{A\text{f}} = 28.6 \text{ m/s}$$

NOW WE CAN USE THIS IN (1)

$$v_{A0} = \left( \frac{m_B}{m_A} v_{A\text{f}}^2 + 2m g d \right)^{1/2}$$

$$v_{A0} = 31.6 \text{ m/s} = 114 \text{ km/hr} > 90 \text{ km/hr}$$