Physics 113 - Solutions to problem set 9

10-29

$\tau = 75 \text{ N}\cdot\text{m} = rF_\perp$

IF $r = 0.28$ THEN

$$F_\perp = \frac{75 \text{ N}\cdot\text{m}}{0.28 \text{ m}} = \boxed{270 \text{ N} = F_\perp}$$

AT THE BOLT $r = \frac{15 \text{ mm}}{2} = 0.0075 \text{ m}$

SO $F_\perp = \frac{\tau}{r} = \frac{75 \text{ N}\cdot\text{m}}{0.0075 \text{ m}} = \boxed{10,000 \text{ N} = F_\perp}$

THIS FORCE IS DISTRIBUTED ON 6 POINTS, SO THE FORCE ON EACH POINT IS

$$\frac{10,000 \text{ N}}{6} = \boxed{1700 \text{ N}}$$

10-39 (a) $\alpha = \frac{a}{r} = \frac{\left(\frac{8.5 \text{ m/s}}{0.35 \text{ s}}\right)}{0.31 \text{ m}} = 78 \text{ rad/s}^2$

(b) $I_{ARM} = \frac{1}{3} M_{arm} \ell^2$ $I_{BALL} = m_{BALL} \ell^2$

$$\Sigma \tau = \tau_{ARM} = I\alpha = \left(\frac{1}{3} M_{ARM} \ell^2 + m_{BALL} \ell^2\right)\alpha$$

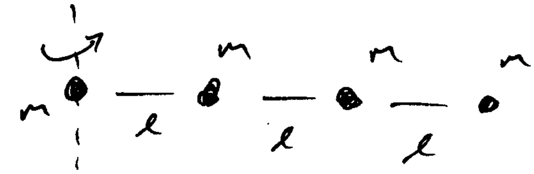
$m_{ARM} = 3.7 \text{ kg}$, $m_{BALL} = 1 \text{ kg}$, $\ell = 0.31 \text{ m}$

$\tau = 16.7 \text{ N}\cdot\text{m}$

THEN THE FORCE FROM THE TRICEPS MUSCLE IS

$$F = \frac{\tau}{r} = \frac{16.7 \text{ N}\cdot\text{m}}{0.025 \text{ m}} = \boxed{670 \text{ N} = F}$$

10-45



(a) $I = \sum_i m_i r_i^2 = m(l)^2 + m(2l)^2 + m(3l)^2 = \boxed{14ml^2}$

(b) $\tau = rF_\perp = (3l)F_\perp = (14ml^2)\alpha$

$$\boxed{F_\perp = \left(\frac{14}{3} ml\right)\alpha}$$

(c) THE FORCE REQUIRED IS MINIMIZED WHEN IT IS PERPENDICULAR TO THE ROD

10-58

(a) $I_{pm} = MR_0^2$

(b) $I_{cm} = \frac{2}{5}Mr_1^2$

THEN, BY P.A.T $I_{AB} = \frac{2}{5}Mr_1^2 + MR_0^2$

(c) % DIFFERENCE $= \dfrac{I_{AB} - I_{pm}}{I_{AB}}$

$$= \frac{\frac{2}{5}mr_1^2 + mR_0^2 - mR_0^2}{\frac{2}{5}mr_1^2 + mR_0^2} = \frac{\frac{2}{5}mr_1^2}{\frac{2}{5}mr_1^2 + mR_0^2}$$

$$= \frac{\frac{2}{5}r_1^2}{\frac{2}{5}r_1^2 + R_0^2} = \boxed{0.3\%}$$

10-62

$$W_{ROT} = \int \tau \cdot d\theta \quad \text{OR} \quad \tau\Delta\theta$$

AND POWER $\rightarrow$ $P = \frac{W}{\Delta t}$

SO $P = \frac{\tau\Delta\theta}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t}$

AT 3750 rpm $\frac{\Delta\theta}{\Delta t} = \frac{(3750 \text{ rev})}{(60 \text{ sec})}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 392 \text{ rad/s}$

$P = (255 \text{ N}\cdot\text{m})(392 \text{ rad/s}) = \boxed{99{,}960 \text{ W}}$

BUT 1 H.P. = 746W

SO $P = \frac{99{,}960}{746} = \boxed{134 \text{ H.P}}$

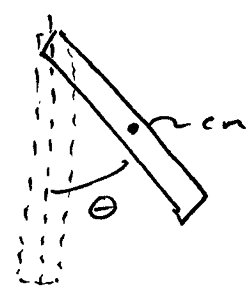
10-66 WE CAN USE CONSERVATION OF ENERGY.

$E_i = mgh_{cm}$

$E_f = \frac{1}{2}I\omega^2$

THE ~~GRAV~~ CENTER OF MASS ~~STARTS~~ ENDS AT $h_{cm_f} = \frac{L}{2}$ ~~cos θ~~. INITIALLY IT IS AT A HEIGHT

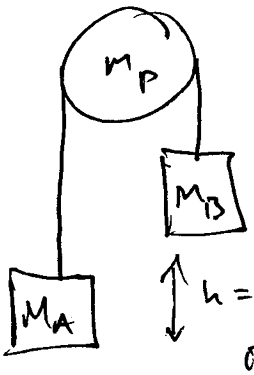
$h_{cm} = \frac{L}{2}(1-\cos\theta)$



SO $E_i = E_f \rightarrow mg\frac{L}{2}(1-\cos\theta) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$

$\boxed{\omega = \left(\frac{3g}{L}(1-\cos\theta)\right)^{1/2}}$ AND $\boxed{V = \omega L = \left(3gL(1-\cos\theta)\right)^{1/2}}$

10-67



$M_A = 35\text{ kg}$

$M_B = 38\text{ kg}$

$M_p = 3.1\text{ kg}$

$R_p = 0.381\text{ m}$

$I_p = \frac{1}{2} M_p R_p^2$

$h = 2.5\text{ m}$

USE CONSERVATION OF ENERGY

$E_i = m_B g h$

$$E_f = \frac{1}{2}(m_B + m_A) v_f^2 + \frac{1}{2}\left(\frac{1}{2} m_p \underbrace{R_p^2)\omega_f^2}_{v_f^2}\right.$$

So

$$m_B g h = \frac{1}{2}\left(M_A + m_B + \frac{1}{2} m_p\right) v_f^2$$

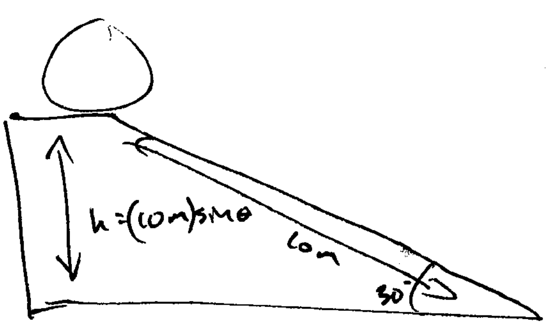
$$v_f = \left(\frac{4 m_B g h}{2M_A + 2m_B + m_p}\right)^{1/2} = \boxed{1.4\text{ m/s}}$$

10-73

A) $E_i = mgh$

$$E_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2$$

$$I_{\text{SPHERE}} = \frac{2}{5} m R^2$$



So, $mgh = \frac{1}{2} m v_f^2 + \frac{1}{2}\left(\frac{2}{5} m R^2\right)\omega_f^2$

$$= \frac{1}{2} m v_f^2 + \frac{1}{5} m v_f^2 = \frac{7}{10} m v_f^2$$

$$v_f = \left(\frac{10}{7} g h\right)^{1/2} = \boxed{8.4\text{ m/s}}$$

$$\omega_f = \frac{v_f}{R} = \boxed{34.5\text{ rad/s}}$$

B) $KE_T = \frac{1}{2} m v_f^2$

$KE_R = \frac{1}{5} m v_f^2$ so $\boxed{\frac{KE_T}{KE_R} = \frac{5}{2}}$

C) NEITHER DEPENDS ON MASS, ω_f DEPENDS ON RADIUS

10-75

$E_i = mgh = mgR_0$

$E_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$ $\qquad I = \frac{2}{5}mr_0^2$

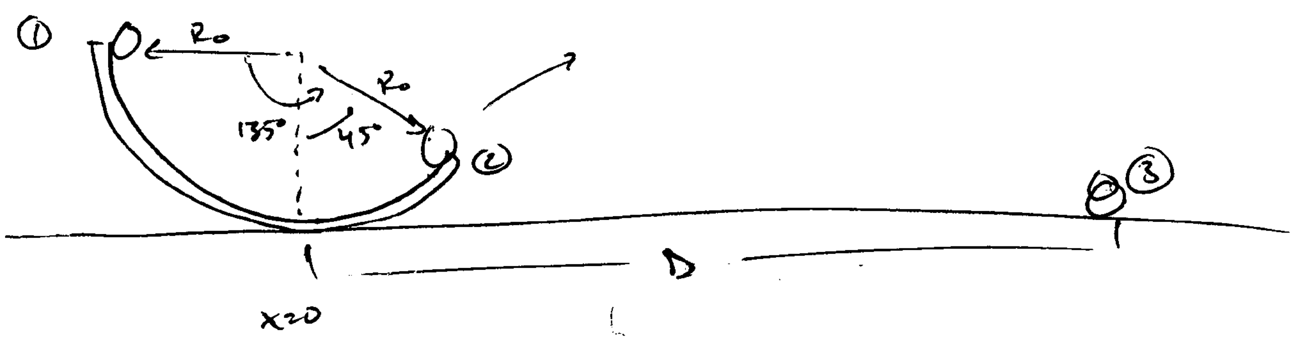
so

$mgR_0 = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mr_0^2\right)\omega_f^2$

$gR_0 = \frac{1}{2}v_f^2 + \frac{1}{5}v_f^2 = \frac{7}{10}v_f^2$

$$\boxed{v_f = \sqrt{\frac{10}{7}gR_0}}$$

10-81



A) WHAT IS ITS SPEED?

CONSERVE ENERGY.

$E_{①} = E_{②}$

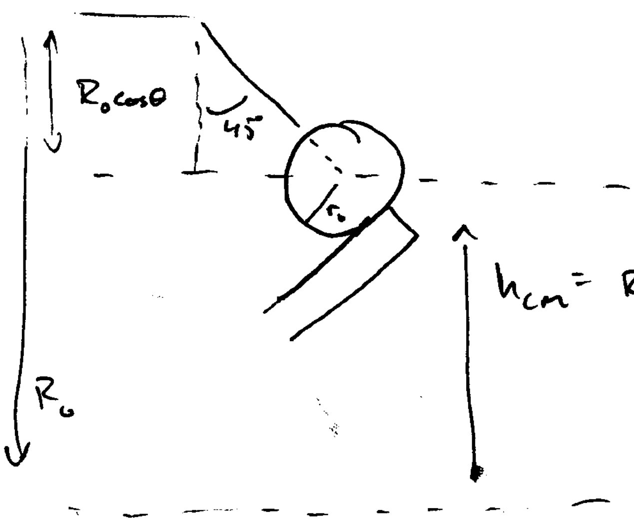
INITIAL ENERGY

ENERGY AS IT LEAVES RAMP

$E_{①} = mgR_0$

$E_2 = mg(R_0 - (R_0 - r_0)\cos\theta) + \frac{1}{2}mv_{②}^2 + \frac{1}{2}I\omega_{②}^2$

$I = \frac{2}{5}mr_0^2, \quad \omega_{②}^2 = \frac{v_{②}^2}{r_0^2}$



$h_{cm} = R_0 - R_0\cos\theta + r_0\cos\theta$

$= R_0 - (R_0 - r_0)\cos\theta$

SO ...

$mgR_0 = mgR_0 - mgR_0\cos\theta + mgr_0\cos\theta + \frac{1}{2}mv_{②}^2 + \frac{1}{5}m\frac{v_{②}^2}{r_0^2}$

PLUG IN KNOWNS AND SOLVE FOR

$v_{②}^2 \rightarrow |\vec{v}_{②}| = \sqrt{v_{②}^2}$

$\rightarrow \boxed{|\vec{V}_{②}| = 1.6\ m/s}$

10-81 (CONT)

B) BALL MOVES LIKE A PROJECTILE.

THE END OF THE RAMP IS AT

$x_0 = R_0 \sin 45^\circ - r_0 \sin 45^\circ$ AND

$y_0 = R_0 - R_0 \cos 45^\circ + r_0 \cos 45^\circ$

AND $V_{0x} = V_{(2)} \cos 45^\circ$, $V_{0y} = V_{(2)} \sin 45^\circ$

$a_x = 0$, $a_y = -g$

USE $y = y_0 + V_{0y} t + \frac{a_y}{2} t^2$

$$0 = R_0 - R_0 \cos 45^\circ + r_0 \cos 45^\circ + V_{(2)} \sin 45^\circ \, t - \tfrac{1}{2}(9.8 \, m/s^2) t^2$$

SOLVE QUADRATIC FOR t

$\rightarrow \boxed{t = 0.277 \, s}$, OR $\cancel{-0.0528 \, s}$

NEGATIVE TIME DOESN'T MAKE SENSE

SO IN THIS TIME

$x_f = x_0 + V_{0x} t + \cancel{\tfrac{1}{2} a_x t^2}$

$\rightarrow x_f = R_0 \sin 45^\circ - r_0 \sin 45^\circ + V_{(2)} \cos 45^\circ \, t$

$\boxed{x_f = 0.48 \, m}$

10-94

(a) FOR $r << R$

AT THE TOP OF THE LOOP WE HAVE

$\Sigma F = mg + N = m\frac{V^2}{R}$, BUT WE WANT N TO VANISH;

SO $g = \frac{V^2}{R} \to V = \sqrt{gR}$

SO NOW WE CAN USE ENERGY CONSERVATION

$E_i = mgh$, $E_f = \frac{1}{2}mV_f^2 + \frac{1}{2}I\omega_f^2 + mg2R$

$I = \frac{2}{5}mr^2$

$mgh = \frac{1}{2}mV_f^2 + \frac{1}{5}mV_f^2 + mg2R$

$gh = V_f^2\left(\frac{1}{2} + \frac{1}{5}\right) + 2gR$ BUT $gR = V_f^2$

SO $h = \frac{gR}{g}\left(\frac{7}{10} + \frac{2gR}{g}\right) = \frac{27}{10}R$

$\boxed{h = 2.7R}$

11-3

(a) HIS MOMENT OF INERTIA INCREASES

$\rightarrow L_1 = L_2 \rightarrow I_1\omega_1 = I_2\omega_2 \rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$

SO $\omega_2 < \omega_1$ SINCE $I_2 > I_1$

(b) $I_1\omega_1 = I_2\omega_2 \rightarrow \frac{I_2}{I_1} = \frac{\omega_1}{\omega_2} = \frac{0.9 \text{ rev/s}}{0.7 \text{ rev/s}} = \boxed{1.3}$

11-10

AGAIN $I_i\omega_i = I_f\omega_f \rightarrow \omega_f = \frac{I_i}{I_f}\omega_i$

$I_i = \frac{1}{2}mR^2$, $\omega_i = 3.7 \text{ rev/s}$

$I_f = \frac{1}{2}mR^2 + \frac{1}{12}m(2R)^2 = \frac{5}{6}mR^2$

SO $\omega_f = \frac{\frac{1}{2}mR^2}{\frac{5}{6}mR^2}\omega_i = \frac{3}{5}\omega_i = \boxed{2.2 \frac{\text{rev}}{\text{s}} = \omega_f}$

11-111 AGAIN, CONSERVE ANGULAR MOMENTUM.

A) $I_i = \frac{1}{2} M R^2$ $\qquad \omega_i = 0.95 \text{ rad/s}$

$$I_f = \underbrace{\frac{1}{2} M R^2}_{\text{MERRY-GO-ROUND}} + \underbrace{m R^2}_{\text{PERSON}}$$

$$\omega_f = \frac{\frac{1}{2} M R^2}{\frac{1}{2} M R^2 + m R^2} (0.95 \text{ rad/s})$$

$$\boxed{\omega_f = 0.55 \text{ rad/s}}$$

B) $KE_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{4} M R^2 \omega_i^2 = \boxed{420 \text{ J}}$

$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}\left(\frac{1}{2} M R^2 + m R^2\right) \omega_f^2 = \boxed{240 \text{ J}}$