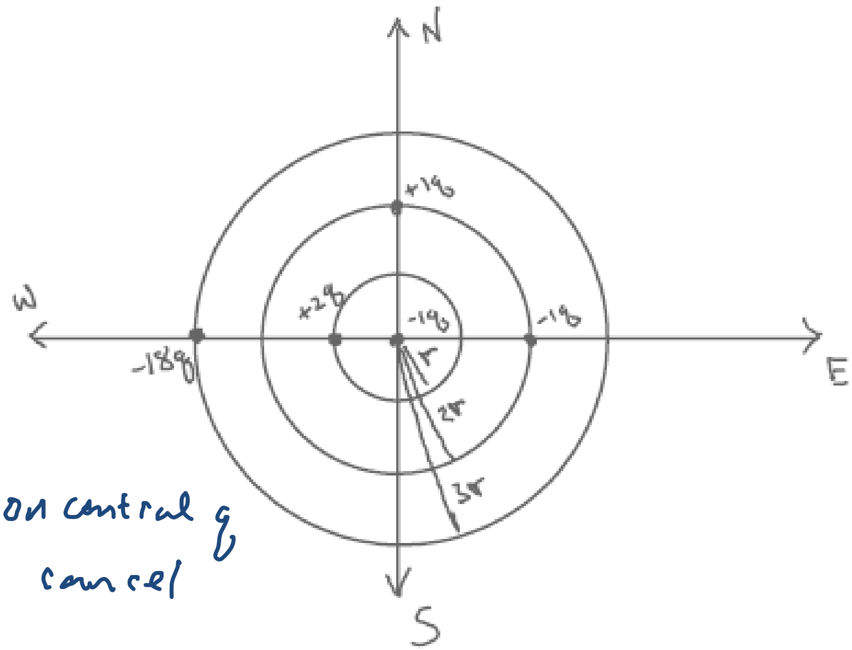
P114 University of Rochester NAME Solution key – SM
S. Manly Spring 2015

Exam 1 (February 12, 2015)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show your work unless otherwise indicated.

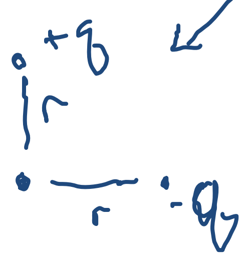
Problem 1 (10 pts, show work):
Consider the situation sketched to the side. An electric charge –q is situated at the origin of the 2-dimensional system. Four other charges are shown at distances of 1r, 2r or 3r from the origin. The charge sign and magnitude and positions of those four charges are shown in the sketch. Determine the net force on the charge at the origin. Give your answer in terms of q, r and k(or ε_o).



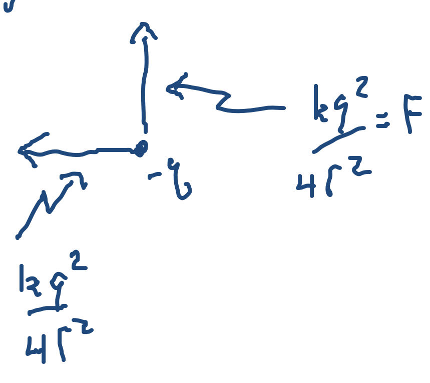
The force contributions on central q from the −18q and +2q cancel

$$\frac{k(+2q)(-1q)}{r^2} + k\frac{(-18q)(-1q)}{(3r)^2} = 0$$

This leaves only contributions from



this gives



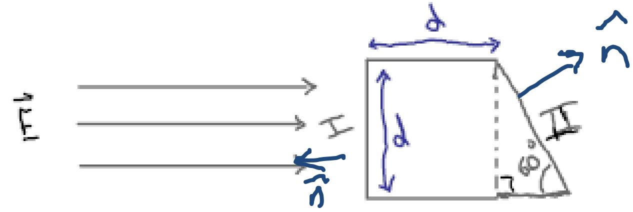
$$F_{NET} = \sqrt{2\left[\frac{kq^2}{4r^2}\right]^2} = \frac{kq^2}{4r^2}$$

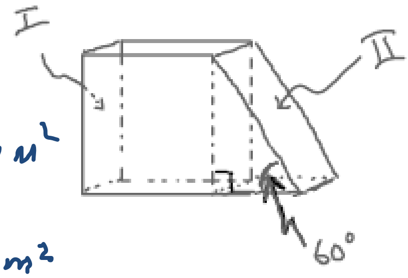
direction is Northwest

$$\sqrt{\frac{2}{16}} = \sqrt{\frac{1}{4\cdot 4}} = \frac{1}{4}$$

Problem 2 (15 points, show work):

Consider a cubic (nonconducting) surface with sides of length d where one of the sides has been pushed out (with a corresponding increase in the area of that side) to make a 60 degree angle with the plane of one of the other sides as shown in the sketch. A constant electric field of magnitude E=4 N/C is incident at a normal angle on the side designated as "I" on the sketch. Let d=2 meters.





a) What is the electric flux through surface I?

area of I $= (2\,m)^2 = 4\,m^2$ $\quad \vec{E}\cdot\hat{n} = -1$

$\Phi_I = -4\,\frac{N}{C}\,4\,m^2$

$= -16\,\frac{Nm^2}{C}$

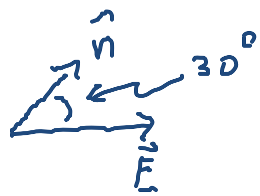
b) What is the electric flux through surface II?

Area of II

$\sin 60° = \frac{2}{d'}$

$d' = \frac{2}{\sin 60°} = 2.3$

Area II $= d\,d' = 4.61\ m^2$



$\Phi_{II} = |\vec{E}| \cos 30$ Area$_{II}$

$= 4 \cos 30\ 4.61 = +16\ \frac{Nm^2}{C}$

c) What is the total electric flux through the surface shown in the figure?

closed surface

Gauss tells us $\quad \Phi_{Total} = \frac{Q_{enclosed}}{\epsilon_0}$

There is no enclosed charge.

So total flux through surface is zero.

Problem 3 (10 pts, short answer/essay):

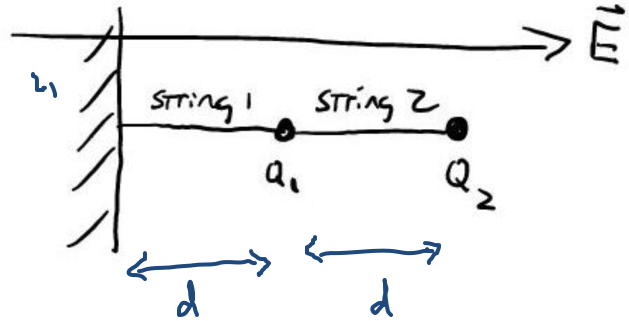
Biff Hamilton takes on a night job at the Suds R Us brewpub and laundromat. He was very excited about being a bartender but he made the unfortunate error of chatting up the owner's daughter and now Biff finds himself working on the laundry side of the business, folding the clothes that come out of the dryers. While at work one day, feeling sorry for himself, Biff notices that when he takes clothes out of the dryer most of the items seem to stick together. Interestingly, he observes that two identical socks coming out of a dryer load repel each other. Briefly explain the physics behind Biff's observation.

The attraction comes about because charge is exchanged when dissimilar materials are rubbed together due to differing "electronegativities" (Tendency for material to attract/hold onto electrons). Each of the two socks will become charged when rubbing other materials. Since they are identical they will each tend to have the same sign of electric charge and repel one another.

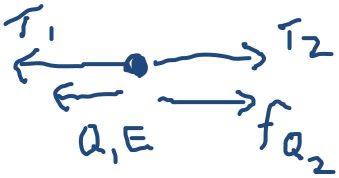
Problem 4 (20 pts, show work):

μC $\quad 8\times10^{4}$ N/C $\quad$ μC

Way out in interstellar space, far from any planets or stars, a charge, Q_1= -2C and mass M_1=0.1 kg, is connected by massless strings, each of length 0.5 m, to a wall and a second charge Q2 = +10C. A constant electric field of magnitude ~~8~~ N/C permeates the region in a direction that is perpendicular to the wall as shown in the sketch. The charges are in equilibrium, i.e. in a static (nonmoving) situation. What is the tension in the two strings?



FBD of Q_1



$$f_{Q_2} = \frac{kQ_1Q_2}{d^2}$$

$$\Sigma F_{Q_1} = 0 = T_2 - T_1 - Q_1E + f_{Q_2}$$

$$0 = T_2 - T_1 - Q_1E + \frac{kQ_1Q_2}{d^2} \qquad \text{I}$$

FBD of Q_2 $\quad f_{Q_1} = f_{Q_2}$

$$\Sigma F_{Q_2}: 0 = Q_2E - T_2 - f_{Q_2} = Q_2E - T_2 - \frac{kQ_1Q_2}{d^2}$$

$$T_2 = Q_2E - \frac{kQ_1Q_2}{d^2}$$

$$T_2 = (8\times10^{4})(10\times10^{-6}) - \frac{(9\times10^{9})(2\times10^{-6})(10\times10^{-6})}{0.5^2}$$

80×10^{-2}

0.8

$180\times10^{-3}\times4 = 720\times10^{-3}$

$= .72$

$$T_2 = 0.08\ \text{N}$$

from I

$$T_1 = T_2 - Q_1E + 0.72$$

$$T_1 = 0.08 - (2\times10^{-6})8\times10^{4} + .72$$

$$T_1 = 0.08 - (16\times10^{-2}) + .72$$

$$T_1 = 0.08 - .16 + .72$$

$$T_1 = 0.64\ \text{N}$$

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Problem 5 (20 points, no need to show work):

A very small, charged, metal sphere is placed inside a thin conducting spherical shell of radius B without touching it. Two Gaussian spheres of radius A and C are used to find the net electric flux inside and outside of the shell. The situation is illustrated in the sketch below.

a) Suppose the electric flux through the two Gaussian surfaces is the same. What is the charge on the inner and outer surfaces of the conducting shell?

inner surface $-Q$ (so $E=0$ in conductor)

outer surface $+Q$ (otherwise net flux thru outer Gaussian surface is zero)

b) Suppose the electric flux through the outer Gaussian surface is three times that through the inner Gaussian surface. What is the charge on the inner and outer surfaces of the conducting shell?

inner surface $-Q$ outer surface $+3Q$

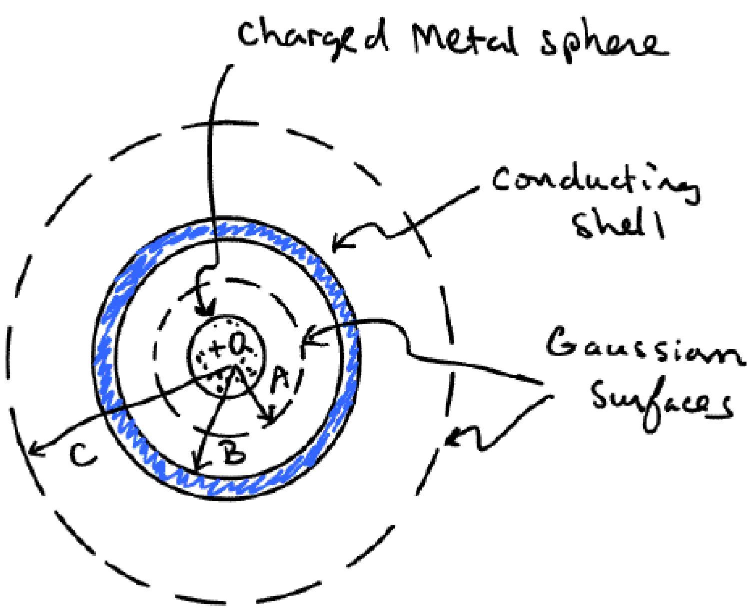
c) If the electric flux through the outer Gaussian surface is zero, what is the electric flux through the inner Gaussian surface?

$+Q/\varepsilon_0$

d) If the electric flux through the outer Gaussian surface is zero, what is the charge on the inner and outer surfaces of the conducting shell?

$-Q$ on inner

No charge on outer



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Problem 6 (25 points, show work):
An infinite, straight, non-conducting cylindrical cable of radius R carries a volume charge density $\rho(r)=A\sqrt{r}$ for r<R, where A is a constant, and ρ=0 for r>R.

(a) Determine the total charge per unit length along the cable in terms of the constant A.

$$\frac{Q}{L} = \frac{1}{L}\int \rho\, dv = \frac{1}{L}\int_0^R A\sqrt{r}\, 2\pi r L\, dr$$

$$\frac{Q}{L} = A2\pi\int_0^R r^{3/2}\, dr = 2A\pi \frac{R^{5/2}}{5/2} = \boxed{\frac{A4\pi R^{5/2}}{5}}$$

check units

$$\frac{C}{m^3} = A\, m^{1/2}$$

$$A \sim \frac{C}{m^{7/2}} \checkmark$$

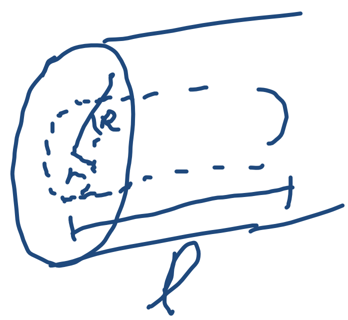
1)	/10
2)	/15
3)	/10
4)	/20
5)	/20
6)	/25
tot	/100

(b) Find the electric field in all space as a function of r.

for r<R

choose Gaussian surface w/ r<R length l

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$



$$|\vec{E}|2\pi r l = \frac{1}{\epsilon_0}\int_V \rho\, dv$$

$$|\vec{E}|2\pi r l = \frac{1}{\epsilon_0}\int_0^r A r^{1/2}\, 2\pi r l\, dr = \frac{2\pi l A}{\epsilon_0}\int_0^r r^{3/2}\, dr$$

$$|\vec{E}|2\pi r l = \frac{2\pi l A}{\epsilon_0}\frac{2}{5} r^{5/2}$$

$$|\vec{E}|_{r<R} = \frac{A}{\epsilon_0}\frac{2}{5} r^{3/2}$$ in $\hat{r}$ direction

For r>R

$$\oint E\cdot dA = \frac{Q_{encl}}{\epsilon_0}$$ For Gaussian surface w/ r>R

$$|\vec{E}|2\pi r l = \frac{Q}{L}\frac{l}{\epsilon_0}$$

$$|\vec{E}|_{r>R} = \frac{A4\pi R^{5/2}}{5\, 2\pi r\, \epsilon_0} = A\frac{2}{5\epsilon_0}\frac{R^{5/2}}{r}$$

in $\hat{r}$ direction