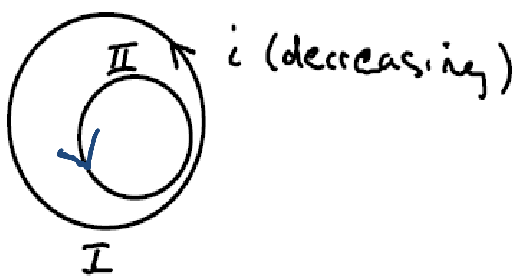
P114 University of Rochester NAME Soln key – SM
S. Manly Spring 2015

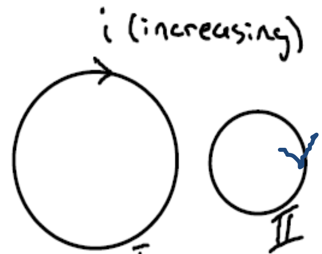
Exam 3 (April 16, 2015)

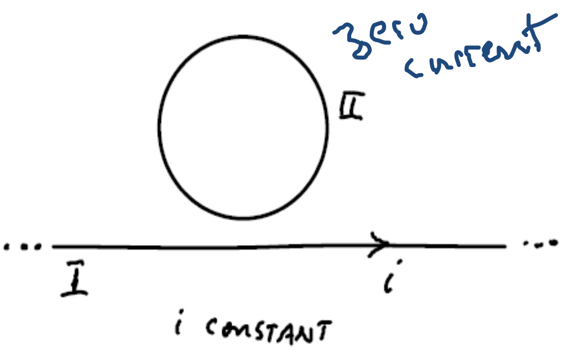
Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show your work unless otherwise indicated.

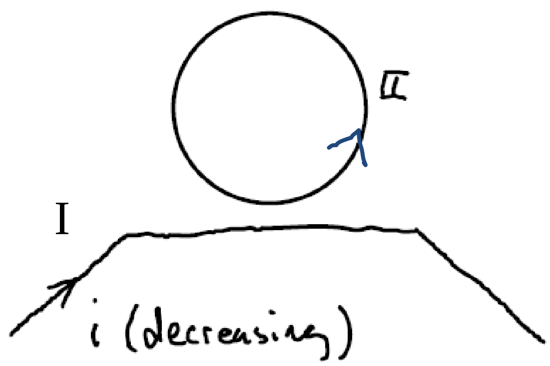
<u>Problem 1 (15 pts, 3 points per part):</u>

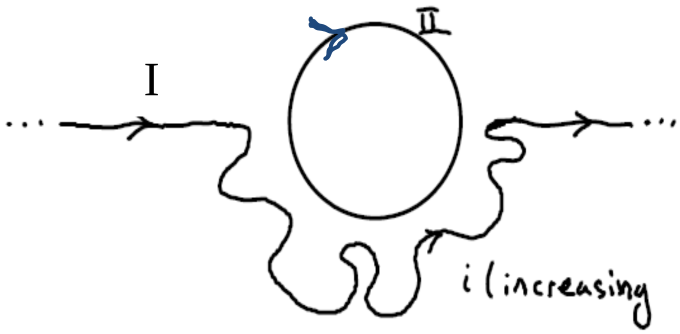
In each of the sketches below is a loop or wire (labeled "I") that carries a current as specified in each drawing. The current is increasing or decreasing or constant in time as specified. Determine the direction of the induced current in the loop labeled "II". On each loop II, indicate clearly the direction of the induced current. If there is no induced current, write "zero current" on loop II.











Problem 2 (8 pts, 4 points per part):
Consider the three currents shown in the sketch below. I1 is a current of 3 A going into the paper. Also shown are four curves (a,b,c,d) in the plane of the paper.

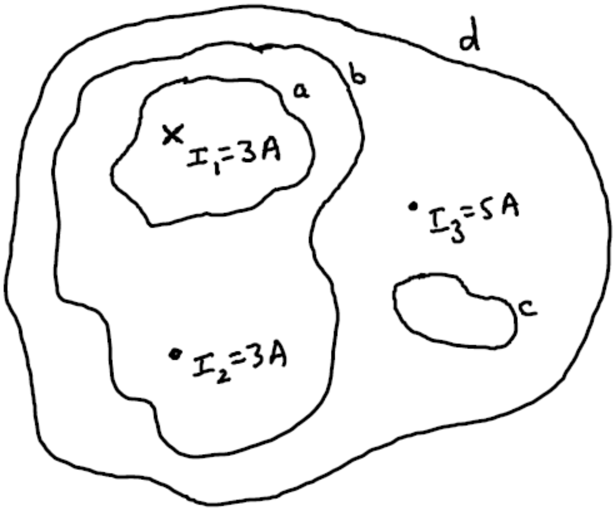
a) Which curve(s) has the largest value of

$$\oint \vec{B} \cdot \vec{dl}$$

$\boxed{d}$ $\oint B \cdot dl = 5\mu_0$

b) Which curve(s) has the smallest value of

$$\oint \vec{B} \cdot \vec{dl}$$



$\boxed{c, b}$

both have $\oint \vec{B} \cdot \vec{dl} = 0$

1)	/15
2)	/8
3)	/10
4)	/8
5)	/12
6)	/12
7)	/10
8)	/12
9)	/13
tot	/100

Problem 3 (10 pts):
After graduation, you get a dream job at Al's Suds, which is a combination laudromat/bar/car wash. One day Al comes looking for you since he knows you have a deep knowledge of basic physics. Al says that one of the motors in his laundry is acting up whenever it gets hot. He says, "I'm pretty sure it happens because the magnetic permeability of one of the paramagnetic parts changes when the motor gets warm; but I don't know how it changes." Briefly explain to Al whether you would expect the magnetic permeability of a paramagnetic material to increase or decrease with rising temperature and why.

Paramagnetism arises when dipoles in the material line up with an external B field. Thermal agitation can reduced the degree of alignment. As the material heats up, thermal motion increases and this knocks a larger proportion of dipoles out of alignment with the field ... reducing the size of the paramagnetic contribution to $\vec{B}$.

Problem 4 (8 pts, 4 points per part):
During normal beating, the heart creates a maximum 4 mV potential difference across 0.3 m of a person's chest, creating a 1 Hz electromagnetic wave.

a) What is the maximum electric field strength in this electromagnetic wave?

$$E_{max} = \frac{4\,mV}{0.3\,m} = 0.013\ V/m$$

b) What is the wavelength of this electromagnetic wave?

$1\ Hz = \nu \qquad c = \nu\lambda \qquad \frac{3\times10^8\ m/s}{1\ 1/s} = \boxed{3\times10^8\ m = \lambda}$

Problem 5 (12 pts, 4 points per part):
The evil emperor of the Zorgon League, Joeloid Seligmort, uses his great magical powers to fire two **huge** blasts of electromagnetic energy from his spaceship toward earth. His aim is to destroy earth and all the Kardashians before reality shows go interstellar. The two blasts are sent simultaneously toward earth. One blast is made of radio waves (of wavelength 100 m) and the other is made of visible light (of wavelength 500x10^{-9} m).

a) Which blast arrives at earth first?

Both blasts arrive at the same time.

b) If the sun is 1.5x10^{11} meters from the earth, how long after Seligmort does his trick does it take before people on earth are vaporized? (Assume the vaporization process is instantaneous once the first blast arrives at earth.)

Speed of light is $c = 3\times10^8$ m/s

$$\frac{1.5\times10^{11}\ m}{3\times10^8\ m/s} = 500\ s = 8\tfrac{1}{3}\ \text{Minutes}$$

c) Suppose at the peak of the visible light blast, the total power emitted by Seligmort is 1000 times that of the typical output of the sun, i.e., Seligmort emits 4000x10^{26} Watts in all directions uniformly. If the sun is 1.5x10^{11} m from the earth, what is the intensity of the light at the peak of Seligmort's pulse at earth?

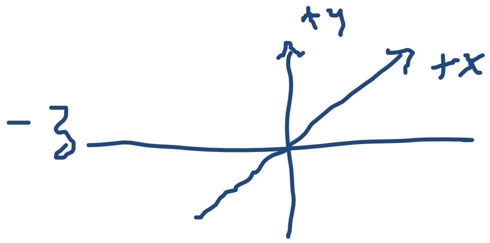
$$\text{Intensity} = \frac{4000\times10^{26}\ W}{4\pi\,(1.5\times10^{11})^2} = 1.4\times10^{6}\ W/m^2$$

1.4 million watts/m^2

Problem 6 (12 pts):

Write the equations for the electric and magnetic fields that make up a plane electromagnetic wave propagating in a vacuum in the negative z direction. Assume the electric field is oriented along the x-direction, the electric field has magnitude E_o=0.2 volts/meter and the frequency of the electric wave is $20x10^6$ Hz.

*



$$\vec{E} = E_0 \sin(kz + \omega t)\,\hat{i}$$

$$\vec{E} = 0.2 \sin(0.4z + 1.2\times10^8 t)\,\hat{i}$$

$$\vec{B} = \frac{E_0}{c} \sin(kz + \omega t)\,(-\hat{j})$$

$$\vec{B} = 6.7\times10^{-10} \sin(0.4z + 1.2\times10^8 t)\,(-\hat{j})$$

if $\vec{E}$ along $+\hat{x}$

$\vec{B}$ must be along $-\hat{y}$ ($\vec{E}\times\vec{B}$ is in direction of prop)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{15} = 0.4 \qquad \omega = 2\pi\nu = 2\pi\, 20\times10^6 = 1.2\times10^8$$

$$c = (20\times10^6\ Hz)\lambda \qquad \lambda = 15\,m$$

Problem 7 (10 pts, 5 points per part):

A clinical MRI (magnetic resonance imaging) magnet has a diameter of 1 m and a length of 1.5 m. It has a uniform magnetic field of 3T in this region and (we'll assume) has no stray field outside that region.

a) How much energy is stored in this magnet when it is on?

$$E_B = (\text{Volume})\,u_B = (\text{volume})\,\frac{B^2}{2\mu_0} = \pi(.5)^2 1.5\,\frac{3}{2(4\pi\times10^{-7})} = 1.4\times10^6\ J$$

b) If this energy were to be released over a period of one second through a massive string of light-bulbs, how many 100 Watt light-bulbs could be lit during that second?

$$\frac{1.4\times10^6\ J}{s} = 1.4 \text{ million watts}$$

could light $\frac{1.4\times10^6}{100} = 14000$ bulbs

P114 University of Rochester NAME Soln key - SM
S. Manly Spring 2015

Problem 8 (12 pts):
Consider the two sketches below. In each case there is a set of parallel conducting rails attached by a third conducting rail of length 2L on the left and L on the right (here L is a length and not an inductance). In each situation a fourth conducting rail slides on the two parallel rails in the direction shown while maintaining electrical contact. The speed v is the same in both the left and the right diagrams. There is a uniform magnetic field B (not shown) that is oriented into or out of the page.

a) (3 pts) For the left diagram, the induced emf is clockwise. What is the direction of B (into or out of the page)?

$\vec{B}$ must be out of page

b) (3 pts) What is the direction of the induced current on on the right sketch (clockwise or counter-clockwise)?

clockwise

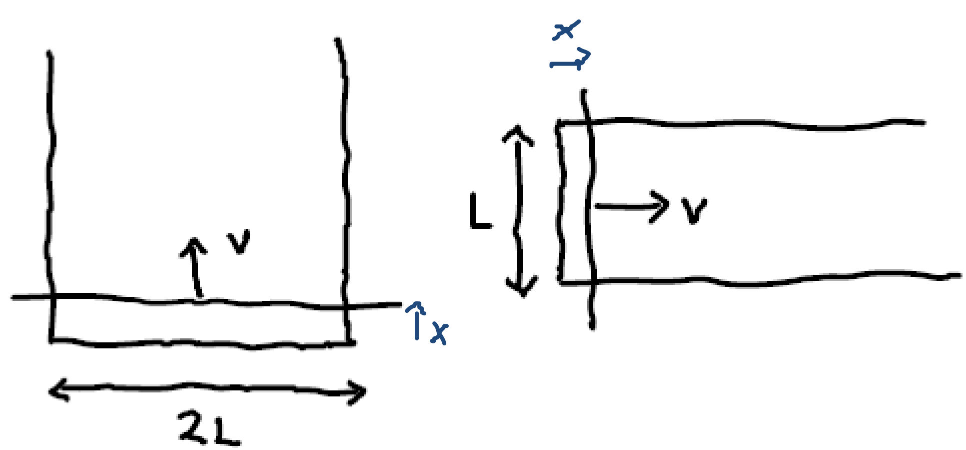
c) (6 pts) What is the relative size of the emf induced in the situation on the left as compared to the situation on the right (show your work)?

$$|\mathcal{E}| = \frac{d\phi}{dt}$$

$$EMF_{Left} = (2)\, EMF_{Right}$$

$$\phi = L \times B \qquad \frac{d\phi}{dt} = LBV$$

$$\phi = 2L \times B \qquad \frac{d\phi}{dt} = 2LBV$$

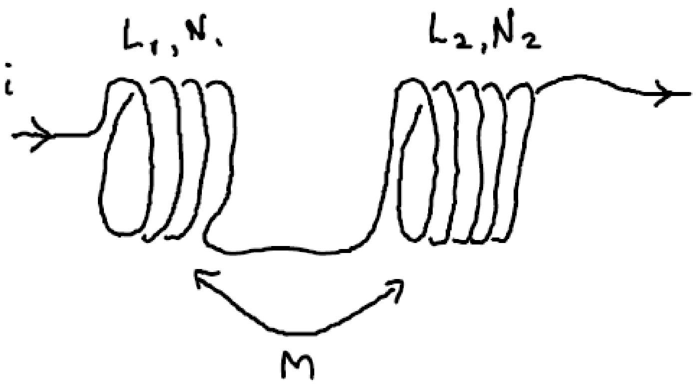


P114 S. Manly — University of Rochester, Spring 2015 — NAME Soln key - Slm

Problem 9 (13 pts):

Consider the situation where two coils, one with N_1 turns and self-inductance L_1 and one with N_2 turns and self-inductance L_2, are connected as shown in the sketch. The mutual inductance between the two coils is M. Determine (derive) the self-inductance of the connected structure L_{eq} (as shown) in terms of L1, L2, and M. Provided are six possibilities, including one that is the correct expression, for your reading enjoyment.

L cost *



$L_{eq} = \frac{2M^2}{L_1 + L_2}$ $\quad$ $L_{eq} = \frac{L_1^2 + L_2^2}{M}$

$L_{eq} = 2L_1 + 2L_2 + M$ $\quad$ $L_{eq} = L_1 + L_2 - 2M$

$L_{eq} = 2L_1 + 2L_2 - M$ $\quad$ $L_{eq} = L_1 + L_2 + 2M$

$\phi_1 = L_1 i$ $\quad$ $\phi_2 = L_2 i$ $\quad$ $\phi_{1 \text{ due to } i \text{ in } 2} = Mi = \phi_{2 \text{ due to } i \text{ in } 1}$

when connected

$$\text{Total flux} = \phi_{1 \text{ due to } 1} + \phi_{2 \text{ due to } 2} + \phi_{1 \text{ due to } 2} + \phi_{2 \text{ due to } 1}$$

$$\text{Tot flux} = \phi_1 + \phi_1 + 2Mi$$

$$\text{Tot flux} = L_1 i + L_2 i + 2Mi = (L_1 + L_2 + 2M)i$$

So $L_{eq} = L_1 + L_2 + 2M$

Exam 3 Formulas

$\vec{F} = q\vec{E}$

$\vec{F} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$

$\phi_E = \oint \vec{E}\cdot d\vec{A}$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$

$\vec{E} = \int_{vol} \frac{k\,dQ}{r^2}\hat{r}$

$V = \text{work}/\text{charge}$

$V_{\text{point charge}} = \frac{kQ}{r}$

$V = \int_{vol} \frac{k\,dQ}{r}$

$E_s = -dV/ds$

$V = IR$

$Q = CV$

$U = \frac{1}{2}CV^2$

$P = IV = I^2R = \frac{V^2}{R}$

$|e| = 1.6\times10^{-19}$ coulombs

$k = 8.99\times10^9\ \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$

$\mu_0 = 4\pi\times10^{-7}\ \frac{Tm}{A}$

$E = E_0/\kappa$

$C = \kappa C_0$

$Q(t) = C\mathcal{E}(1-e^{-t/RC})$

$Q(t) = Q_0 e^{-t/RC}$

$R_{eq} = \Sigma r_i$
$1/C_{eq} = \Sigma \frac{1}{C_i}$ } For series geometry

$\vec{F} = q\vec{v}\times\vec{B} = \ell\vec{i}\times\vec{B}$

$\vec{\mu} = nIA$

$\vec{\tau} = \vec{\mu}\times\vec{B}$

$B_{solenoid} = \mu_0 nI$

$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{i\,d\vec{\ell}\times\hat{r}}{r^2}$

$\oint_{curve} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$

$\mathcal{E} = -d\Phi_B/dt$

$\Phi_B = \oint \vec{B}\cdot d\vec{a}$

$\Phi_B = Li$

$\mathcal{E} = -L\,di/dt$

$u_B = B^2/2\mu_0$

$B = E/c$ For plane wave

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

$\langle S\rangle = \frac{EB}{2\mu_0}$

$P_{radiation} = S/c$ (Absorbed)

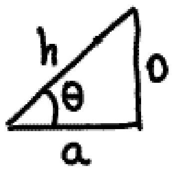
$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

$v = \lambda\nu$

$\vec{B} = \mu_0(1+\chi_M)\vec{B}_{ext}$

$\mu_0\chi_M = \mu$

P114 University of Rochester NAME ____
S. Manly Spring 2015



$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$

$\tan\theta = o/a$

Sphere: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

cylinder: $A = 2\pi r L + 2\pi r^2$

$V = \pi r^2 L$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

$$\int \frac{du}{u} = \ln|u|$$

$$\int e^u du = e^u$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = \sqrt{x^2+a^2}$$

$a_c = \frac{mv^2}{r}$

$S = r\theta$

$KE = \frac{1}{2}mv^2$

$PE_{spring} = \frac{1}{2}kx^2$

$V = V_0 + at$

$x = x_0 + V_0 t + \frac{1}{2}at^2$

$V^2 = V_0^2 + 2a(x - x_0)$

$x = x_0 + \frac{1}{2}(V + V_0)t$