Final Exam (May 6, 2015)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show your work unless otherwise indicated.

Problem 1 (6 pts, 2 points per part):

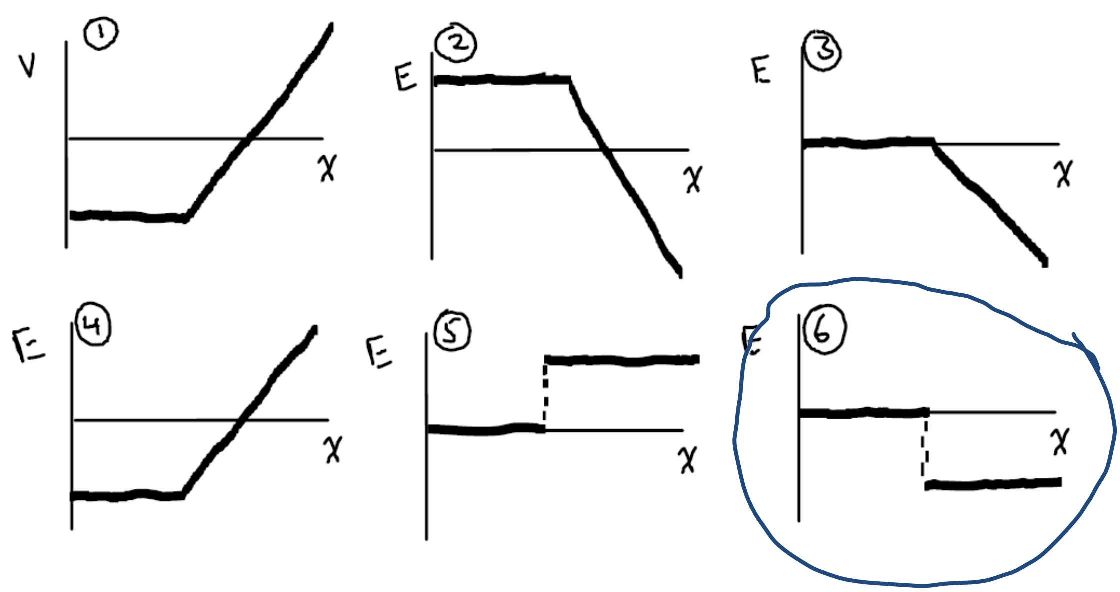
For the three situations below, mark the direction of the induced current in the loops denoted with "?".

Problem 2 (5 pts):

If the potential V of an array of charges versus the distance from the charges (in x direction) is as shown in graph 1, which (of graphs 2-6) shows the electric field as a function of distance x?

a) 2
b) 3
c) 4
d) 5
e) 6

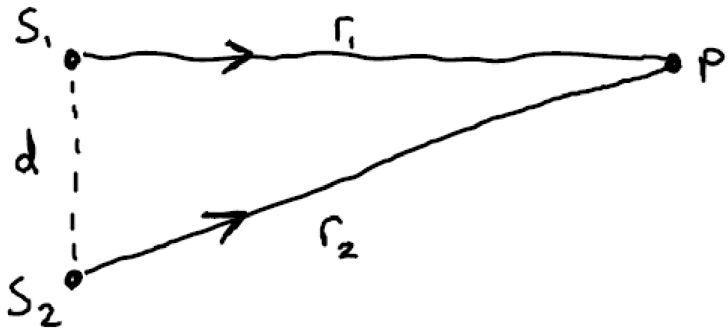
$E = -\frac{dV}{dx}$



Problem 3 (5 pts):

Two sources of monochromatic (single frequency) light are located at positions S_1 and S_2 as shown. If the two sources emit light in phase, the intensity at point P is a maximum when

a) $d=\lambda$
b) $r_2+r_1=\lambda$
c) $r_2-r_1=\lambda$
d) $r_2+r_1=\lambda/2$
e) $r_2-r_1=\lambda/2$



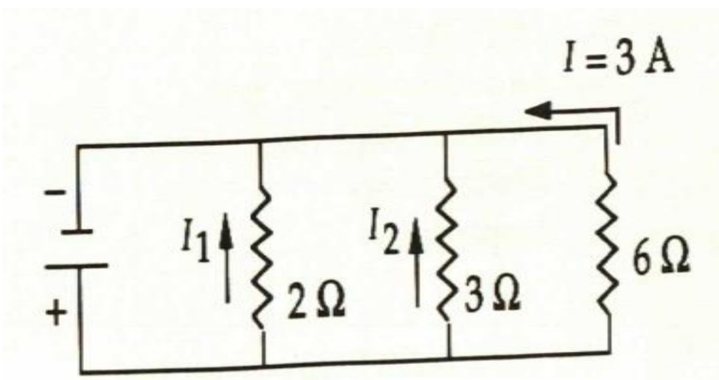
More credit

Problem 4 (6 pts, show your work):

You connect resistors of 2Ω, 3Ω, and 6Ω in parallel across a battery. The current through the 6Ω resistor is 3 A. The power dissipated in the 3Ω resistor is

a) 0.24 kW
b) 0.11 kW
c) 0.36 kW
d) 0.54 kW
e) 0.86 kW

$\mathcal{E}$



$\mathcal{E} = (3A)(6\Omega) = 18\text{ V}$

$18 = \mathcal{E} = (3\Omega) I_2 \qquad I_2 = 6\text{ A}$

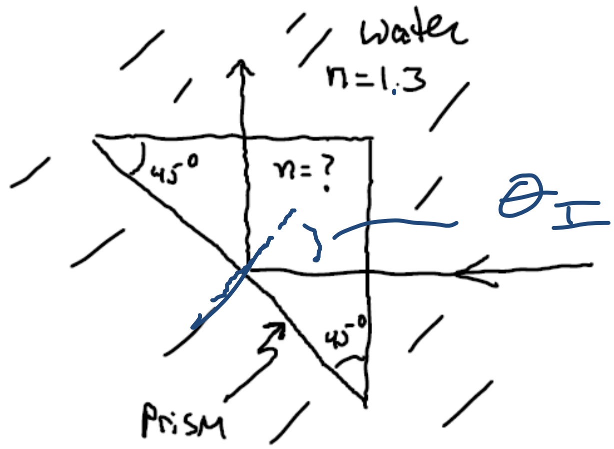
$P_{3\Omega} = i^2 R = 6^2 3 = 108$ WATTS

1.3

Problem 5 (7 pts, show your work):

For the prism immersed in water (n=1.33), the minimum index of refraction that will produce total reflection of the indicated ray is approximately

a) 0.94
b) 1.28
c) 1.50
d) 1.65
e) 1.88



Min index will be when

$45^\circ = \theta_{critical}$

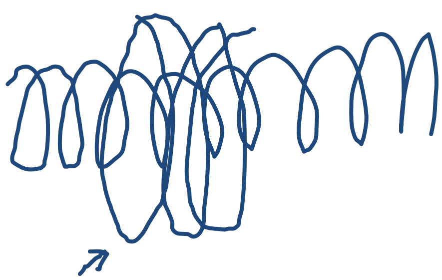
$n \sin\theta_c = 1.33 \sin 90^\circ$

$\sin 45^\circ = \frac{1.33}{n}$

$n = \frac{1.33}{\sin 45^\circ} = 1.88$

Problem 6 (8 pts, show your work):

A 120-turn conducting coil with radius 2 cm and resistance 5 Ω is placed outside (and co-axial with) an infinite solenoid with 220 turns per cm, radius 1.3 cm, and current 1.5 A (at t=0). Beginning at t=0, the current in the solenoid changes from 1.5 A to zero in 25 ms at a constant rate. What is the current in the 120-turn coil during the period the solenoid current is changing?



120 Turns
r = 2 cm
5 Ω

∞ solenoid
n = 220
r = 1.3 cm
i = 1.5 A (t=0)

$di/dt = \frac{1.5-0}{25\times10^{-3}} = 60 \text{ A/s}$

$B_{solenoid} = \mu_0 i n$

$dB/dt = \Phi_{coil\ 1\ turn} = \mu_0 n \frac{di}{dt}$

$\frac{dB}{dt}\Big|_{120\ Turns} = (120)\mu_0 n \frac{di}{dt}$

$\mathcal{E}\Big|_{120} = -\frac{d\Phi_B}{dt} = \left[\pi(.013)^2\right]\frac{dB}{dt}\Big|_{120}$

$i\Big|_{120} = \frac{\mathcal{E}|_{120}}{R} = \frac{\pi(.013)^2(120)\mu_0(220)(60)}{5}$

$4\pi\times10^{-7}$

$i\Big|_{120} = 2\times10^{-4}\ A$

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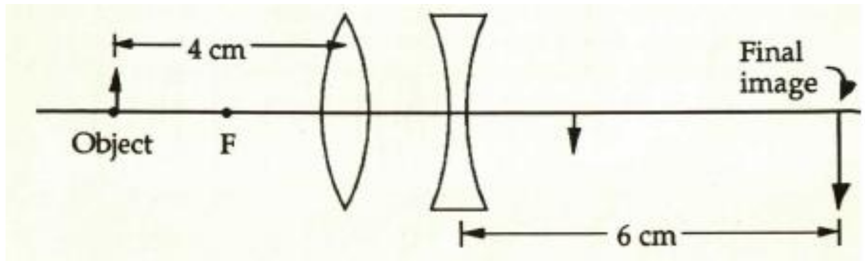
Problem 7 (8 pts, show your work):

An object placed 4 cm to the left of a converging lens of focal length 2 cm produces an image 4 cm to the right of the lens. A diverging lens placed at the focal point of the positive lens as shown produces a final image 6 cm to the right of the diverging lens. The diverging lens has a focal length of

a) -1.5 cm
b) -3.0 cm
c) -6.0 cm
d) 10 cm
e) -2.0 cm



$$\frac{1}{-2} + \frac{1}{6} = -\frac{1}{f}$$

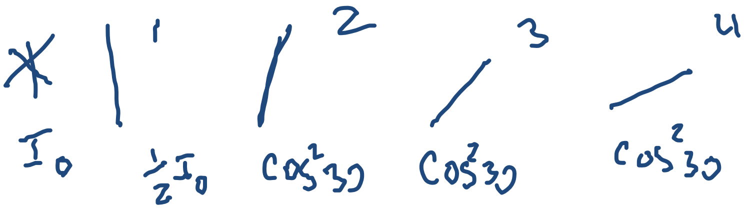
$$\frac{-6+2}{12} = -\frac{1}{f}$$

$$f = -3$$

So f of lens is -3.0 cm

Problem 8 (8 pts, show your work):

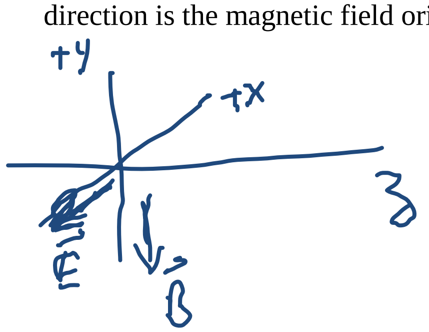
(5 pts) Four perfectly polarizing plates are stacked so that the axis of each plate is turned 30 degrees with respect to that of the plate above it. Calculate the approximate percentage of the incident unpolarized light that is transmitted by the stack.



$$I = \frac{1}{2} I_0 \left[\cos^2 30\right]^3 = 0.21 I_0$$

$\boxed{21\%}$

(3 pts) Suppose the light transmitted by the stack is a plane wave that propagates along the +z-axis. If, at a certain moment in time, the electric field is oriented along the –x-axis, in what direction is the magnetic field oriented?



$\vec{E} \times \vec{B}$ in $+\hat{z}$

if

$\vec{E}$ along $(-\hat{x})$

then

$\vec{B}$ along $(-\hat{y})$

Problem 9 (7 pts, show your work):

A 20 µF capacitor is charged to 200 V and it connected across a 1000 Ω resistor. What is the initial current just after the capacitor is connected to the resistor?

$$Q = Q_0 e^{-t/RC} \qquad Q_0 = CV$$

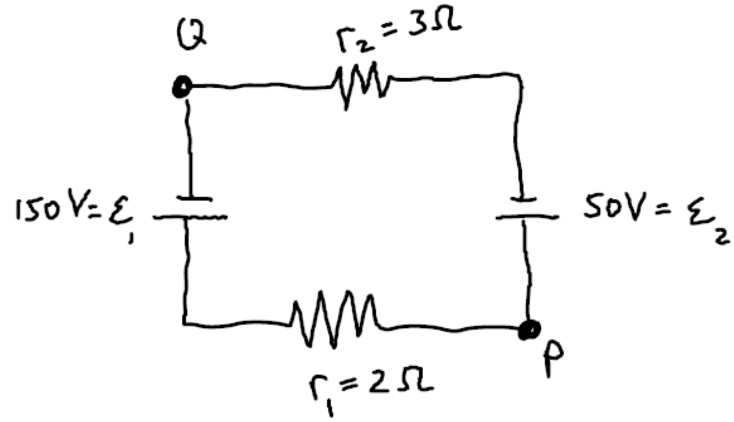
$$\frac{dQ}{dt} = -\frac{Q_0}{RC} e^{-t/RC} \qquad Q_0 = 20\times10^{-6}\,\text{F}\ 200\,\text{V}$$

$$Q_0 = 4\times10^{-3}\,\text{C}$$

$$|i(t)| = \frac{Q_0}{RC} e^{-t/RC} \qquad i(0) = \frac{Q_0}{RC} = \frac{4\times10^{-3}}{(1000)\ 20\times10^{-6}} = 0.2\,\text{A}$$

Problem 10 (8 pts, show your work):

Consider the circuit shown. If the absolute potential at point P is 100V, what is the absolute potential at point Q?



$$0 = 150 - 50 - i r_1 - i r_2$$

$$100 = i(r_1 + r_2)$$

$$i = 20\,\text{A}$$

if $V_P = 100$

i

$$V_Q = V_P - 50 - (20)(3) =$$

$$V_Q = 100 - 50 - 60 = -10\,\text{V}$$

$\boxed{\text{Absolute potential at point Q is } -10\text{ V}}$

Problem 11 (10 pts, show your work):

Flash forward a few years. Because of your amazing expertise in nuclear physics, you are hired as an analyst for Jethro's Southern Ribs and Nuclear Physics Consulting restaurant/firm that is situated near the State Department office building in Wasshington, DC. This niche place is known to do a booming business during weekday lunches. While you are working one day, President Obama comes in for lunch. While he's eating he asks you about something that one of his nuclear negotiators told him. He says, "I hear that once a nuclear bomb is made, it ages due to the radioactive decay of the uranium. How long is the bomb functional after it has been made?"

Let's help out the President. As it turns out, what he says is true. Let us suppose that a nuclear bomb can function if the purity of the main isotope of uranium (^{235}U) is 90% or better (probably not the real number). As this isotope decays in the bomb, the purity of the isotope inside the bomb drops and the bomb may cease to function as designed. Suppose on the day of manufacture the purity of the uranium in a bomb is 100%. ~~Let the nuclear decay constant for~~ the half life for ^{235}U ~~be~~ is $7\text{x}10^8$ years. *

a) (6 pts) How long after manufacture (in years) will a bomb remain viable(functional)? *(I know we did not have time to study how nuclear bombs work in this class. You do not need to know anything about that to answer this question.)*

$$N(t) = N_0 e^{-t\lambda} \qquad \lambda = \frac{.693}{t_{1/2}} = \frac{.693}{7\times10^8} = 9.9\times10^{-10}$$

$$\frac{N}{N_0} = 0.9 = e^{-t\lambda} \qquad t = 1\times10^8 \text{ yrs}$$

$$\ln .9 = -t\,(9.9\times10^{-10})$$

b) (4 pts) If ^{235}U has an atomic number of 92 and decays by emitting an alpha particle, how many protons and ~~and~~ neutrons will be present in the decay product nucleus? *

how many

$$^{235}_{92}U \rightarrow {}^{231}_{90}X + \alpha$$

decay product has
90 protons
and 143 neutrons

1)	/6
2)	/5
3)	/5
4)	/6
5)	/7
6)	/8
7)	/8
8)	/8
9)	/7
10)	/8
11)	/10
12)	/11
13)	/11
tot	/100

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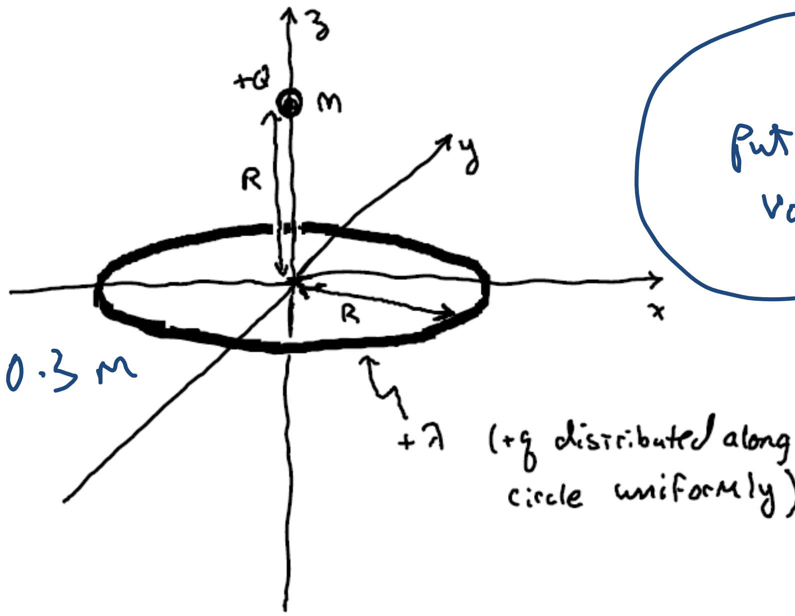
Problem 12 (11 pts, show your work):

Consider a mass M that is free to slide frictionlessly up and down a thin vertical rod at the surface of the earth. Assume the upward direction is along the +z-axis. The rod along which the mass can slide (the z-axis) is the symmetry axis for a circle of charge that lies in the x-y plane, centered at the origin. The circle of charge has a uniform charge/length and total charge +q. It also has a radius R. The mass carries an electric charge +Q and is in stable equilibrium at a distance R above the origin, as shown in the sketch. Determine the mass M.

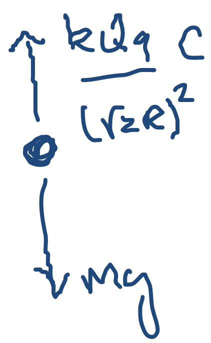
1×10^{-6} C

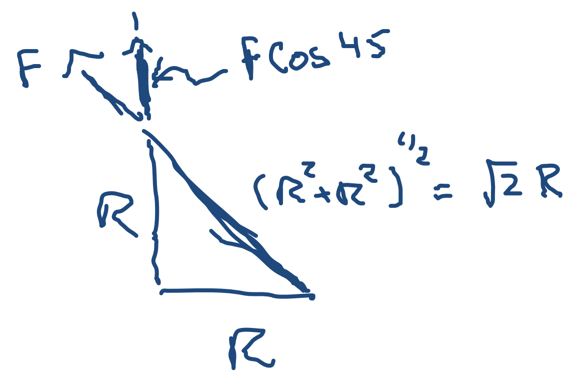
0.3 m

2×10^{-6} C



Put in values





$$mg = \frac{kQq}{(\sqrt{2}R)^2}\cos 45$$

$$m = \frac{1}{g}\frac{kQq}{(\sqrt{2}R)^2}\cos 45$$

$$M = \frac{9\times10^{9}}{9.8}\,\frac{2\times10^{-6}\;1\times10^{-6}}{2(.3)^2}\cos 45 = 7\times10^{-3}\ \text{kg}$$

Problem 13 (11 pts, show your work):

Consider a spherical charge distribution of radius with a volume charge density given by

$r>R$, $\rho(r)= 0$
$r<R$, $\rho(r)=(\rho_1 - \rho_2 r)$,

where ρ_1 and ρ_2 are positive constants with appropriate units (so that both terms in $\rho(r)$ h ave units of coulombs/m^3).

Assume the total charge for r<R is +Q. Determine the electric field in all space. For simplicity you should leave your answer in terms of ρ_1 and ρ_2. In other words, you are not required to solve for of ρ_1 and ρ_2 in terms of Q.

for $r>R$ $\vec{E} = \frac{kQ}{r^2}\hat{r}$

For $r<R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = \frac{\int \rho\, dv}{\epsilon_0}$$

$$\vec{E}\, 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^r \rho\, 4\pi r^2 dr$$

$$E\, 4\pi r^2 = \frac{4\pi}{\epsilon_0}\left[\int_0^r \rho_1 r^2 dr - \int_0^r \rho_2 r^3 dr\right] = \frac{4\pi}{\epsilon_0}\left[\rho_1 \frac{r^3}{3} - \rho_2 \frac{r^4}{4}\right]$$

$$|\vec{E}| = \frac{1}{r^2\epsilon_0}\left[\rho_1 \frac{r^3}{3} - \rho_2 \frac{r^4}{4}\right] = \frac{1}{\epsilon_0}\left[\rho_1 \frac{r}{3} - \rho_2 \frac{r^2}{4}\right]$$

in radial direction