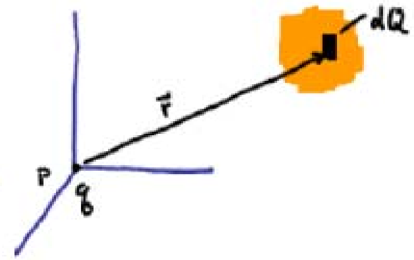
Physics 114 – January 29, 2015

- No lecture meetings in Hoyt next week (2/3, 2/5)
- 2 Lectures will be posted on class website (Slides + Audio)
- 2/10 Lecture will begin where 2/5 lecture left off
- Prob sets, workshops, labs all continue as normal
- I will be in email contact except for time in transit
- Exam I is 2/12 ... So, I will be in touch about details concerning that Next week

Last Time

$\vec{E}_{\text{at } P}$ due to system of discrete charges $= \displaystyle\sum_{i=1}^{N} \frac{kQ_i}{r_i^2}\,\hat{r}_i$

$\vec{E}_{\text{at } P}$ due to continuous charge distribution

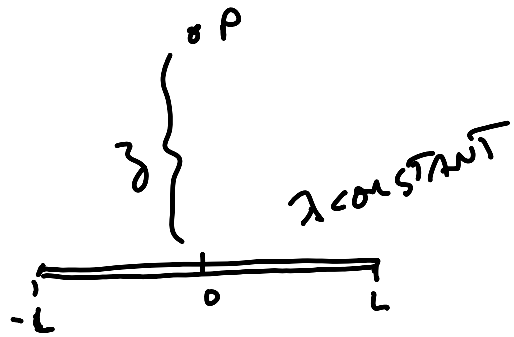


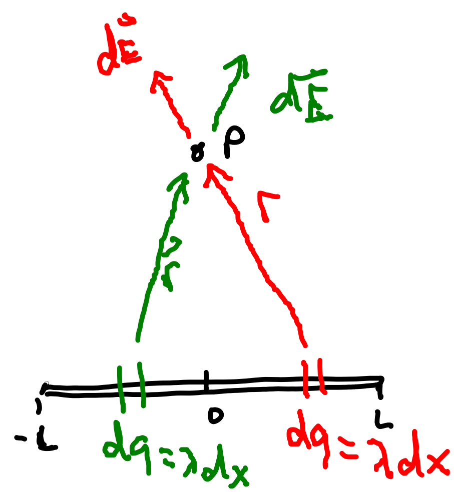
$$= \int_{\text{vol}} \frac{k\,dQ}{r^2}\,\hat{r} = \int_{\text{volume}} \frac{k\,\rho(\vec{r})\,dv\,\hat{r}}{r^2}$$

All of these things might change as you integrate over the volume

- might be hard.
- might not be so bad.

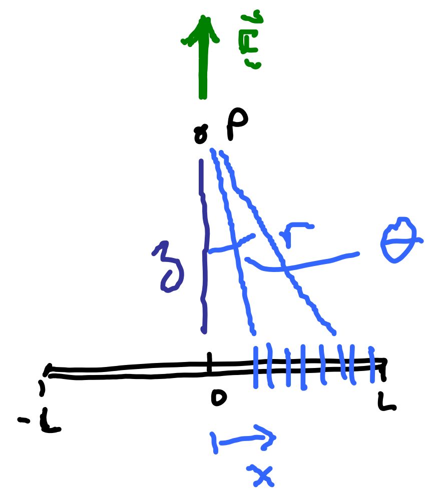
a doable example





See MIT Teal demos

line + loop integration



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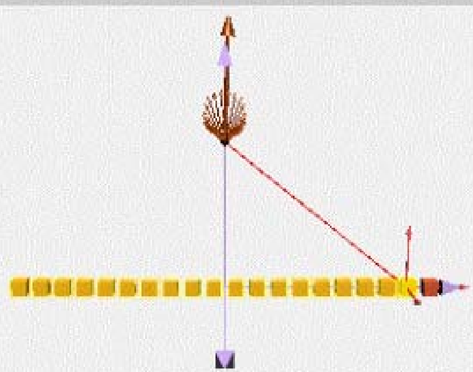
MIT Physics 8.02
ELECTRICITY & MAGNETISM

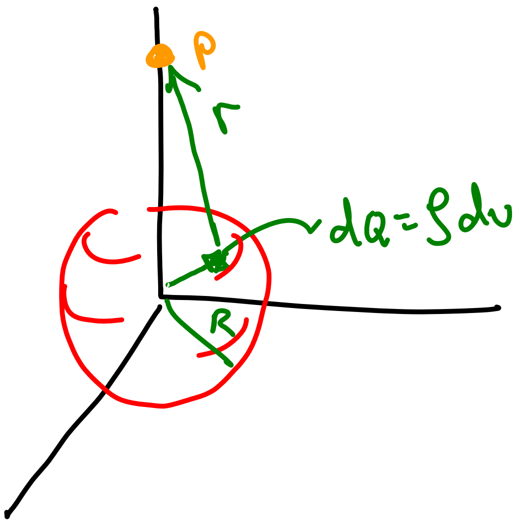
SECTION : Electrostatics



SUBJECT: *Integrating Along a Line of Charge*
Fullscreen Version

DESCRIPTION:

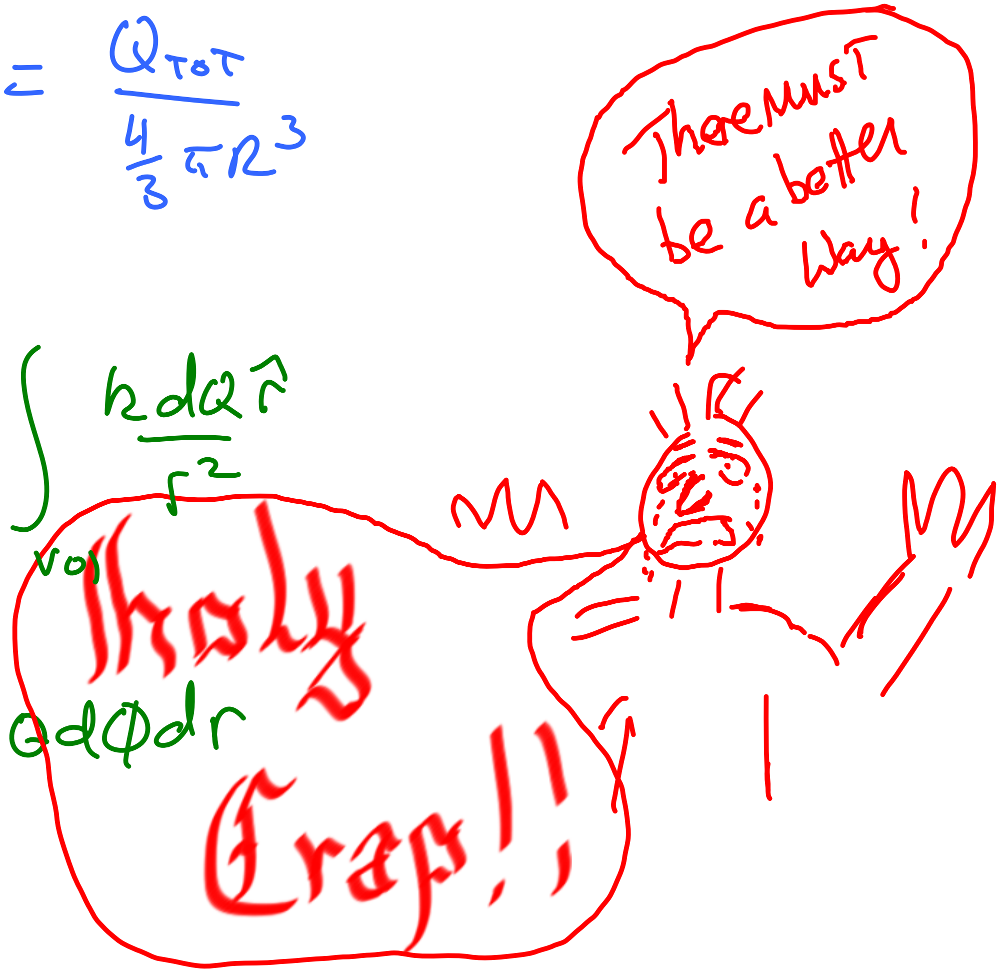
This simulation illustrates the electric field generated by a line of charge, and shows how, by the principle of superposition, a continuous charge distribution can be thought of as the sum of many discrete charge elements. Each element generates its own field, described by Coulomb's Law (and represented here by the small vectors attached to the observation point), which, when added to the contribution from all the other elements, results in the total field of the line (given by the large resultant vector). In this animation, each



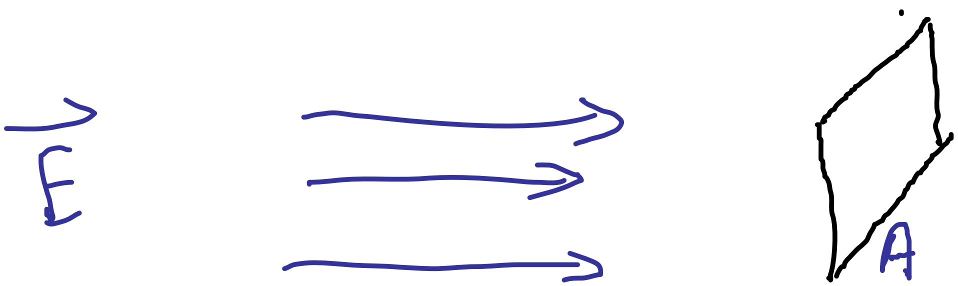
$$\rho = \frac{Q_{TOT}}{\frac{4}{3}\pi R^3}$$

$$\vec{E} = \int_{vol} \frac{k\, dQ\, \hat{r}}{r^2}$$

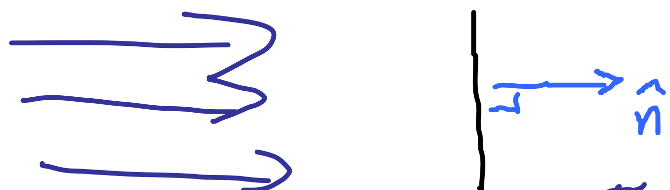
$$E = \int_0^R \int_0^\pi \int_0^{2\pi} \frac{k \rho r^2 \sin\theta \, d\theta \, d\phi \, dr}{r^2}$$



Electric Flux

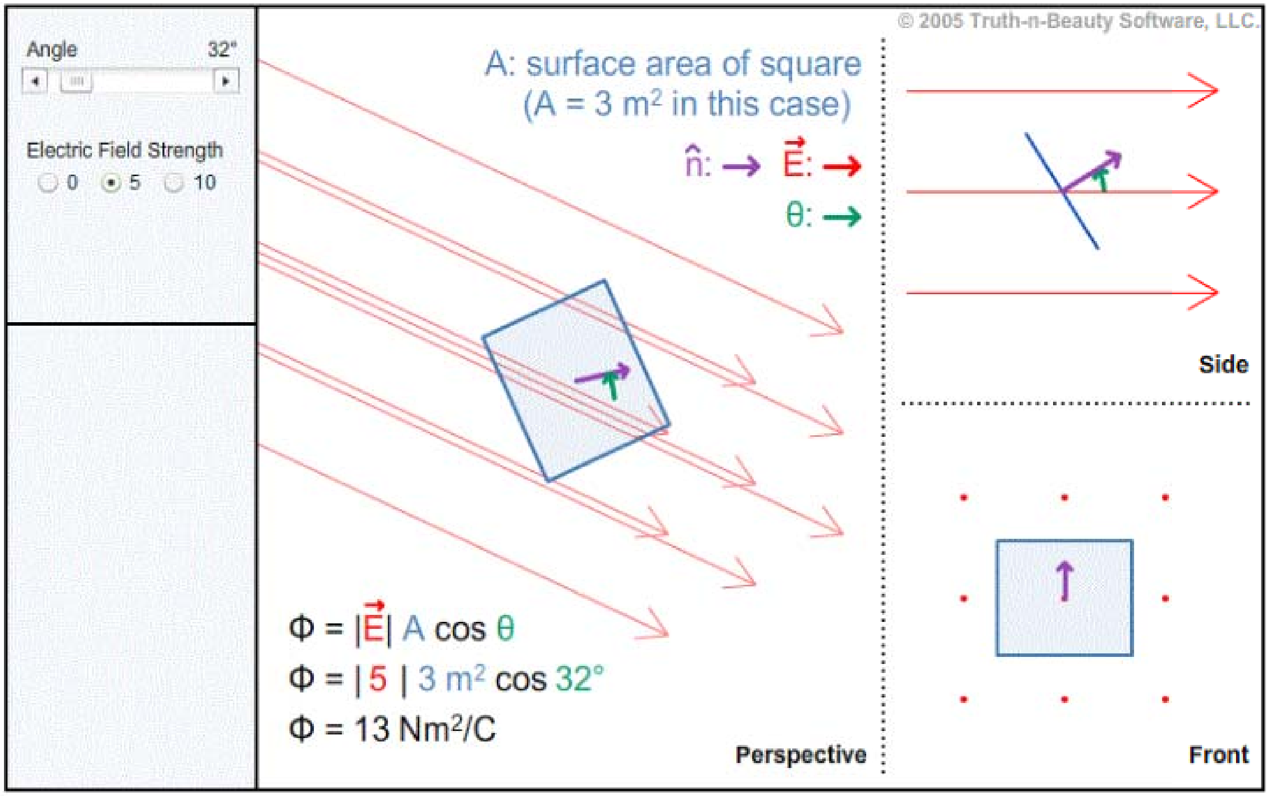


$$\phi = |\vec{E}| A$$



$$\phi = \vec{E} \cdot \hat{n} A$$

$$\phi = \underbrace{\vec{E} \cdot \hat{n} A}_{|E| A \cos\theta}$$



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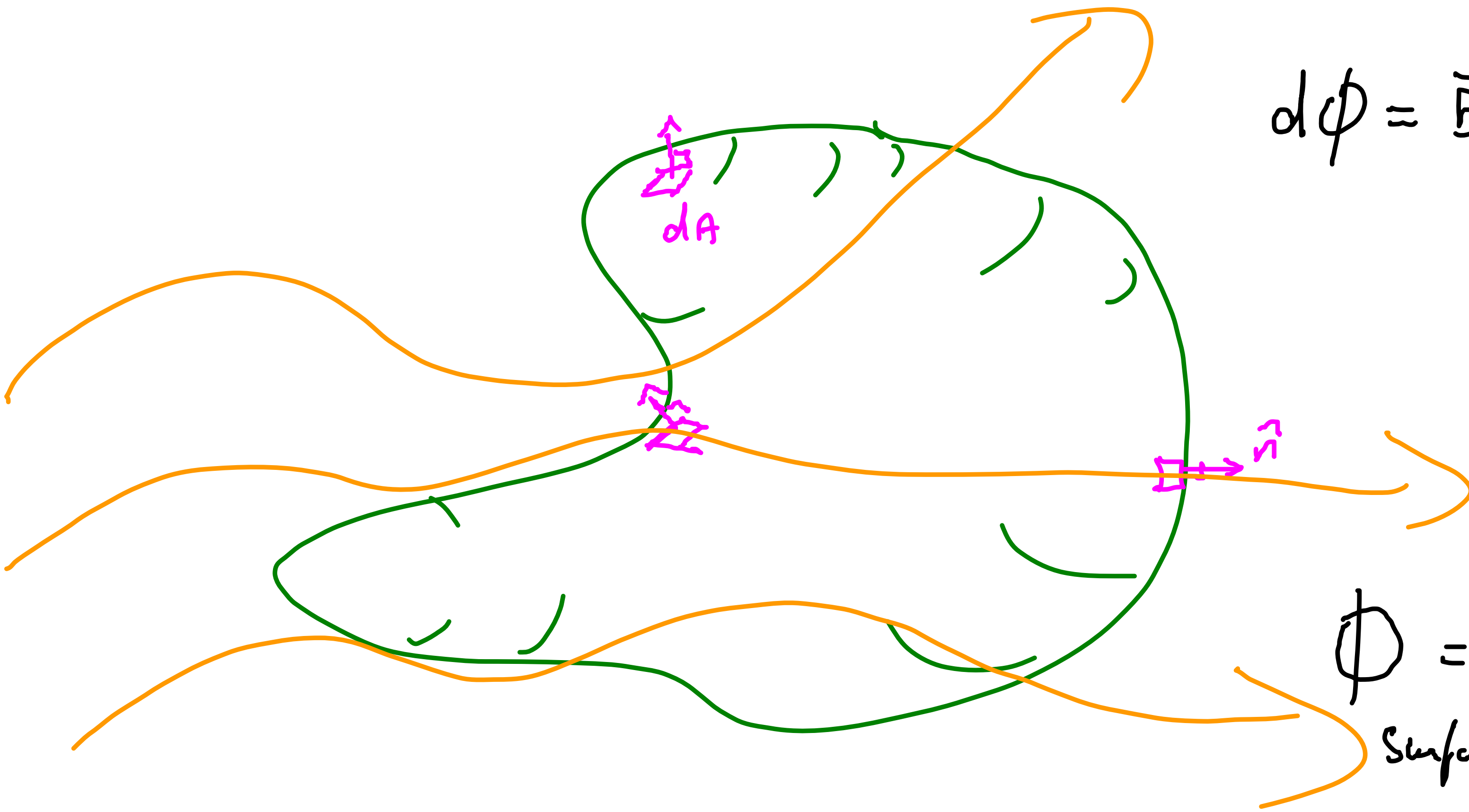
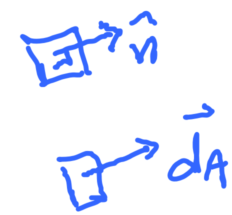
Angle 32°
Electric Field Strength
0 5 10
A: surface area of square
(A = 3 m² in this case)
n̂: E⃗:
θ:
Side
Front
Perspective
Φ = |E⃗| A cos θ
Φ = | 5 | 3 m² cos 32°
Φ = 13 Nm²/C

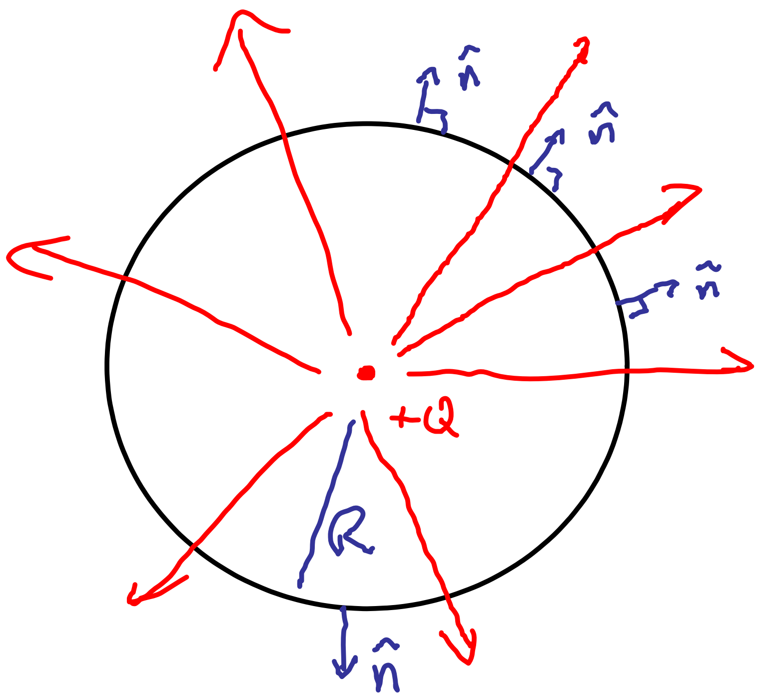


$$d\phi = \vec{E}\cdot\underbrace{\hat{n}\,dA}_{\vec{dA}}$$



$$\phi = \oint_{s} \vec{E}\cdot\hat{n}\,dA$$

Surface



$\vec{E}$ radial symmetry

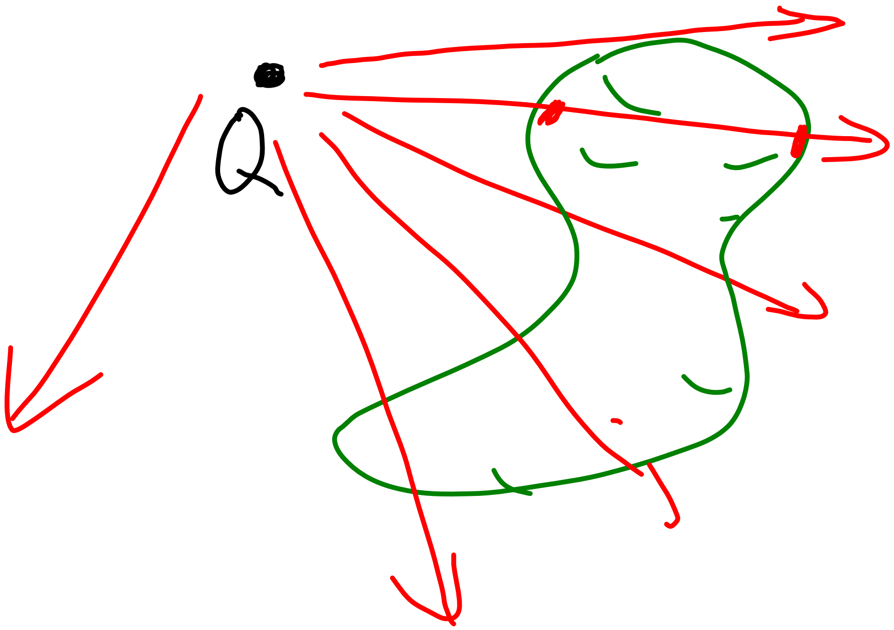
$$\phi = \oint_{\text{surface}} \vec{E}\cdot\hat{n}\, dA$$

$$= \oint |\vec{E}|\, dA$$

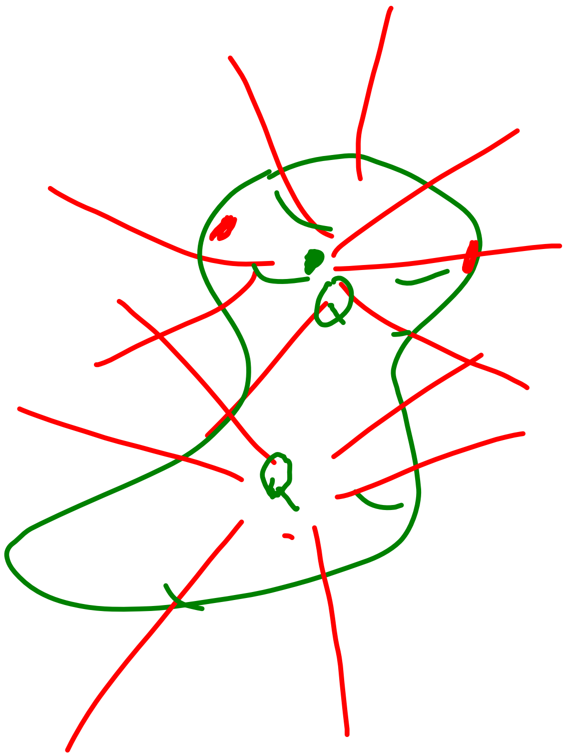
$$= \oint \frac{kQ}{R^2}\, dA$$

$$\phi = \frac{kQ}{R^2} \underbrace{\oint dA}_{4\pi R^2}$$

$$\phi = kQ\,4\pi = \frac{Q}{\epsilon_0}$$



$\phi = 0$



$$\phi = \frac{Q_{Net}}{\epsilon_0}$$

$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}}$$

Gauss' Law

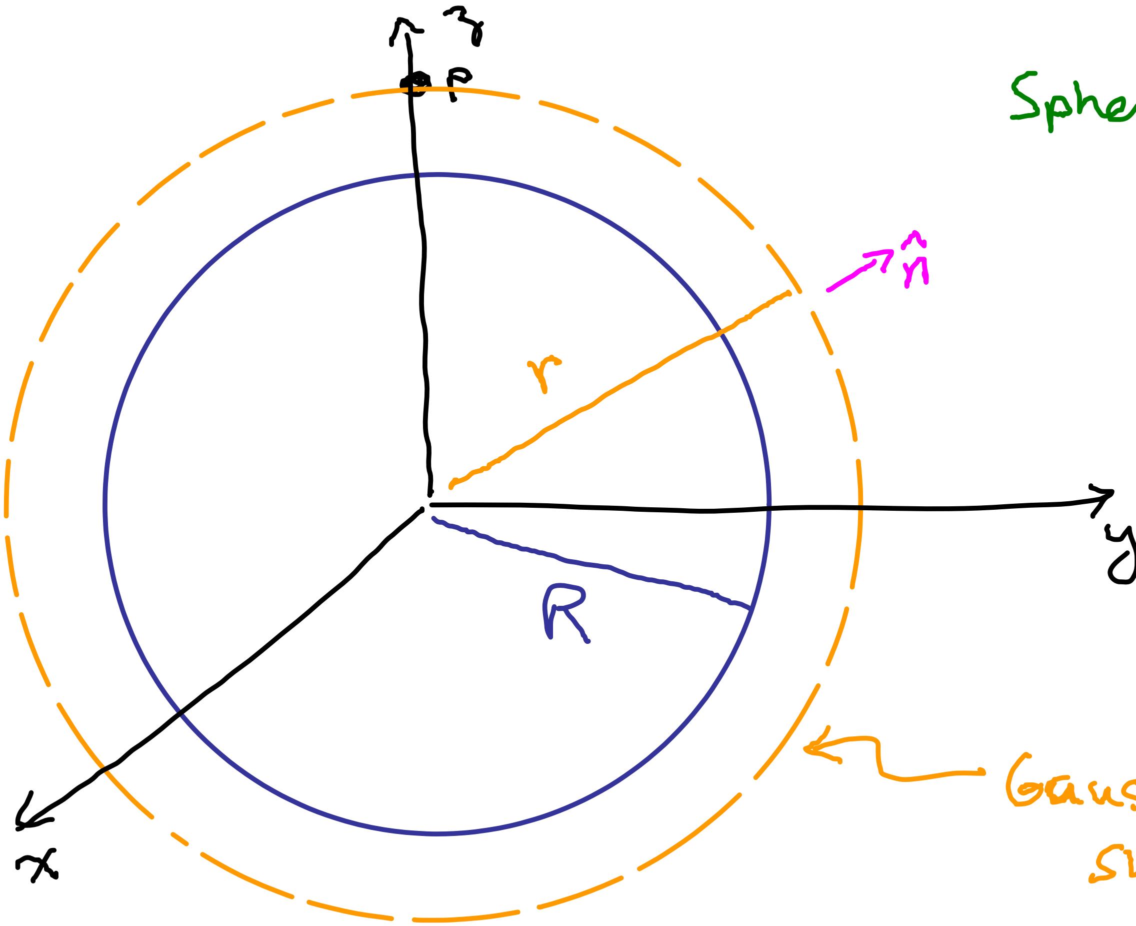
Important

Johann Carl Friedrich Gauss
(1777-1855, Germany)

very Smart
Plus

A mad cool sense of fashion!

When a philosopher says something that is true then it is trivial. When he says something that is not trivial then it is false



Spherical dist of charge

$\rho =$ CONSTANT

what is $\vec{E}_P$

What is $\vec{E}$ in all space

Outside sphere $r > R$

$$\Phi_{\text{surf at } r>R} = \oint \vec{E}\cdot\vec{n}\,dA = \oint E\,dA = E\underbrace{\oint dA}_{4\pi r^2}$$

$$\frac{Q_{\text{net}}}{\epsilon_0} = E\,4\pi r^2$$

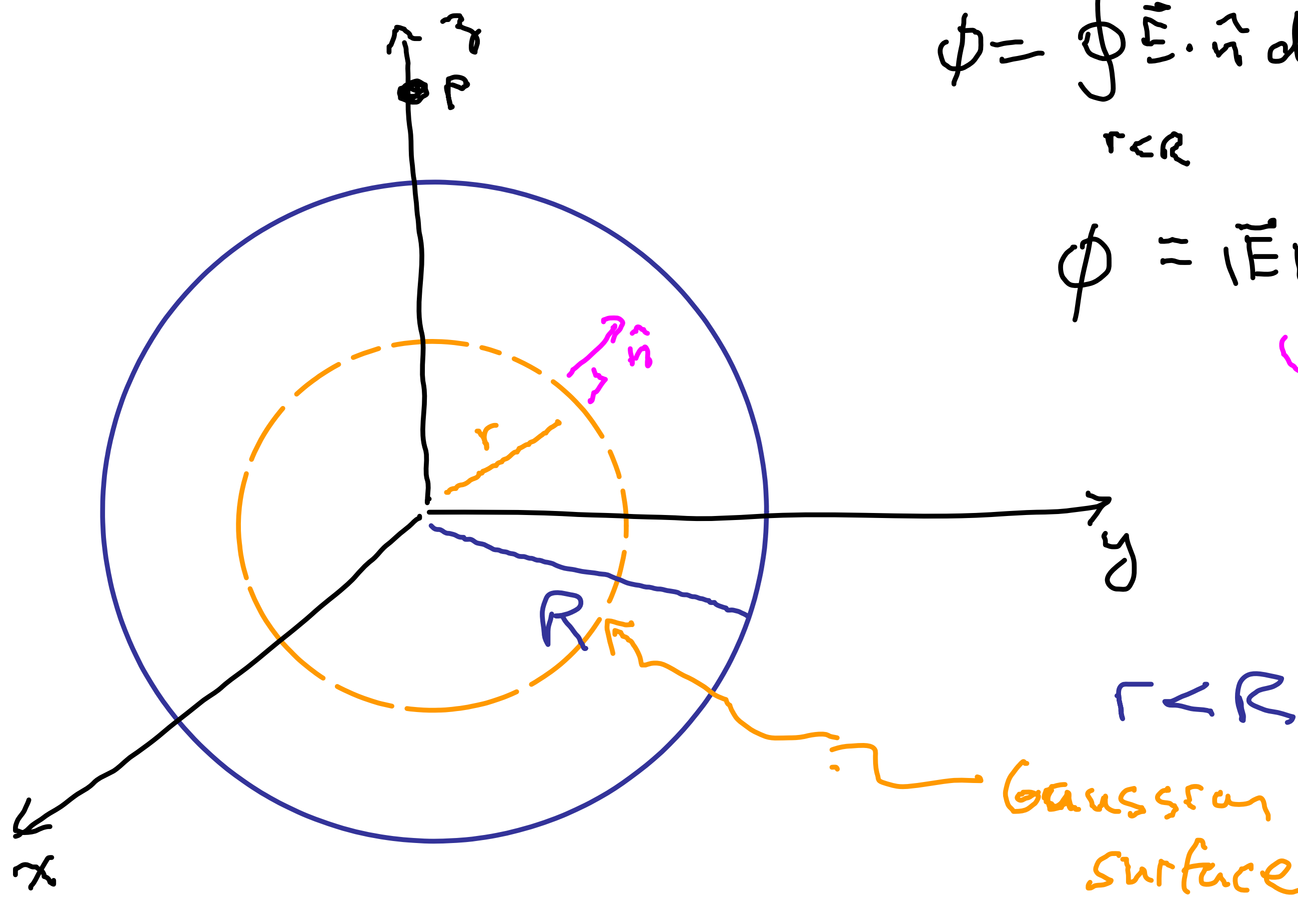
$$\oint \rho\,dV = \frac{4}{3}\pi R^3 \rho$$

$$\rho = \frac{Q_{\text{net}}}{\frac{4}{3}\pi R^3}$$

$$|\vec{E}|_{r>R} = \frac{1}{\epsilon_0}\,\frac{\frac{4}{3}\pi R^3 \rho}{4\pi r^2}$$

$$|\vec{E}|_{r>R} = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$= \frac{kQ}{r^2}$$



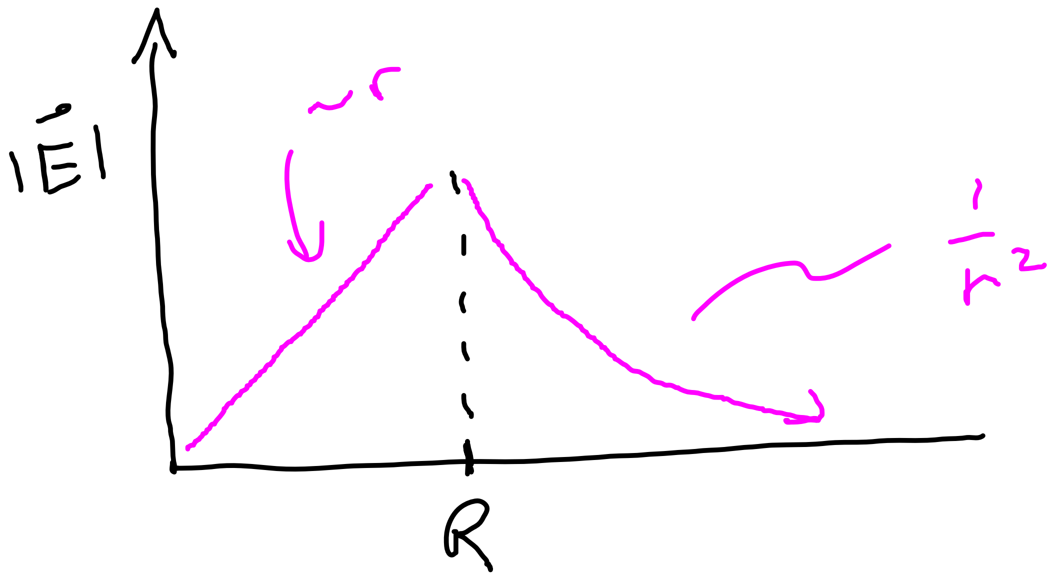
$$\phi = \oint_{r<R} \vec{E}\cdot\hat{n}\,dA$$

$$\phi = |\vec{E}|\,4\pi r^2 = \frac{Q_{encl}}{\epsilon_0}$$

$$\frac{Q_{encl}}{\epsilon_0} = \frac{\rho \frac{4}{3}\pi r^3}{\epsilon_0} = |\vec{E}|\, 4\pi r^2$$

$$|\vec{E}|_{r<R} = \frac{\rho}{\epsilon_0}\frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\rho\, r}{\epsilon_0\, 3}$$

$$= \frac{Q\, r}{\frac{4}{3}\pi R^3\, \epsilon_0\, 3} = \frac{k Q r}{R^3}$$



|E|
~r
1/r²
R